Physics 6C
Introduction to Physics III
Electricity and Magnetism

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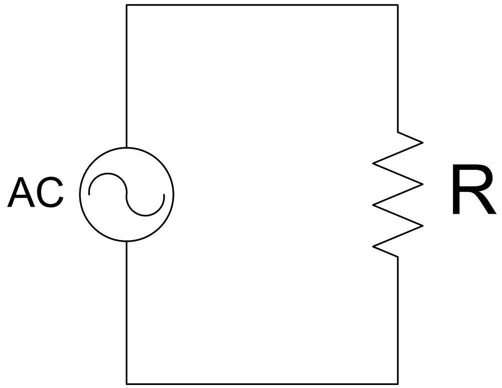
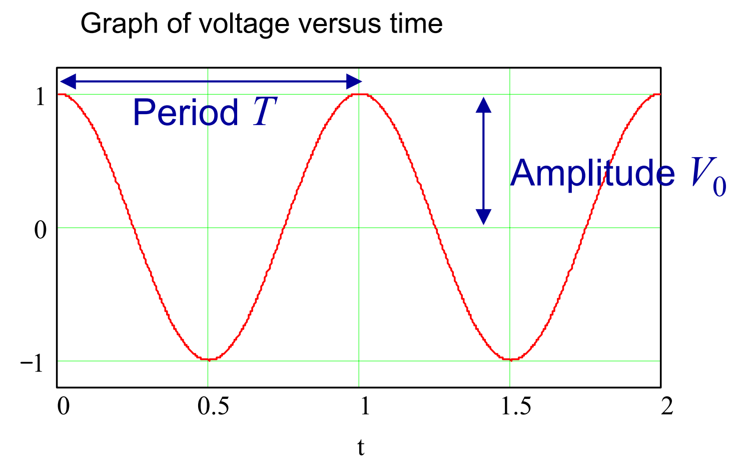
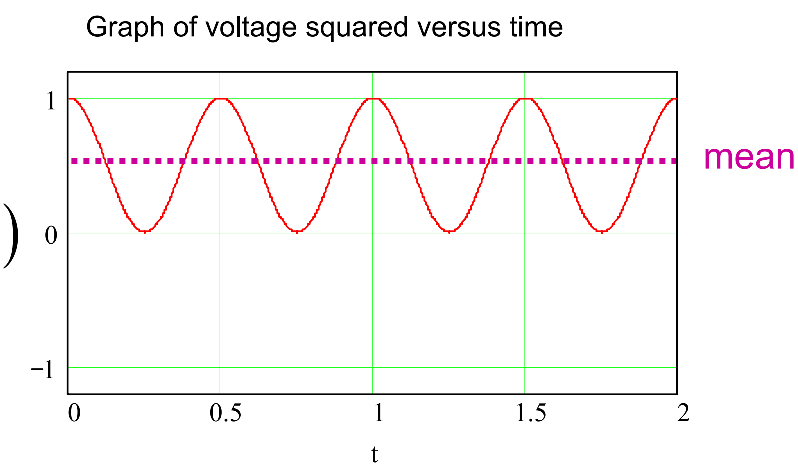
AC Current and Voltage



$$V(t) = V_0 \cos \omega t$$

mean voltage is zero!

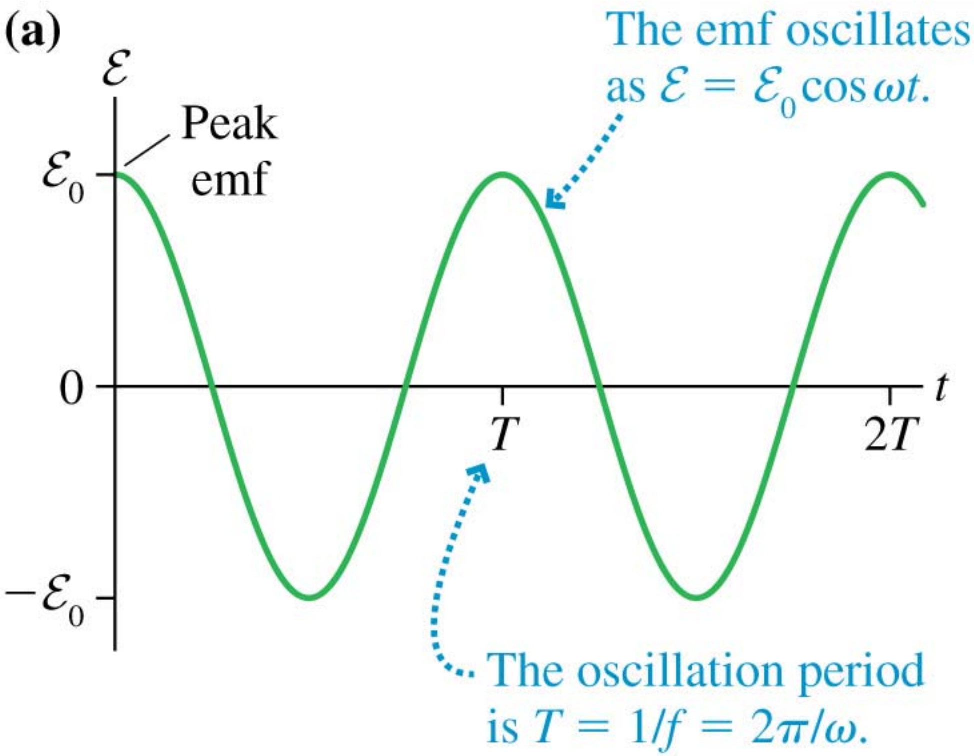


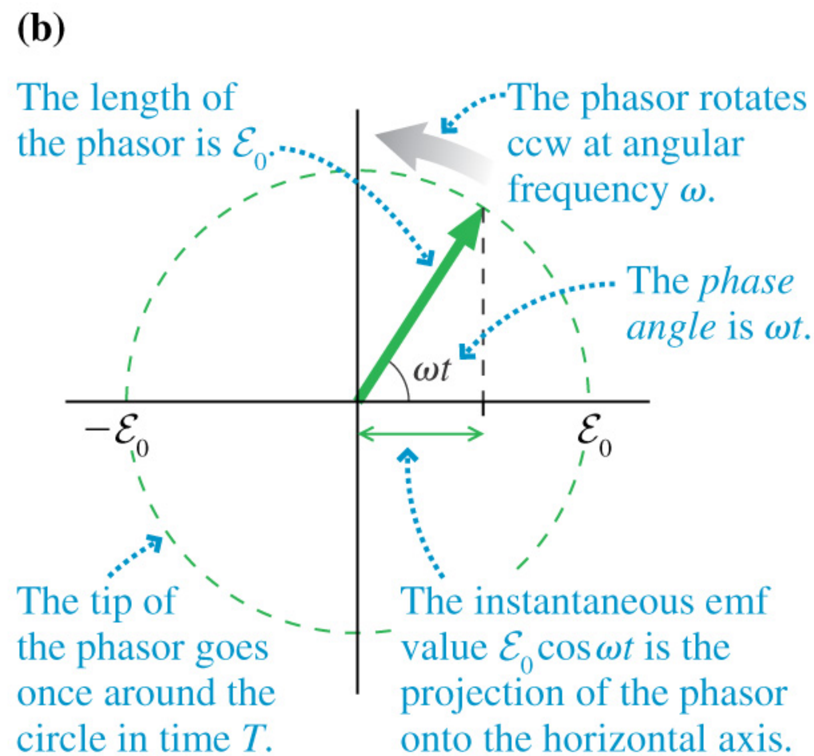


$$V(t)^2 = V_0^2 \, \tfrac{1}{2}\left(1 + \cos 2\omega t\right)$$

The rms voltage is $V_{rms} \equiv \sqrt{\langle V^2 \rangle} = \dfrac{V_0}{\sqrt{2}}$

Phasors

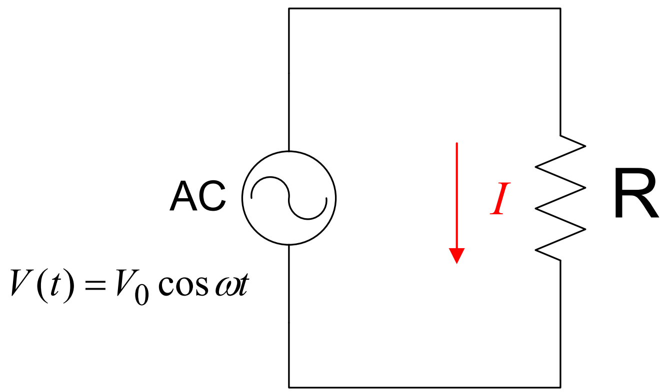




Represent the cosine function as the x-component of a rotating vector. This provides a graphical means to keep track of both amplitude and phase.;

(Note for the mathematically inclined: this is equivalent to representing the cosine function as the real part of a complex number.)

AC Circuits: Resistance



$$V(t) - IR = 0$$ Kirchhoff's rule

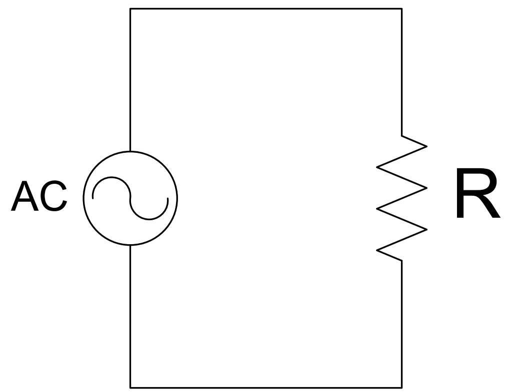
$$I(t) = \frac{V(t)}{R}$$

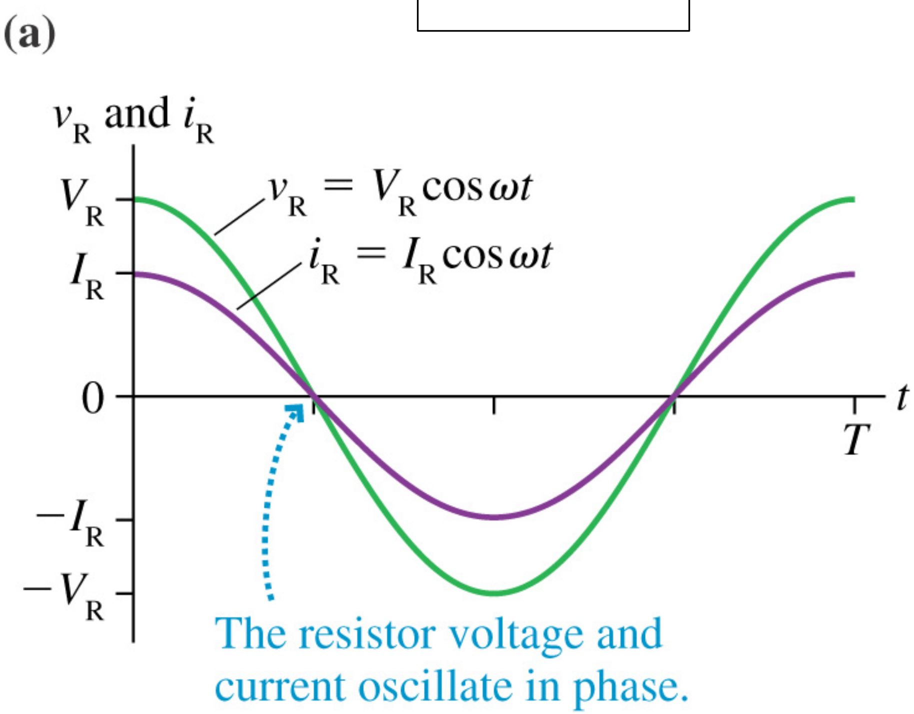
$$I(t) = \frac{V_0}{R} \cos \omega t$$

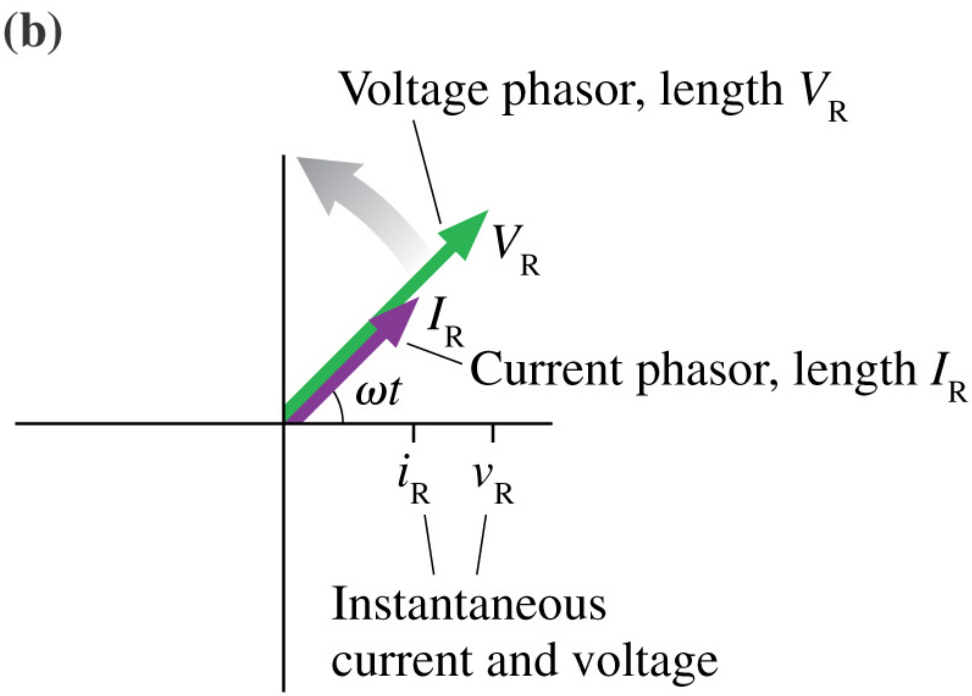
Energy is dissipated (turned into heat) by the resistor!

$$P = I^2 R = \frac{V_0^2}{R} (\cos \omega t)^2 \Rightarrow P_{av} = \frac{1}{2} \frac{V_0^2}{R} = \frac{V_{rms}^2}{R}$$

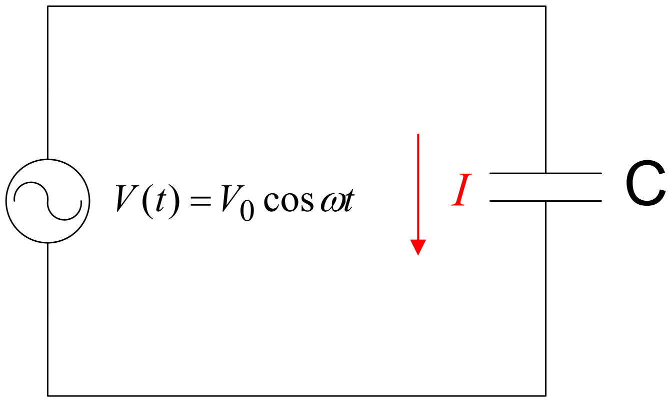
Phasors for Resistive Circuit







AC Circuits: Capacitance



$$V(t) - \frac{Q}{C} = 0$$ Kirchhoff's rule

$$\frac{dV}{dt} = \frac{1}{C}\frac{dQ}{dt} = \frac{1}{C} I$$

$$I(t) = C\frac{dV}{dt} = -C\omega V_0 \sin \omega t$$

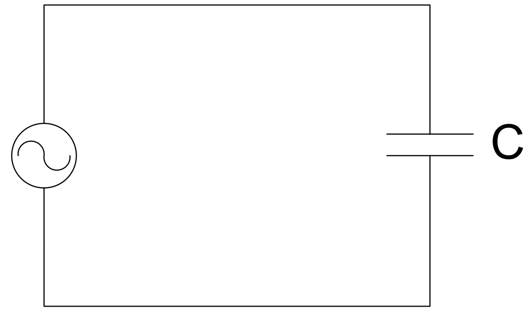
$$I(t) = \frac{V_0}{1/\omega C} \cdot \cos(\omega t + \tfrac{\pi}{2})$$

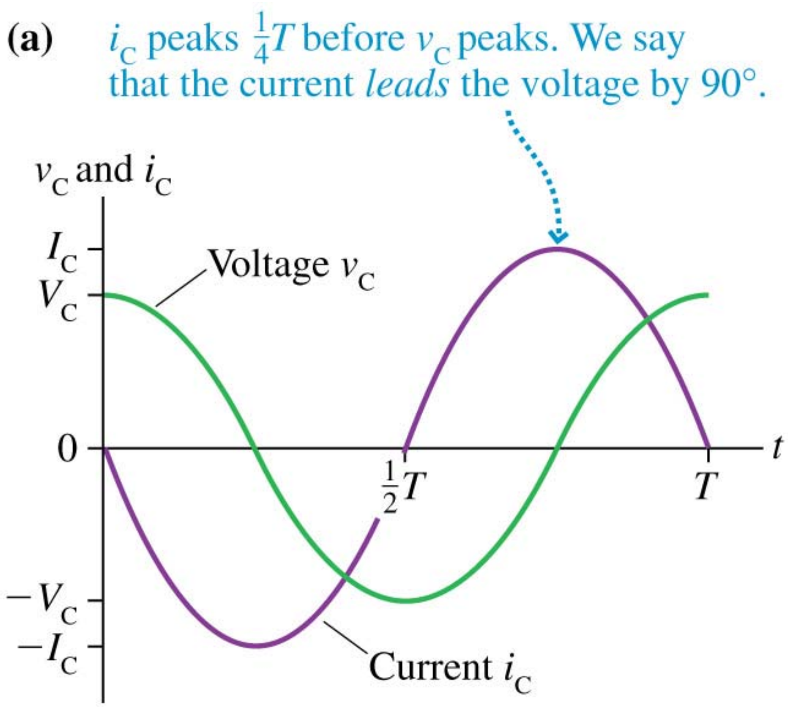
Capacitive "reactance": $$X_C \equiv \frac{1}{\omega C}$$

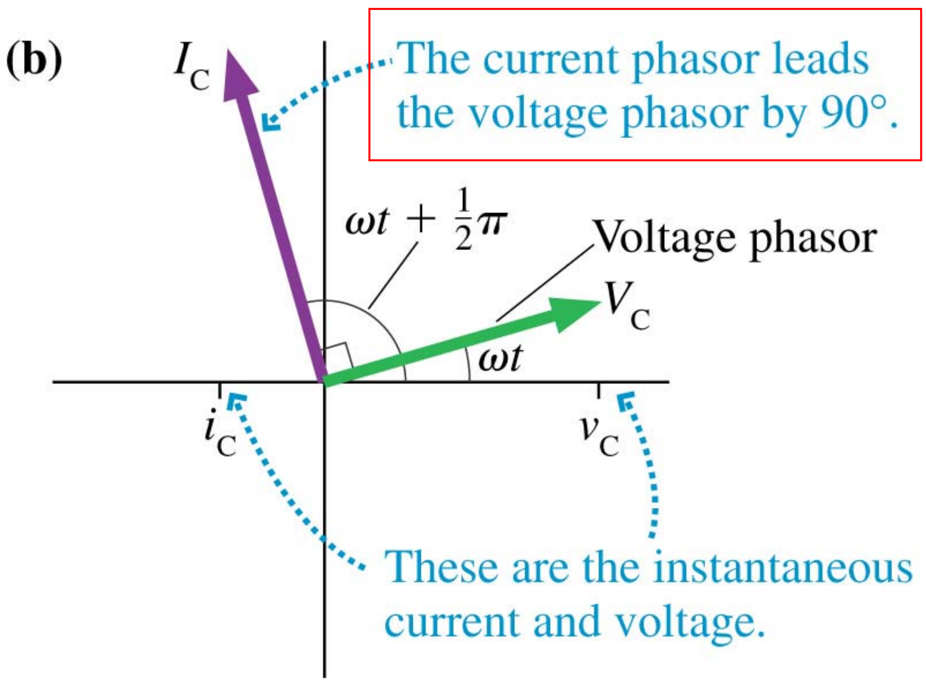
Energy is stored, NOT dissipated!

$$I_{\max} = \frac{V_0}{X_C}$$

Phasors for Capacitive Circuit



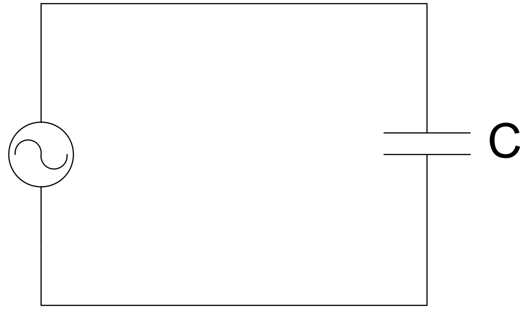




Exercise 35-9

- A 20 nF capacitor is connect across an AC generator that produces a peak voltage of 5.0 V.
  a) At what frequency f is the peak current 50 mA?
  b) What is the instantaneous value of the emf at the instant when $i_C = I_C$?

$$v(t) = 5.0\cos(2\pi f t)$$



Notation: our textbook uses i to represent the time-dependent current and I to represent the amplitude:

$$i_C \equiv I(t) = I_C \cos\left(\omega t + \frac{\pi}{2}\right)$$

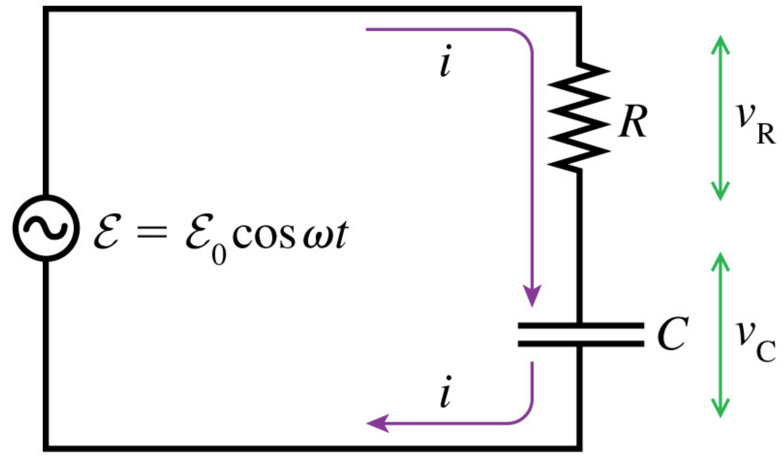
RC Circuit

Kirchhoff's loop rule holds at every instant in time:

$$\mathcal{E}(t) = v_R(t) + v_C(t)$$

We want to find the resulting current, *which generally will not be in phase with the voltage*:

$$i(t) = I_{\max}\cos(\omega t + \phi)$$



We have to find both the amplitude $I_{\max}$ and the phase ϕ.

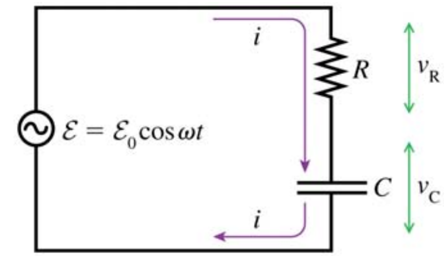
$$v_R = I_{\max} R \cos(\omega t + \phi)$$

The resistor voltage is in phase with current

$$v_C = I_{\max}\frac{1}{\omega C}\cos\left(\omega t + \phi - \frac{\pi}{2}\right)$$

The capacitor voltage lags behind the current by 90 degrees.

RC Circuit



$$\mathcal{E}(t) = v_R(t) + v_C(t)$$

$$\mathcal{E}_0 \cos \omega t = I_{\max} R \cos(\omega t + \phi) + I_{\max} \frac{1}{\omega C} \cos\left(\omega t + \phi - \tfrac{\pi}{2}\right)$$

This equation can be solved for both $I_{\max}$ and ϕ by using trig identities, but it is easier to do it graphically using phasors. The algebra then just looks like vector addition.

$$\mathcal{E}_0 = \sqrt{V_R^2 + V_C^2} = I_{\max} \sqrt{R^2 + (1/\omega C)^2}$$

$$\tan \phi = \frac{V_C}{V_R} = \frac{1}{\omega RC}$$

As in a circuit with just a capacitor, the voltage lags behind the current, but by less than 90 degrees.