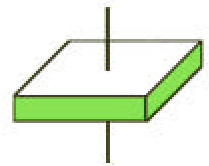
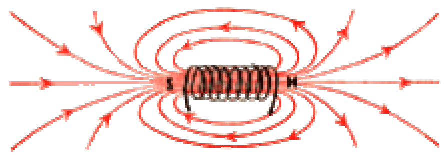
Inductor Energy Storage

- Both capacitors and inductors are energy storage devices
- They do not dissipate energy like a resistor, but store and return it to the circuit depending on applied currents and voltages
- In the capacitor, energy is stored in the electric field between the plates



- In the inductor, energy is stored in the magnetic field around the inductor



Energy Storage Formula

- We write

$$P = IV = I\left(L\frac{dI}{dt}\right)$$

Compare with capacitor

$$E = \frac{1}{2}CV^2$$

- and since energy

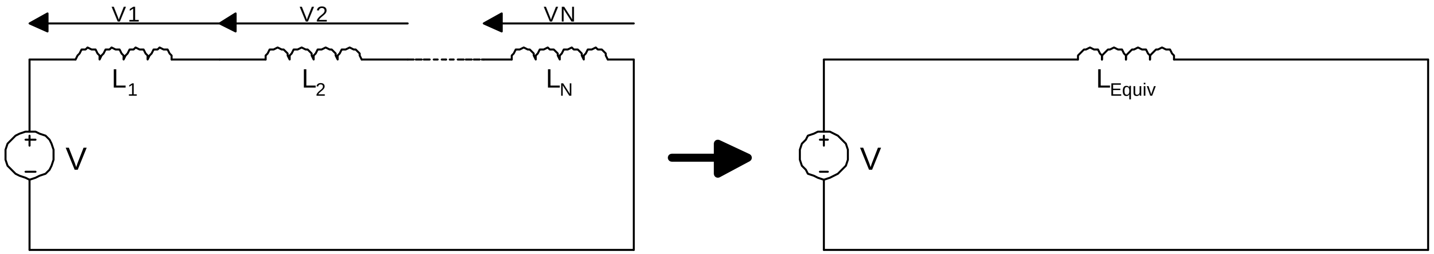
$$E = \int_0^t P dt$$

$$E = \int_0^t IL\frac{dI}{dt}dt = \int_{I_0}^{I_t} LI dI = L\left[\frac{I^2}{2}\right]_{I_0}^{I_t}$$

- and, assuming the initial current $I_0=0$ and the final current $I_t=I$, we have

$$E = \frac{1}{2}LI^2$$

Series Inductors



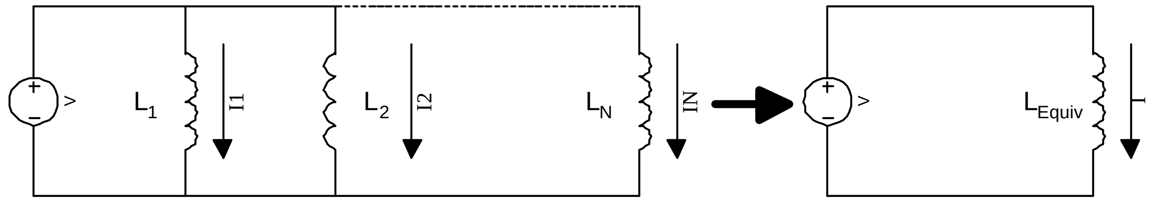
$$V = V_1 + V_2 + \ldots + V_N$$
$$= L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt} + \ldots + L_N \frac{dI}{dt}$$
$$= (L_1 + L_2 + \ldots + L_N) \frac{dI}{dt}$$

$$V = L_{Equiv} \frac{dI}{dt}$$

$$\therefore L_{Equiv} = L_1 + L_2 + \ldots + L_N$$

- So inductors in series add like resistors in series

Parallel Inductors



For any inductor r on the left

$$V = L_r \frac{dI_r}{dt} \Rightarrow I_r = \frac{1}{L_r}\int_0^t V dt \quad [+I_r(0) = 0 \text{ assumed}]$$

$$I = \frac{1}{L_{Equiv}}\int_0^t V dt$$

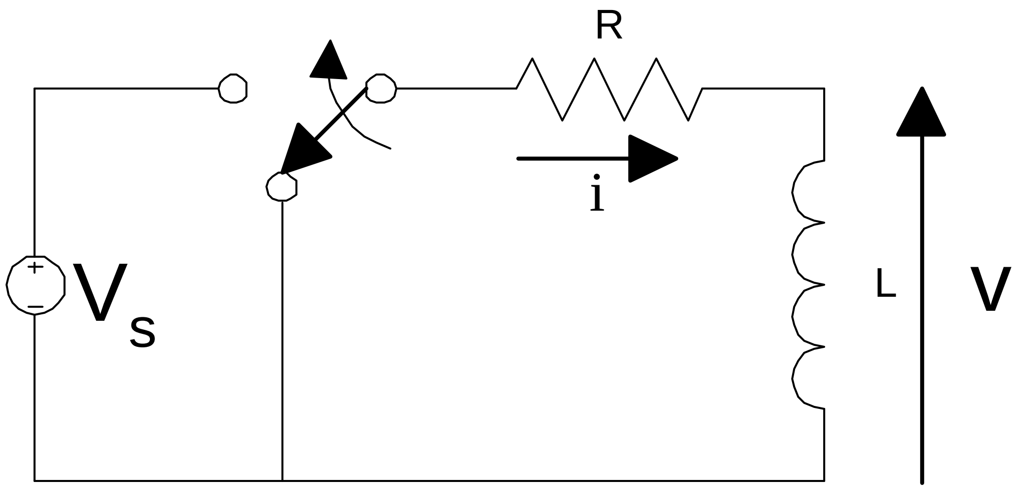
$$I = \sum_{r=1}^{N}\left(\frac{1}{L_r}\int_0^t V dt\right) = \sum_{r=1}^{N}\frac{1}{L_r}\left(\int_0^t V dt\right)$$

$$\frac{1}{L_{Equiv}} = \sum_{i=1}^{N}\frac{1}{L_i}$$

- So inductors in parallel add like resistors in parallel

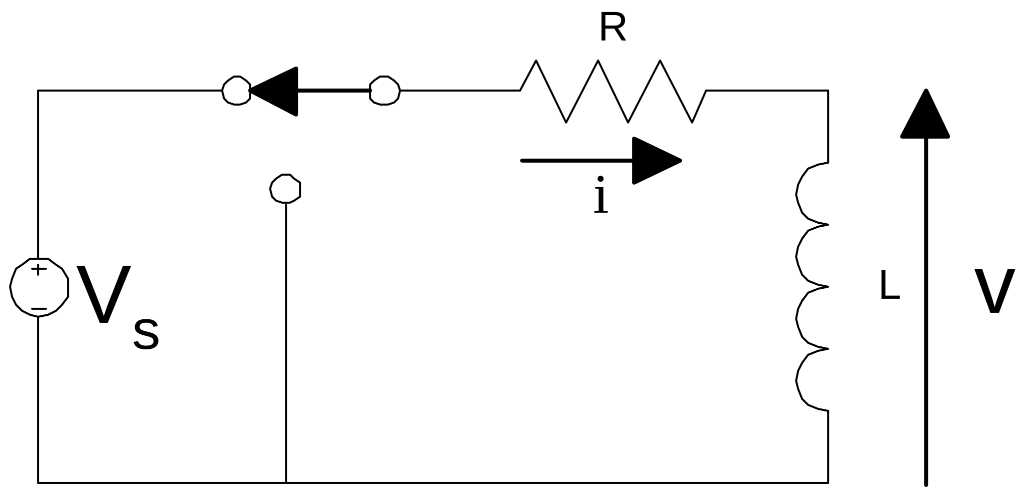
Behaviour of Simple RL Circuits

- Consider that the switch has been in the position shown for a long time so that no current is flowing.
- The switch moves to the other position at time t=0. What is the behaviour of the voltage, v, across the inductor?



Simple RL Circuit

Initially, there will be no current through the inductor because the inductor will create a voltage to oppose a step change in current. Hence a voltage of V_s will initially appear across the inductor.
As the current increases, the voltage across the inductor will decrease.
Eventually, a steady current of V_s/R will be reached and v will fall to zero.



Analysis of RL Circuit

- Assume the inductor is not storing energy at t=0 (no current)
- KVL for this circuit yields

$$V_s = Ri + L\frac{di}{dt}$$

We know the solution has the form

$$i(t) = I_A + I_B e^{-\frac{t}{t}}$$

Initial and final boundary values yields

$$I_{\text{initial}} = I_A + I_B = 0; I_{\text{final}} = I_A = \frac{V_s}{R}$$

$$i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-\frac{t}{t}} \qquad (*)$$

Final Solution

- Now we just need to determine the time constant τ
- Differentiating (*) yields

$$\frac{di}{dt} = \frac{1}{t}\frac{V_s}{R}e^{-\frac{t}{t}} = \frac{1}{t}\left(\frac{V_s}{R} - i\right)$$

Backsubstitution into the KVL equation yields

$$V_s = Ri + \frac{L}{t}\left(\frac{V_s}{R} - i\right)$$

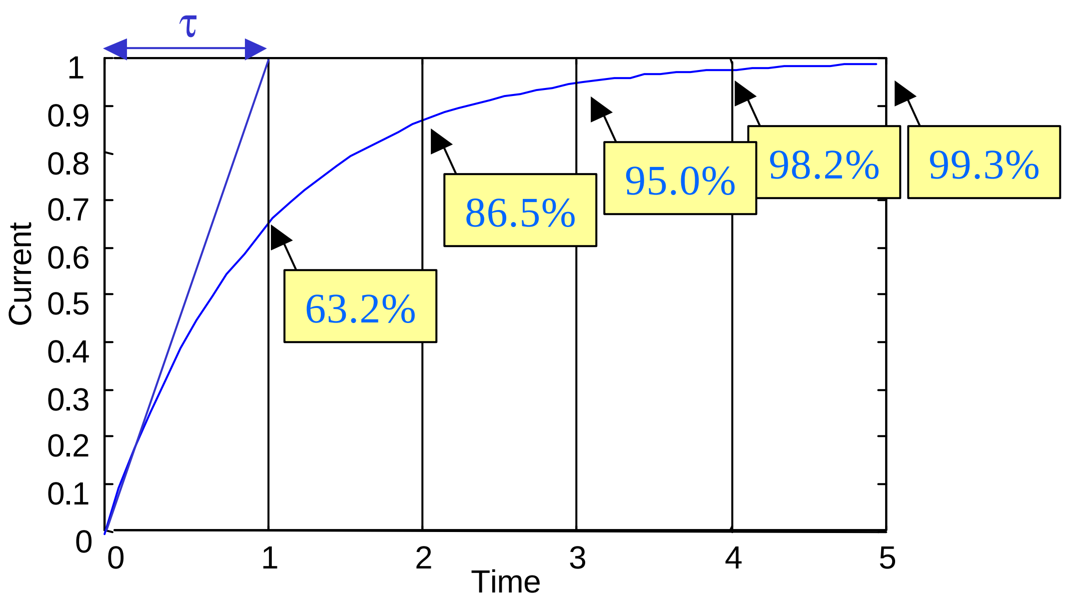
$$V_s - Ri = \frac{L}{Rt}(V_s - Ri)$$

$$t = \frac{L}{R}$$

$$i(t) = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

Current Characteristics

If we plot normalised current (fraction of final current $I_0=V_s/R$) through the inductor against time expressed in multiples of the time constants, $\tau = L/R$, we obtain the following graph.

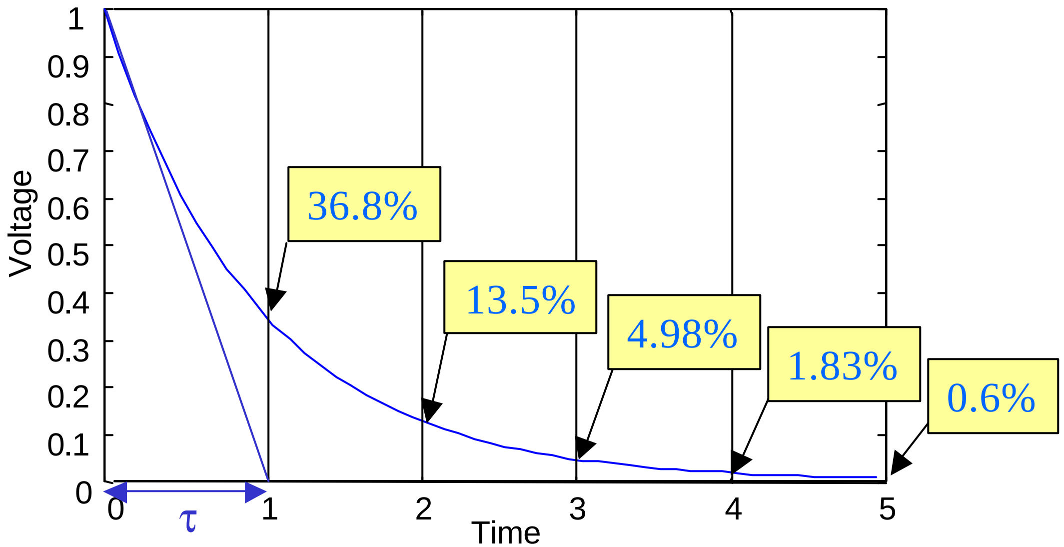


After $t=\tau$ we obtain 63.2% of the final current. After $t=5\tau$, we obtain 99.3% of the final current

Initial slope is I_0/τ. This is a quick way to estimate τ on an oscilloscope.

Voltage Characteristics

Similarly, If we plot normalised voltage (fraction of supply voltage, V_s) across the inductor against time expressed in multiples of the time constant, $\tau = L/R$, we obtain the following graph.



After $t=\tau$ we obtain 36.8% of supply voltage. After $t=5\tau$, we obtain 0.6% of the supply voltage

Initial slope is V_s/τ. This is a quick way to estimate τ on an oscilloscope.