Energy Storage of Capacitor

- Capacitors do not dissipate power, but store energy when charging and restore it to the circuit when discharging.
- How much energy does a capacitor store?

$$E = \int_0^T P dt = \int_0^T VI dt = \int_0^T VC \frac{dV}{dt} dt$$

RHS allows us to integrate wrt V instead of T, but we must change the upper limit of integration to the final capacitor voltage V_T at t=T

$$E = \int_0^{V_T} CV dv = \frac{1}{2} C V_T^{\ 2}$$

In general, just use the formula

$$E = \frac{1}{2} C V^2$$

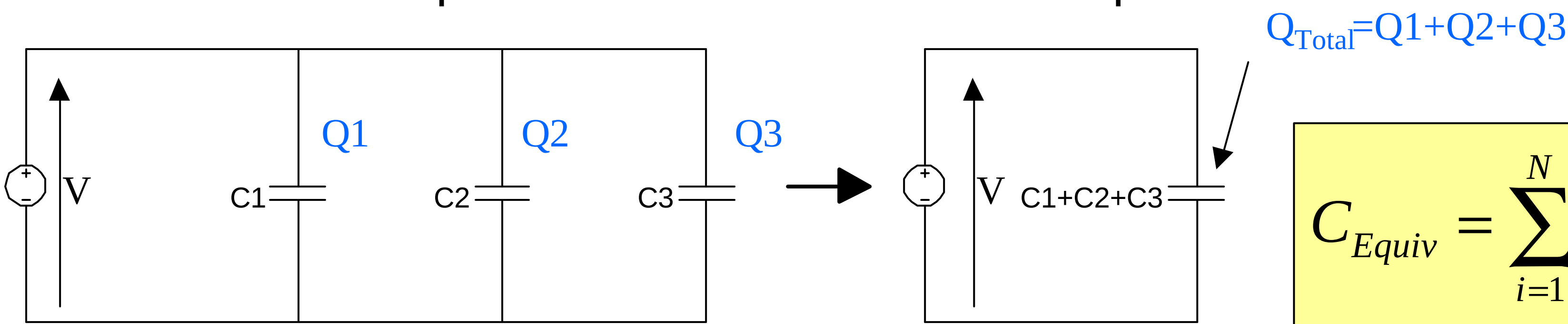
Parallel Capacitors

- Recall that capacitance is given by

$$C = \frac{Q}{V}$$

where Q is the charge and V is the applied voltage.

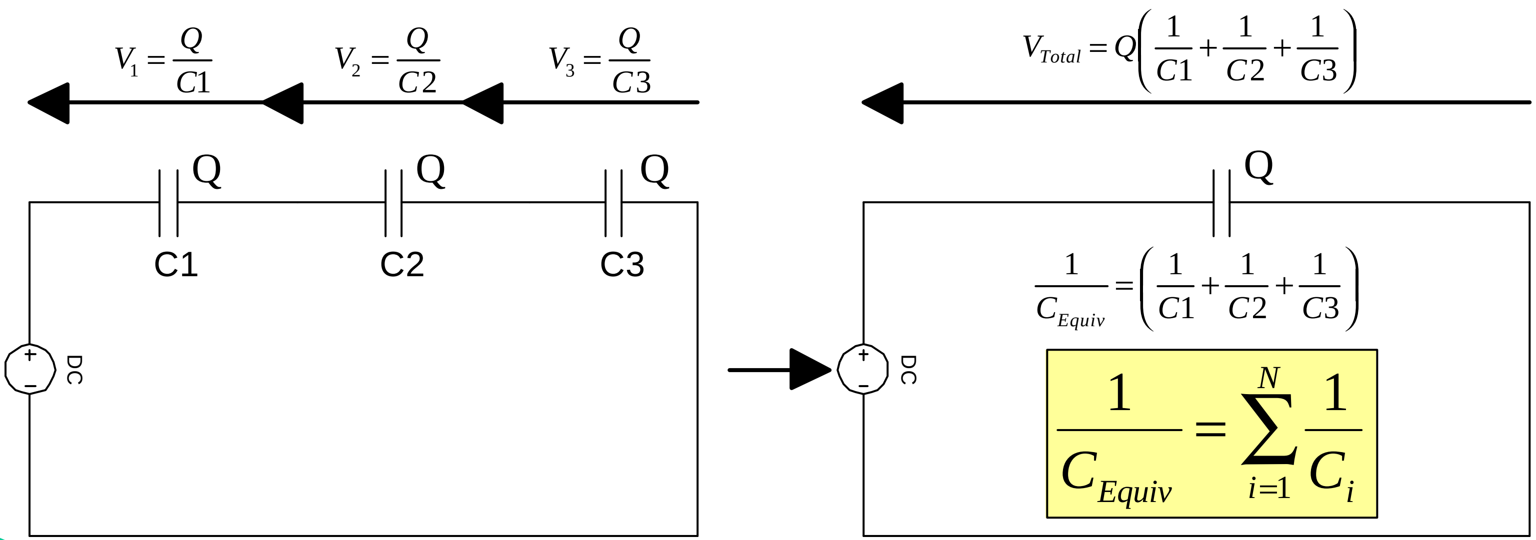
- Thus if capacitors are placed in parallel, the total charge stored will sum and hence the equivalent capacitance will just be the sum of the capacitances of the individual capacitors.



$$C_{Equiv} = \sum_{i=1}^{N} C_i$$

Series Capacitors

- If a current, I, is passed through a set of series capacitors over a period of time, T, each will acquire the same charge Q=IT.
- Each capacitor, C_i, will exhibit a voltage, $V_i=Q/C_i$.

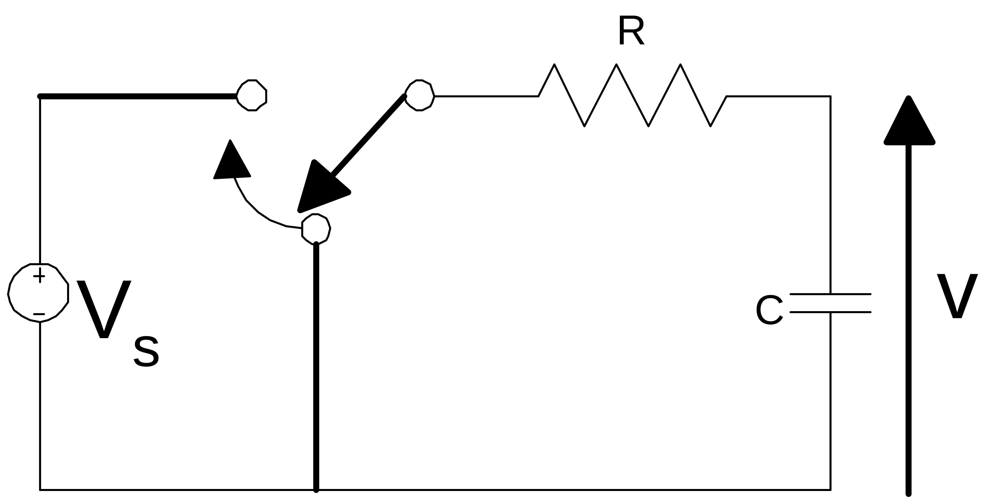


Parallel and Series Capacitors

- In other words
  - Capacitors in parallel are combined like the formula for resistors in series
  - Capacitors in series are combined like the formula for resistors in parallel
- Non-standard capacitance values can be made be combining standard value capacitors in parallel and series.

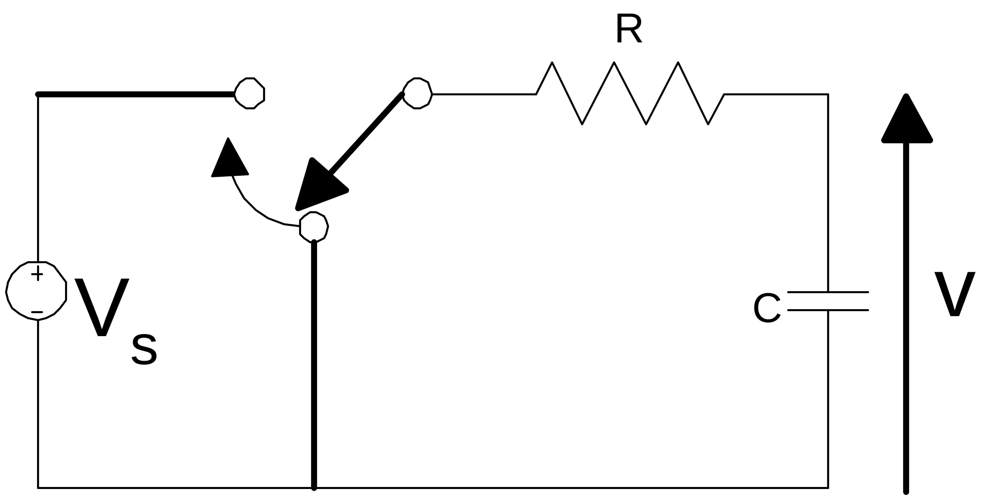
Behavior of Simple RC Circuit

- Consider that the switch has been in the position shown for a long time so that the capacitor is completely discharged.
- The switch moves to the other position at time t=0. What is the behaviour of the voltage, v, across the capacitor?



Simple RC Circuit

- Initially, there will be no charge on the capacitor (hence no voltage across it) and a current of $i=V_s/R$ will flow through the resistor
- As the capacitor charges, the voltage across the capacitor will rise and the current will decrease.
- Eventually, the capacitor will become completely charged to the voltage V_s and no current will flow at all.



Differential Equations

- Using Kirchoff's Voltage Law we have

$$-V_s + iR + \frac{q}{C} = 0 \quad (1)$$

where i=i(t) and q=q(t) are the instantaneous current and charge respectively

- Differentiating wrt t yields

$$R\frac{di}{dt} + \frac{1}{C}i = 0 \Rightarrow i = -RC\frac{di}{dt} \quad (2)$$

- Since this equation says that i(t) is simply a scalar, -RC, times di/dt, we guess that i(t) must take the form of an exponential function. (Recall that the derivative of e^x wrt x is also e^x)
- Thus to solve (2), i(t) must take the form where A is a scalar constant

$$i = Ae^{-\frac{t}{RC}}$$

Solution to Equations

$$i = Ae^{-\frac{t}{RC}}$$

- We solve for A by substituting back into (1) at time t=0.

$$-V_s + iR + \frac{q}{C} = 0 \quad (1)$$

$$\text{at time } t = 0,\ q = 0, \text{ and } i = Ae^{-\frac{t}{RC}} = A, \text{ yielding}$$

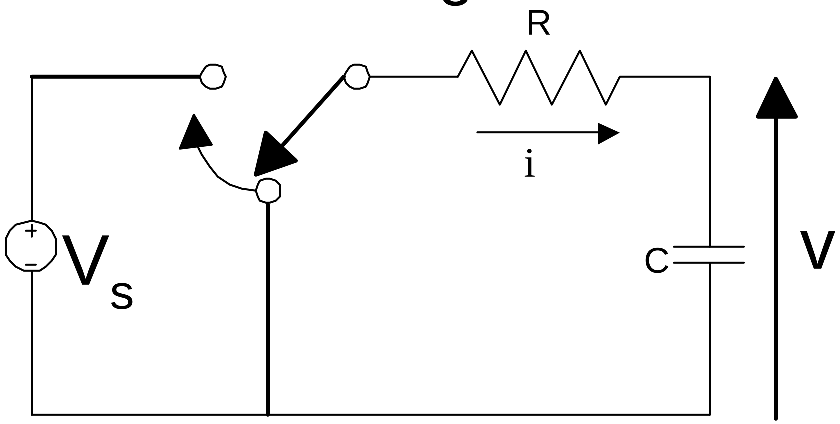
$$-V_s + AR = 0 \quad (1)$$

- Thus A=V_s/R and the final solution is

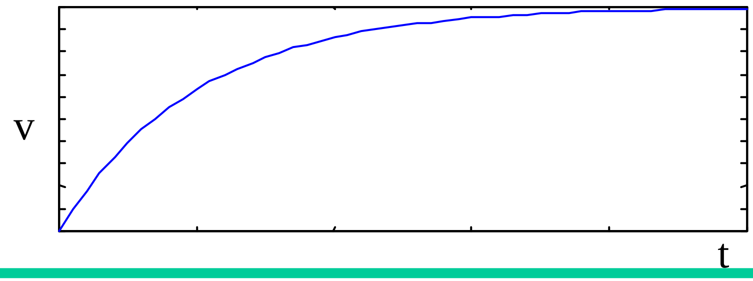
$$i = \frac{V_s}{R} e^{-\frac{t}{RC}}$$

Capacitor Voltage

- The voltage across the capacitor, v, is given by V_s minus the voltage across the resistor given by $V_R=iR$



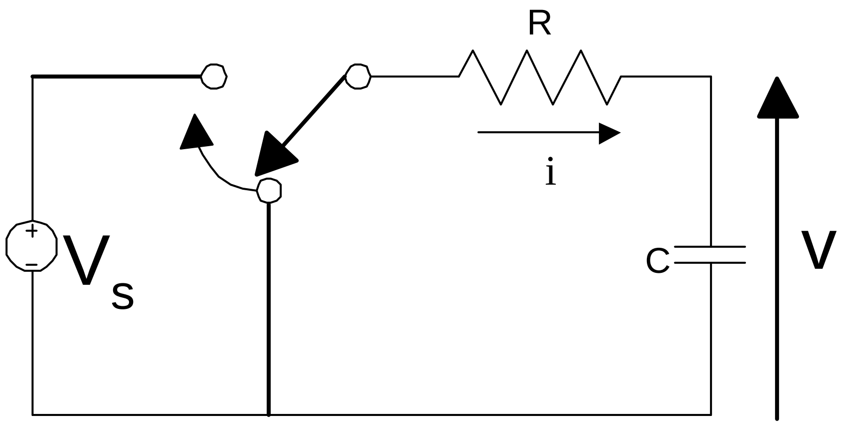
$$v = V_s - iR$$
$$= V_s - \frac{V_s}{R} e^{-\frac{t}{RC}} R$$
$$= V_s\left(1 - e^{-\frac{t}{RC}}\right)$$



$$v = V_s\left(1 - e^{-\frac{t}{RC}}\right)$$

Final Voltages and Currents

- The quantity RC is called the time constant of the circuit denoted by the Greek symbol, τ, (Tau)
- The time constant has the units of time (seconds).

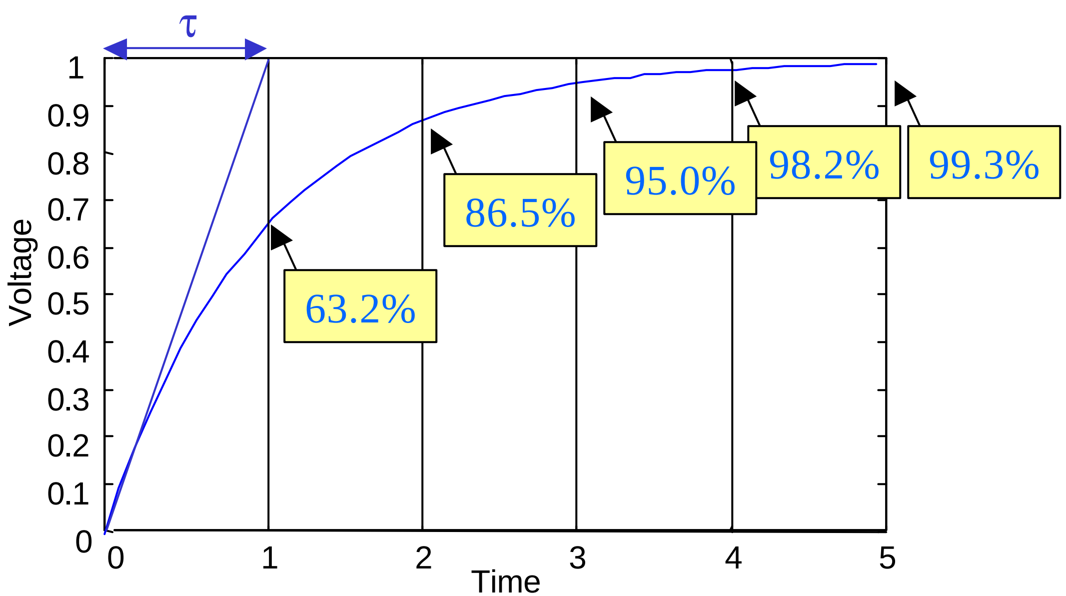


$$v = V_s\left(1 - e^{-\frac{t}{t}}\right)$$

$$i = \frac{V_s}{R} e^{-\frac{t}{t}}$$

Voltage Characteristics

- If we plot normalised voltage (fraction of supply voltage) across the capacitor against time expressed in multiples of the time constants, τ =RC, we obtain the following graph.

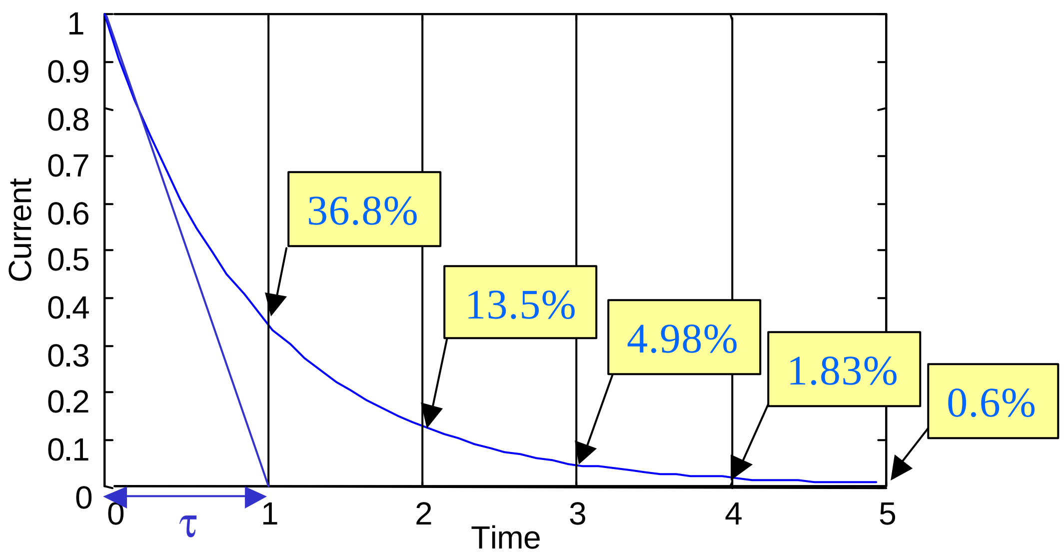


After t=τ time constant we obtain 63.2% of final voltage. After t=5τ, we obtain 99.3% of the final voltage

Initial slope is V_s/τ. This is a quick way to estimate τ on an oscilloscope.

Current Characteristics

Similarly, If we plot normalised current (fraction of initial current, i_0) through the capacitor against time expressed in multiples of the time constants, τ =RC, we obtain the following graph.



After t=τ time constant we obtain 36.8% of initial current. After t=5τ, we obtain 0.6% of the initial current

Initial slope is i_0/τ. This is a quick way to estimate τ on an oscilloscope.