Inductor Energy Storage

- Both capacitors and inductors are energy storage devices.
- They do not dissipate energy like a resistor, but store and return it to the circuit depending on applied currents and voltages.
- In the capacitor, energy is stored in the electric field between the plates.
- In the inductor, energy is stored in the magnetic field around the inductor.
Energy Storage Formula

• We write

\[ P = IV = I \left( L \frac{dI}{dt} \right) \]

• and since energy

\[ E = \int P \, dt \]

\[ E = \int_{0}^{t} IL \frac{dI}{dt} \, dt = \int_{I_0}^{I_t} LI \, dI = L \left[ \frac{I^2}{2} \right]_{I_0}^{I_t} \]

• and, assuming the initial current \( I_0 = 0 \) and the final current \( I_t = I \), we have

\[ E = \frac{1}{2} LI^2 \]

Compare with capacitor

\[ E = \frac{1}{2} CV^2 \]
Series Inductors

\[ V = V_1 + V_2 + \ldots + V_N \]
\[ = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt} + \ldots + L_N \frac{dI}{dt} \]
\[ = (L_1 + L_2 + \ldots + L_N) \frac{dI}{dt} \]

\[ V = L_{\text{Equiv}} \frac{dI}{dt} \]

\[
\therefore L_{\text{Equiv}} = L_1 + L_2 + \ldots + L_N
\]

- So inductors in series add like resistors in series
Parallel Inductors

For any inductor \( r \) on the left

\[
V = L_r \frac{dI_r}{dt} \Rightarrow I_r = \frac{1}{L_r} \int_0^t V dt \quad [+I_r(0) = 0 \text{ assumed}]
\]

\[
I = \frac{1}{L_{\text{Equiv}}} \int_0^t V dt
\]

\[
I = \sum_{r=1}^{N} \left( \frac{1}{L_r} \int_0^t V dt \right) = \sum_{r=1}^{N} \frac{1}{L_r} \left( \int_0^t V dt \right)
\]

\[
\frac{1}{L_{\text{Equiv}}} = \sum_{i=1}^{N} \frac{1}{L_i}
\]

- So inductors in parallel add like resistors in parallel
Behaviour of Simple RL Circuits

- Consider that the switch has been in the position shown for a long time so that no current is flowing.
- The switch moves to the other position at time $t=0$. What is the behaviour of the voltage, $v$, across the inductor?
Simple RL Circuit

Initially, there will be no current through the inductor because the inductor will create a voltage to oppose a step change in current. Hence a voltage of $V_s$ will initially appear across the inductor. As the current increases, the voltage across the inductor will decrease. Eventually, a steady current of $V_s/R$ will be reached and $v$ will fall to zero.
Analysis of RL Circuit

• Assume the inductor is not storing energy at t=0 (no current)
• KVL for this circuit yields

\[ V_s = Ri + L \frac{di}{dt} \]

We know the solution has the form

\[ i(t) = I_A + I_B e^{\frac{t}{\tau}} \]

Initial and final boundary values yields

\[ I_{\text{initial}} = I_A + I_B = 0; \quad I_{\text{final}} = I_A = \frac{V_s}{R} \]

\[ i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{\frac{t}{\tau}} \] (\*)
Final Solution

• Now we just need to determine the time constant $\tau$
• Differentiating (*) yields

$$\frac{di}{dt} = \frac{1}{\tau} \frac{V_s}{R} e^{-\frac{t}{\tau}} = \frac{1}{\tau} \left( \frac{V_s}{R} - i \right)$$

Backsubstitution into the KVL equation yields

$$V_s = Ri + \frac{L}{\tau} \left( \frac{V_s}{R} - i \right)$$

$$V_s - Ri = \frac{L}{R\tau} (V_s - Ri)$$

$$\tau = \frac{L}{R}$$

$$i(t) = \frac{V_s}{R} \left( 1 - e^{-\frac{R}{L} t} \right)$$
Current Characteristics

If we plot normalised current (fraction of final current $I_0=\frac{V_s}{R}$) through the inductor against time expressed in multiples of the time constants, $\tau = \frac{L}{R}$, we obtain the following graph.

After $t=\tau$ we obtain 63.2% of the final current. After $t=5\tau$, we obtain 99.3% of the final current.

Initial slope is $I_0/\tau$. This is a quick way to estimate $\tau$ on an oscilloscope.
Voltage Characteristics

Similarly, if we plot normalised voltage (fraction of supply voltage, $V_s$) across the inductor against time expressed in multiples of the time constant, $\tau = L/R$, we obtain the following graph.

After $t = \tau$ we obtain 36.8% of supply voltage. After $t = 5\tau$, we obtain 0.6% of the supply voltage.

Initial slope is $V_s/\tau$. This is a quick way to estimate $\tau$ on an oscilloscope.