Experiment 5: Frequency Modulation

1   Introduction

Angle modulation includes both *phase modulation* (PM) and *frequency modulation* (FM). The main difference between angle modulation and amplitude modulation is that in angle modulation, the information is contained in the angle of the carrier whereas in amplitude modulation, the information is in the amplitude of the carrier. We will be dealing only with FM in this lab. The theories and concepts are similar for PM.

Why should we use FM instead of AM? The main advantages of using FM over AM are:

1. A Much better signal-to-noise ratio. There is as much as a 25-dB increase in this ratio over AM. You can notice this while listening to the car radio during a thunder storm.

2. When two FM transmitters are nearby operating on the same frequency, there is a much smaller geographical interference area as compared to AM transmitters operating on one frequency.

3. Less radiated power required for the same signal-to-noise ratio for FM over AM.

There are also some serious disadvantages of FM. An FM wave typically requires 15 to 20 times the bandwith of an AM wave. Also, FM systems are much more complicated to analyze and build than AM systems.

2   FM theory

At URI we have a radio station, 90.3 FM (WRIU). WRIU is a frequency modulated radio station with a carrier of 90.3MHz. Each FM station occupies a bandwidth of about 150kHz to 175kHz, but is alloted a bandwidth of about 200kHz; this is why each station is separated by 0.2 (MHz) on your dial. Let's take a look at the modulation process to get a feel for what happens. The formula for an FM signal, $s(t)$, is given by

$$s(t) = A_c \cos \left(2\pi f_c t + 2\pi k_f \int\limits_0^t m(\tau) d\tau \right) \tag{1}$$

$$
\text{where}
\begin{aligned}
m(t) &= && \text{modulating signal} \\
f_c &= && \text{carrier frequency} \\
A_c &= && \text{carrier amplitude}
\end{aligned}
$$

Equation (1) is a general equation for an FM signal. If we let the modulating signal be a pure sinusoid, $m(t) = A_m \, cos2\pi f_m t$, then equation (1) becomes

$$
s(t) = A_c \, \cos\left(2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right) = A_c \, cos\left(2\pi f_c t + \beta \sin 2\pi f_m t\right) \tag{2}
$$

$$
\text{where}
\begin{aligned}
k_f A_m = \Delta f &= && \text{frequency deviation} \\
\beta &= && \text{Modulation index}
\end{aligned}
$$

There are two types of FM depending on the value of β.

NBFM $(\beta \ll 1)$

If β is much less then 1, we have Narrow Band Frequency Modulation (NBFM). Equation (2) can be approximated as

$$
s(t) = A_c \, \cos\left(2\pi f_c t\right) - \beta A_c \, \sin(2\pi f_c t) \, \sin(2\pi f_m t) \tag{3}
$$

See the textbook and the class notes for details.

WBFM (all β)

The Wide Band Frequency Modulation formula is valid for all β. In this lab, we will be dealing with WBFM. The approximations used for NBFM are not accurate here. Using Fourier series coefficients, Equation (2) for WBFM can be written as (see text)

$$
s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) cos\left[2\pi(f_c + nf_m)t\right] \tag{4}
$$

where $J_n(\beta)$ is the nth order Bessel function of the first kind and argument β.

The definition of the Bessel function is shown below. It is an integral that needs to be evaluated using a calculator or software for different values of n and β.

$$
J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta sinx - nx)} dx \tag{5}
$$

Again, for details, see the text.

3 FM vs the Spectrum Analyzer

FM is not an easy concept to understand. Therefore, the first step on the path to complete FM comprehension will a be series of experiments in which you will use a function generator to create an FM signal with a low carrier frequency and relatively narrow bandwidth. You will then use a spectrum analyzer to help you observe how changing the parameters of the FM signal will affect its spectrum. And lastly, you will compare the measurements obtained with the spectrum analyzer to the values predicted by the wideband FM equation.

The following instructions assume that you are using the HP/Agilent 33120A Function Generator and the SR770 Spectrum Analyzer. The 33120A will allow you to modulate a pure tone (sinusoid) and several more complicated signals like a square, triangular, or sawtooth wave. We will use the generator's default modulating signal, a sinusoid.

- With the function generator, create an FM signal with $f_c = \mathbf{10kHz}$ and $f_m = \mathbf{1kHz}$. The latter value, f_m, can be adjusted by pressing **Shift** followed by **Freq**. Set the deviation Δf to **100Hz** by pressing **Shift** and then **Level**. Set up the spectrum analyzer to view the resulting spectrum (use a linear vertical scale). Record the magnitude and frequency of each impulse in the frequency domain. (In addition to the clearly visible impulses, there should be very tiny impulses at 8 and 12kHz. Don't forget to record these.) What is the value of β?

- Increase the frequency deviation Δf in increments of of 100Hz until $\Delta f = \mathbf{1kHz}$. Describe the change in the spectrum of the FM signal. Record the magnitude and frequency of each impulse for $f_m = \mathbf{1kHz}$ and $\Delta f = \mathbf{1kHz}$. What is the value of β now?

- Set Δf back to **100Hz**. Decrease f_m in increments of of 100Hz until $f_m = \mathbf{100Hz}$. Once again, measure the magnitude and frequency of each impulse. What happened to the spectrum? What's the new value of β? What is the relation between the value of β and the signal's spectrum?

- Use Matlab to evalate the bessel function for the first few values of n ($J_0(\beta)$, $J_1(\beta)$, and $J_2(\beta)$) for each of the β values found in the previous steps. The syntax is: $bessel(n, \beta)$. How do these values compare with the meausured magnitudes? If you don't understand how the Bessel function relates to an FM spectrum, go back and read a little bit on the FM theory.

4 Demodulation

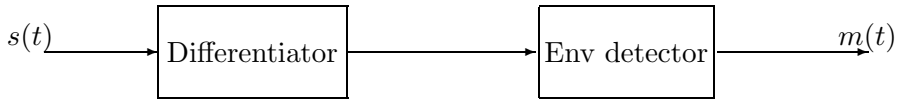One common method for demodulation of an FM signal involves two steps.

Figure 1: Demodulation of FM

The first step involves **differentiating** the FM signal in order to get the modulating signal, $m(t)$, out from the sinusoid as shown below. If we differentiate equation (1), we will get

$$s'(t) = -[2\pi f_c + 2\pi k_f m(t)] \left[A_c \, sin \left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau \right) \right] \qquad (6)$$

Differentiation can be achieved with a special filter designed in MATLAB. Recall that the transfer function of a differentiator is just s, that is, its frequency response is $j2\pi f$, which is a straight line in the frequency domain. Remember also that the frequency response of a differentiator only needs to be linear in the frequency band occupied by the FM signal. Figure 2, below, shows the response of the differentiator used in our FM demodulator.
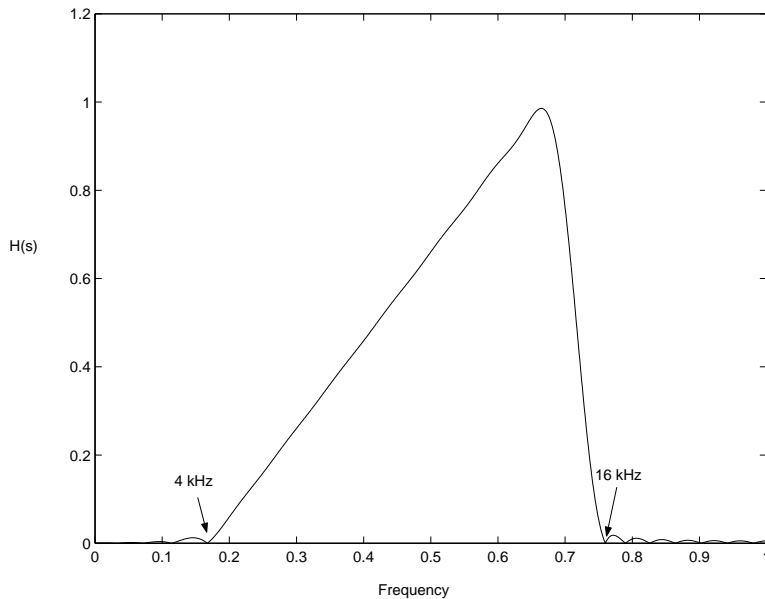

Figure 2: Magnitude response of differentiator

Wouldn't it be nice if we could build our own digital comercial FM reciever using the equipment we have in the lab? Well, we can't. The primary reason is because the bandwidth of comercial (wideband) FM is about 200kHz while the bandwidth of our best DSP board is limited to 24kHz, one half of its highest sampling frequency. The best we can hope to do is to create our own low frequency narrow bandwidth FM signal. This process is not very difficult because the function generators in the lab have built-in frequency modulation capabilities.

4

- Create an FM signal on the function generator with a carrier frequency, $f_c = $ **10 kHz**, modulating frequency, $f_m = $ **5kHz**, and frequency deviation, $\Delta f = $ **100Hz** . The modulating frequency, f_m, will be varied from **5 kHz down to 100 Hz** on the function generator. Remember that $\beta = \frac{\Delta f}{f_m}$. So, the value of β will vary from **0.02 to 1** as f_m is varied from 5 kHz to 100Hz.

- For $f_m = $ 5kHz, and $\Delta f = $ 100Hz, find the bandwidth of the FM signal. Do the same for $f_m = $ 100Hz, and Δf unchanged. If you were to modulate a voice signal that contained frequencies from 100Hz to 5kHz, and you used a frequency deviation of 100Hz, what would you expect the bandwidth of the resulting FM signal to be?

- The differentiator needs to be linear over the frequencies occupied by the FM signal. Based on the maximum bandwidth of FM, derived in the previous question, we designed a differentiator that was linear in the range of **4kHz** to **16kHz**. (See figure 2). The coefficients for the differentiator are stored in the file, 'diff.h'.

- The second step in our FM demodulation scheme is to do envelope detection. We choose to use one of the methods developed in the AM lab. On the DSP, we will square, lowpass filter, and then take the square root of the output of the differentiator in order to recover the modulating signal, $m(t)$ (as well as a DC constant).

- Connect the output of the function generator to the input of the DSP board, and connect the output of the DSP board to the A input of the spectrum analyzer. Under C:\EZ-KIT\EZSHARC\tt, open the file 'tt.c'. You will see that there are two filter coefficient files defined in the program: 'lpf.h' and 'diff.h'. You know what the diff.h file is. The lpf.h file contains the coefficients for the low pass filter used for envelope detection. Scroll down to the middle of the program and make sure you understand the main part of the code.

- On the function generator, set the modulating frequency to 5 kHz. On the computer, double click on Mk.bat and after it has compiled, run the Talkthru demo in the EZ-KIT Lite Host window. You should see the spectrum of the demodulated signal on the spectrum analyzer. Adjust the span, starting frequency, scale etc. to get a good view. Also view the signal in both log and linear scale. Looking at the signal's spectrum, can you see any drawbacks with this method of demodulation?

- Vary f_m in steps from 5 kHz down to 100 Hz and notice the change in the spectrum.