Physics 6C
Introduction to Physics III
Electricity and Magnetism

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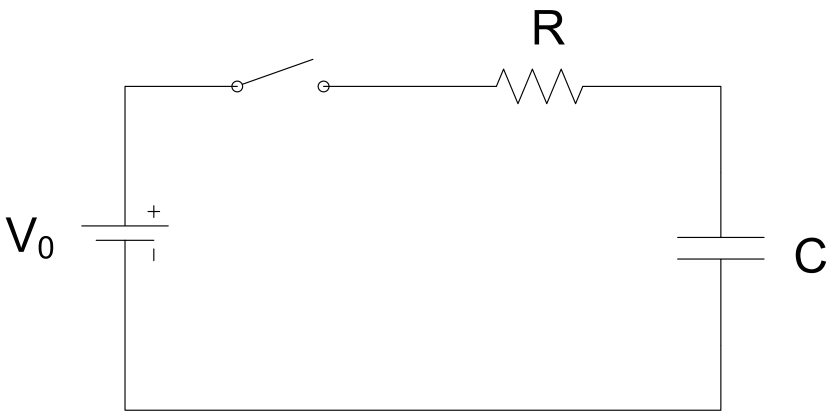
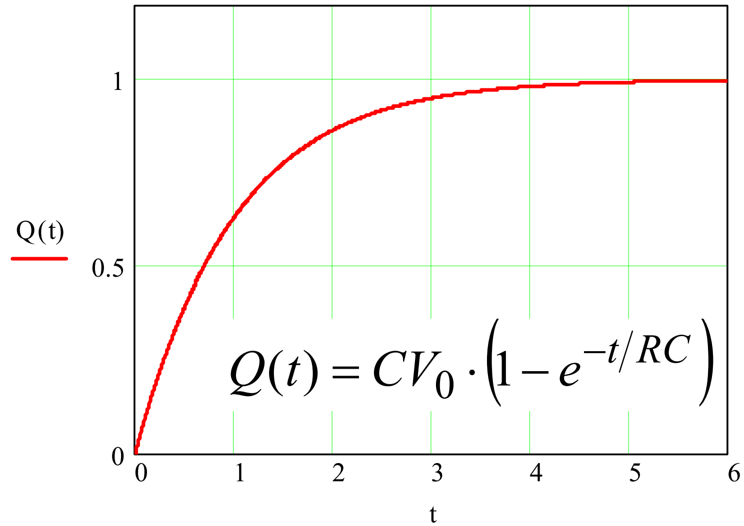
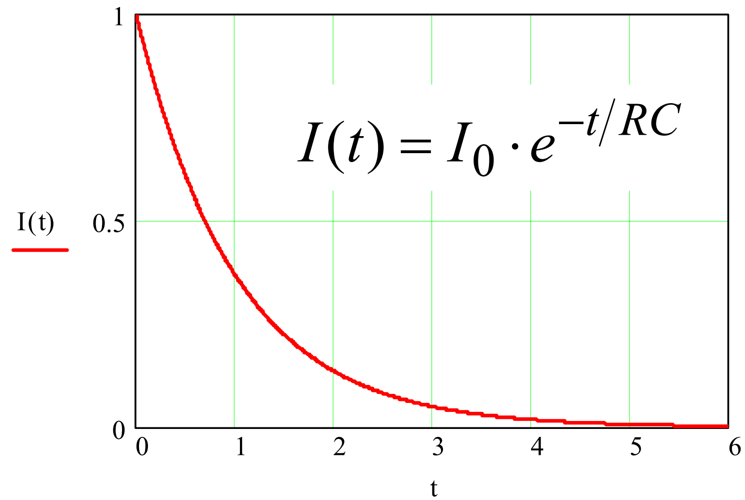
RC Circuit: Charging





This is an example of a "transient" response. The current changes with time after the switch closes and eventually reaches a constant value of zero.

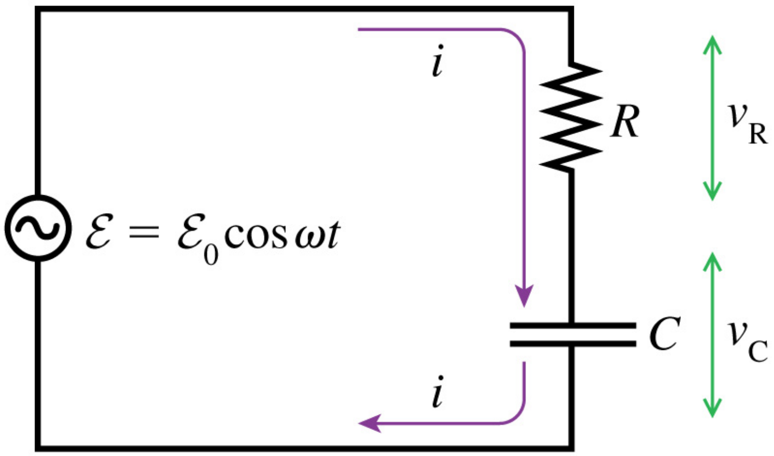


For AC circuits, we are not interested in the transient response, but instead we consider *only the behavior after the circuit has been on for a long time.*

RC Circuit with AC Source

Kirchhoff's loop rule holds at every instant in time:

$$\mathcal{E}(t) = v_R(t) + v_C(t)$$



We want to find the resulting current, *which generally will not be in phase with the voltage*:

$$i(t) = I_{\max} \cos(\omega t + \phi)$$

We have to find both the amplitude $I_{\max}$ and the phase ϕ.

Assume that the source has been turned on for a long time, so all transients have died out.

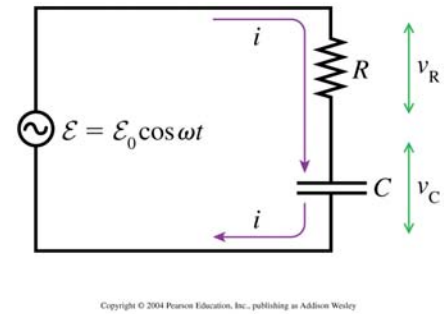
$$v_R = I_{\max} R \cos(\omega t + \phi)$$

The resistor voltage is in phase with current

$$v_C = I_{\max} \frac{1}{\omega C} \cos\left(\omega t + \phi - \frac{\pi}{2}\right)$$

The capacitor voltage lags behind the current by 90 degrees.

RC Circuit



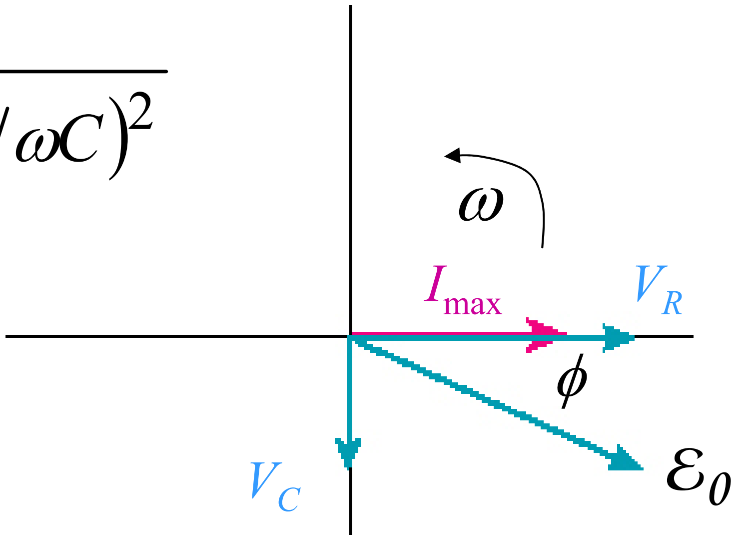
$$\mathcal{E}(t) = v_R(t) + v_C(t)$$

$$\mathcal{E}_0 \cos\omega t = I_{\max} R \cos(\omega t + \phi) + I_{\max} \frac{1}{\omega C} \cos\left(\omega t + \phi - \tfrac{\pi}{2}\right)$$

This equation can be solved for both $I_{\max}$ and ϕ by using trig identities, but it is easier to do it graphically using phasors. The algebra then just looks like vector addition.

$$\mathcal{E}_0 = \sqrt{V_R^2 + V_C^2} = I_{\max}\sqrt{R^2 + (1/\omega C)^2}$$

$$\tan\phi = \frac{V_C}{V_R} = \frac{1}{\omega RC}$$



As in a circuit with just a capacitor, the voltage lags behind the current, but by less than 90 degrees. $\phi > 0$

$$\text{Note: } Z = \sqrt{R^2 + X_C^2}$$

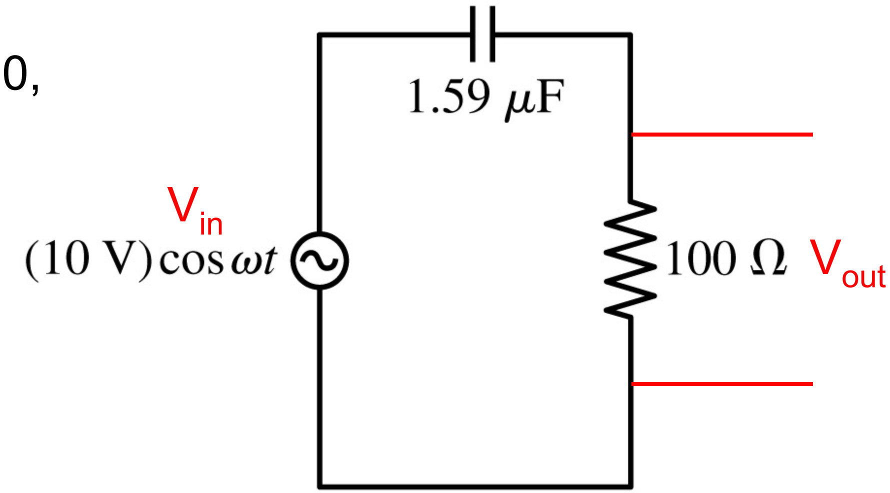
Problem 35-36

a) Evaluate V_R at emf frequencies 100, 300, 1000, 3000, and 10,000 Hz

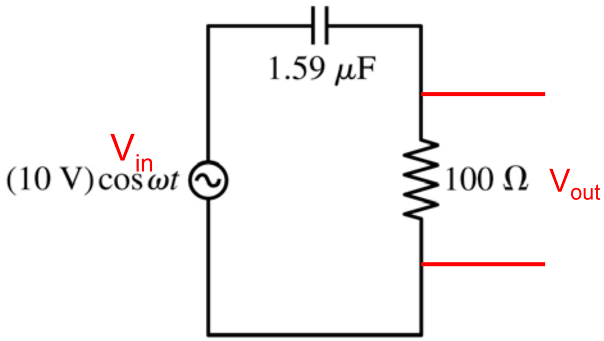
b) Graph V_R vs frequency.

This is an example of a "high pass filter."

Note that for $f=0$ (DC) the current must be zero, and therefore V_{out} is 0.



Problem 35-36



$R := 100 \quad V_{in} := 10 \quad C := 1.59 \cdot 10^{-6}$

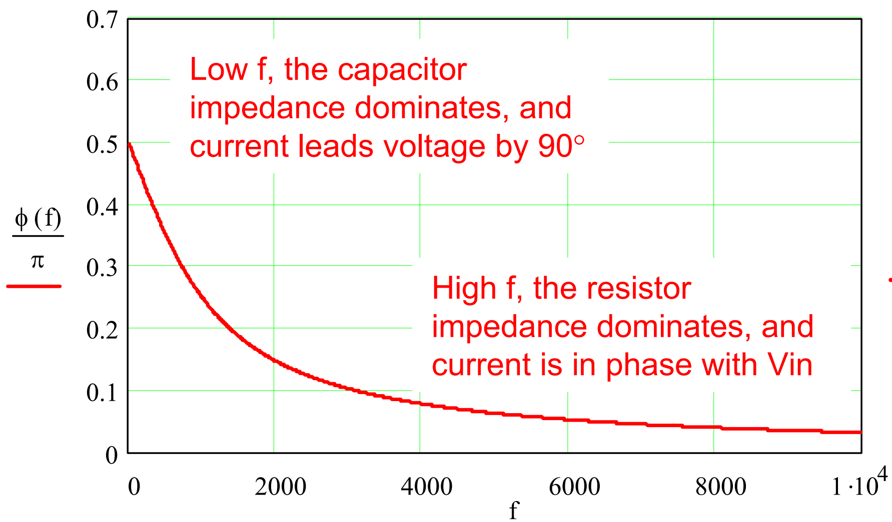
$$V_{out}(f) := V_{in} \cdot \frac{R}{\sqrt{R^2 + \left(\frac{1}{2 \cdot \pi \cdot f \cdot C}\right)^2}}$$

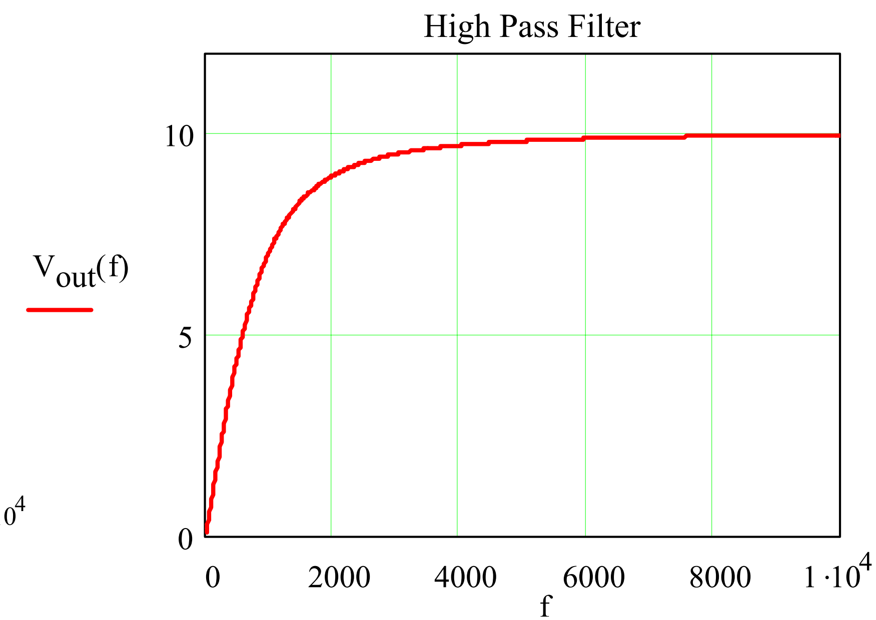
$V_{out}(100) = 0.994 \quad V_{out}(1000) = 7.068 \quad V_{out}(10000) = 9.95$

$V_{out}(300) = 2.871 \quad V_{out}(3000) = 9.486$

The output (voltage across resistor) is in phase with the current, which leads the input voltage by an angle

$$\phi(f) := \operatorname{atan}\left(\frac{1}{2 \cdot \pi \cdot f \cdot R \cdot C}\right)$$





Problem 35-36 continued.

This plot demonstrates for one frequency (1500 Hz) how the resistor and capacitor voltages add together at all times to yield the voltage of the source.

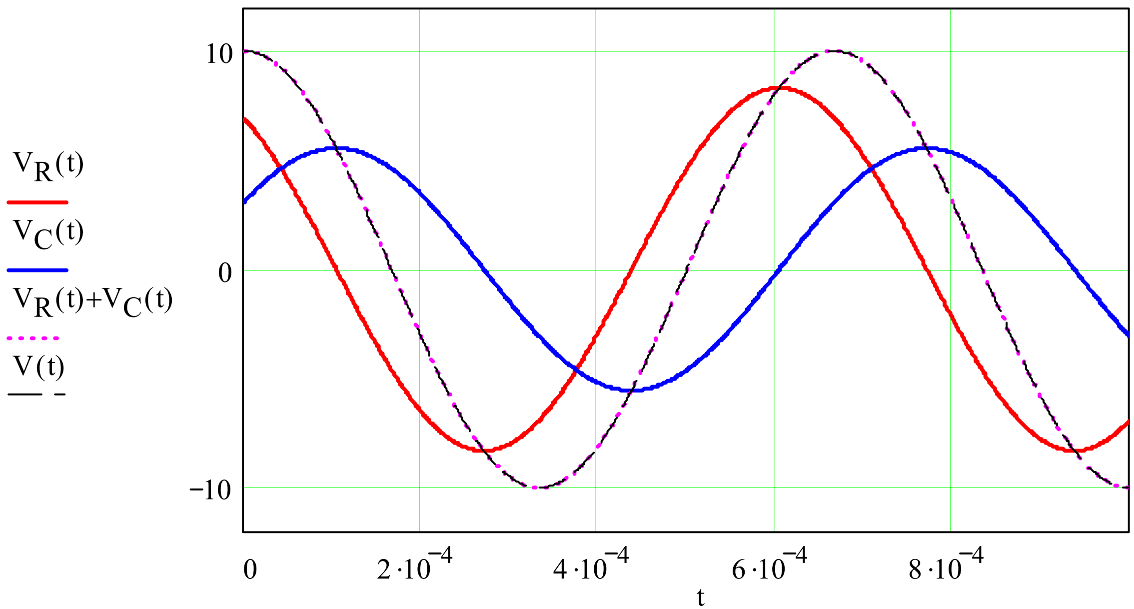
$R := 100 \qquad C := 1.59\cdot 10^{-6} \qquad V_{in} := 10 \qquad \omega := 2\cdot\pi\cdot 1500$

$V(t) := V_{in}\cdot\cos(\omega\cdot t)$

$$I_{max} := \frac{V_{in}}{\sqrt{R^2 + \left(\frac{1}{\omega\cdot C}\right)^2}} \qquad \phi := \operatorname{atan}\left(\frac{1}{\omega\cdot R\cdot C}\right) \qquad \phi = 33.716\,\text{deg}$$

$I(t) := I_{max}\cdot\cos(\omega\cdot t + \phi)$

$$V_R(t) := I_{max}\cdot R\cdot\cos(\omega\cdot t + \phi) \qquad V_C(t) := I_{max}\cdot\frac{1}{\omega\cdot C}\cdot\cos\left(\omega\cdot t + \phi - \frac{\pi}{2}\right)$$



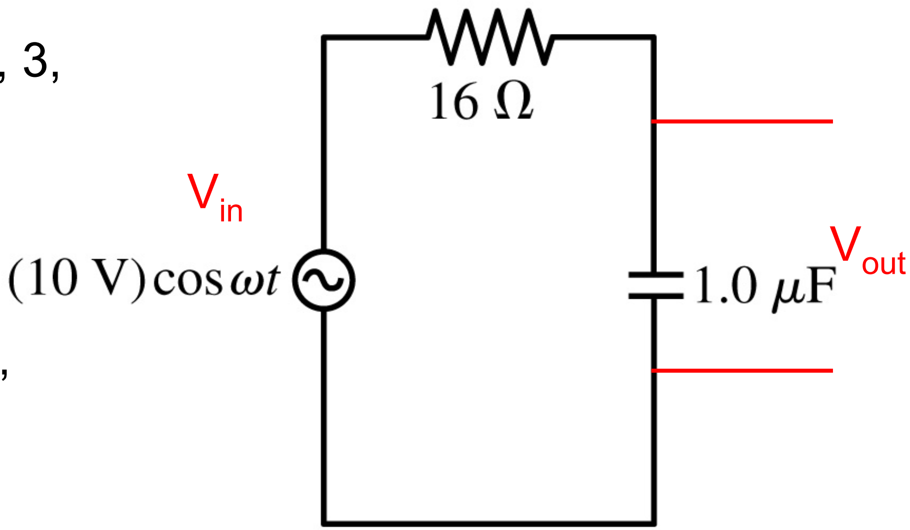
Problem 35-37

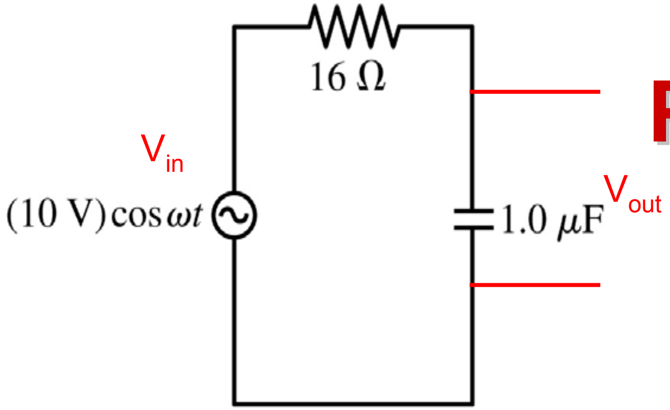
a) Evaluate V_C at emf frequencies 1, 3, 10, 30, and 100 kilo-Hertz

b) Graph V_C vs frequency.

This is an example of a "low pass filter."

Note that for $f=0$ (DC) the current must be zero, and therefore V_{out} is $V_{in.}$





Problem 35-37

$R := 16 \quad V_{in} := 10 \quad C := 1.0\cdot 10^{-6}$

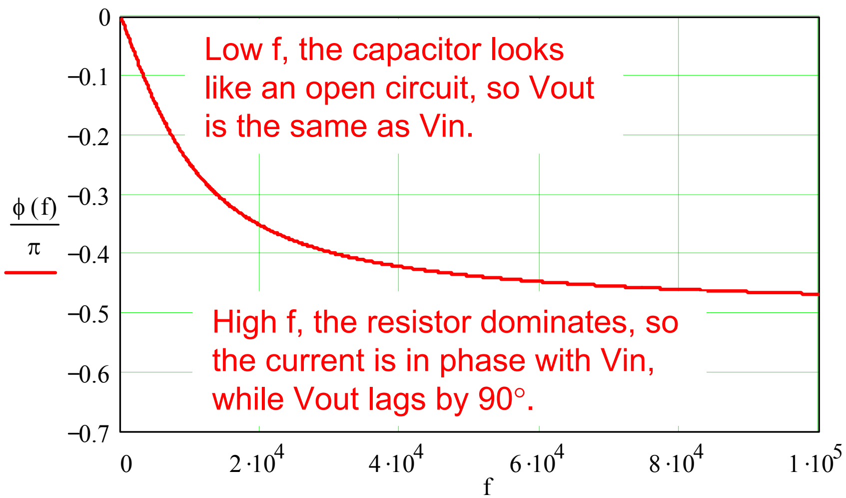
$$V_{out}(f) := V_{in}\cdot\frac{\frac{1}{2\cdot\pi\cdot f\cdot C}}{\sqrt{R^2+\left(\frac{1}{2\cdot\pi\cdot f\cdot C}\right)^2}}$$

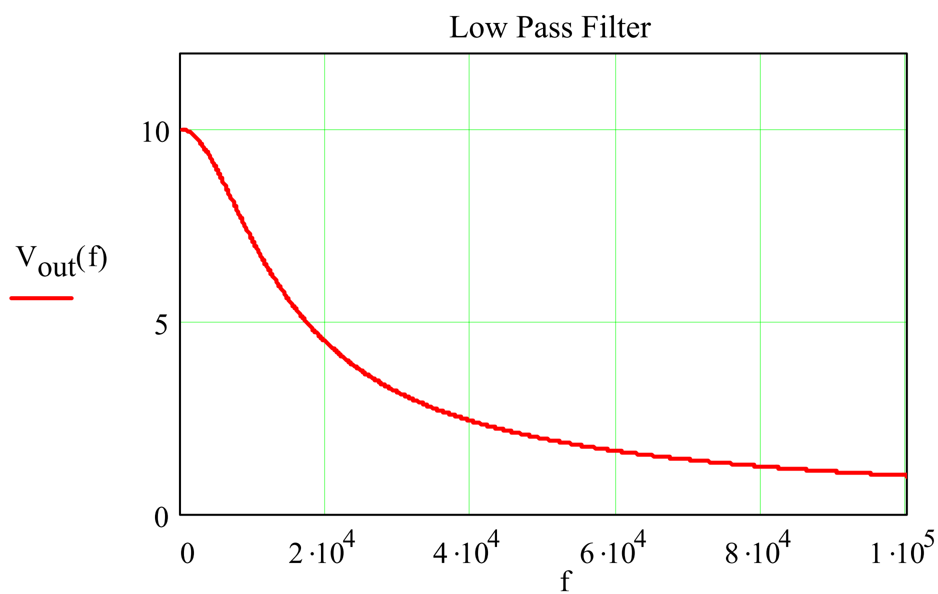
$V_{out}(1000) = 9.95 \quad V_{out}(10000) = 7.052 \quad V_{out}(100000) = 0.99$

$V_{out}(3000) = 9.574 \quad V_{out}(30000) = 3.147$

The voltage across the capacitor lags behind the current by 90°, so V_{out} has a phase angle

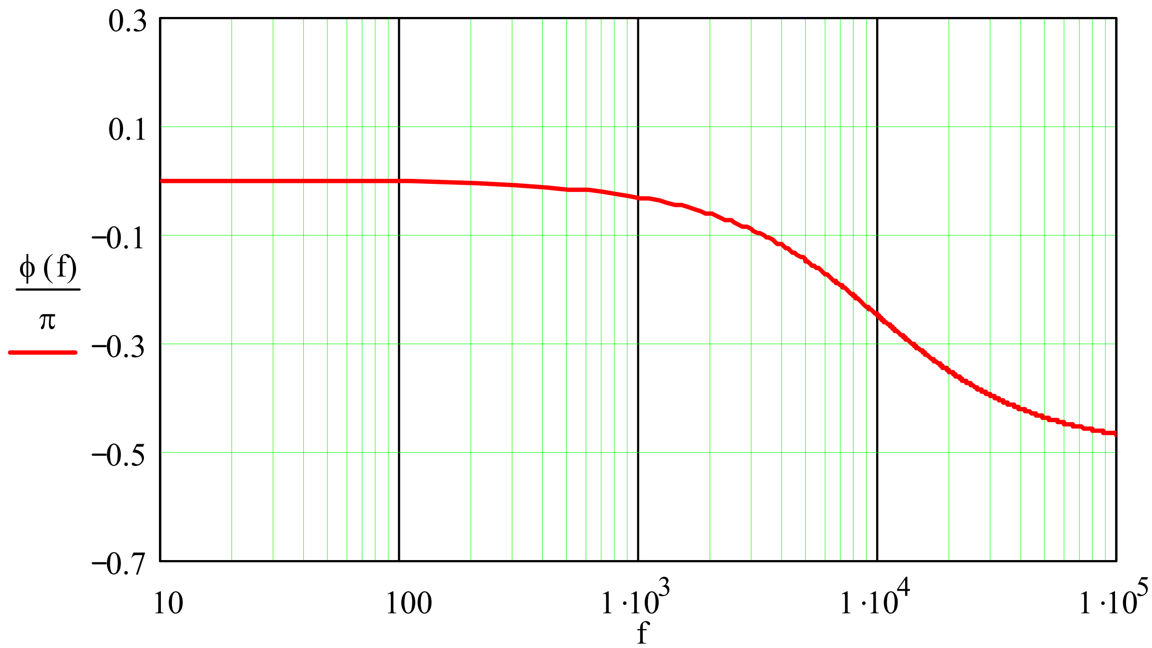
$$\phi(f) := \operatorname{atan}\left(\frac{1}{2\cdot\pi\cdot f\cdot R\cdot C}\right) - \frac{\pi}{2}$$





$$\phi(f) := \operatorname{atan}\left(\frac{1}{2\cdot\pi\cdot f\cdot R\cdot C}\right) - \frac{\pi}{2}$$

Problem 35-37

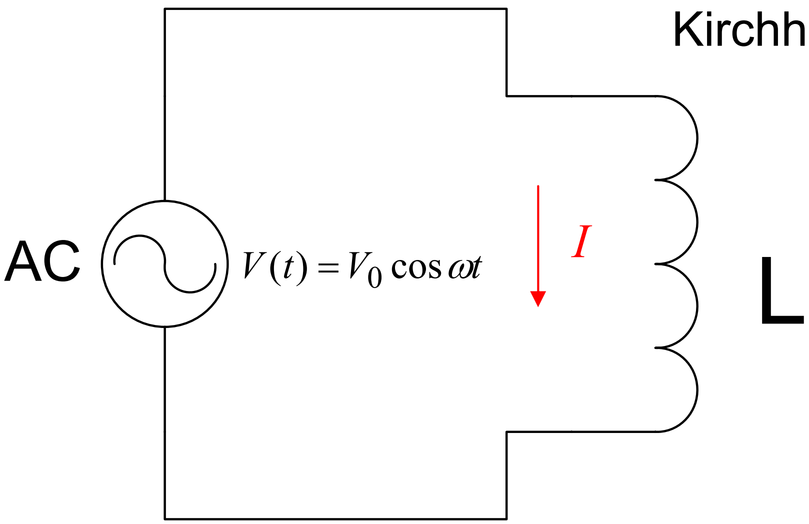


It is very common to plot frequency on a log scale. In that case, the same phase-angle plot looks as shown here.

The point where the phase angle is –45° is called the cross-over frequency. It occurs where

$$\omega_c = \frac{1}{RC} = 62.5\,\text{kHz or } f = 10\,\text{kHz}$$

AC Circuits: Inductance



Kirchhoff's rule: $V(t) + \mathcal{E} = 0 \qquad \mathcal{E} = -L\dfrac{dI}{dt}$

$$V = L\frac{dI}{dt}$$

$$I(t) = \frac{1}{L}\int V(t)dt$$

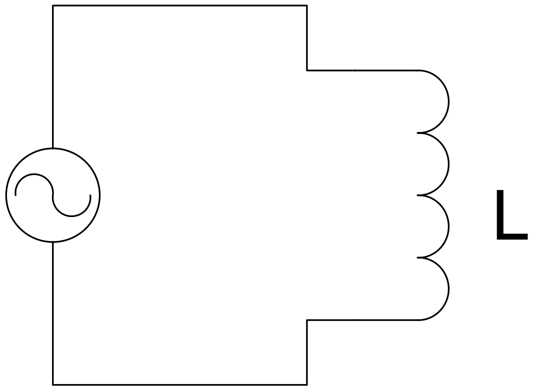
$$I(t) = \frac{V_0}{L}\int \cos \omega t \, dt$$

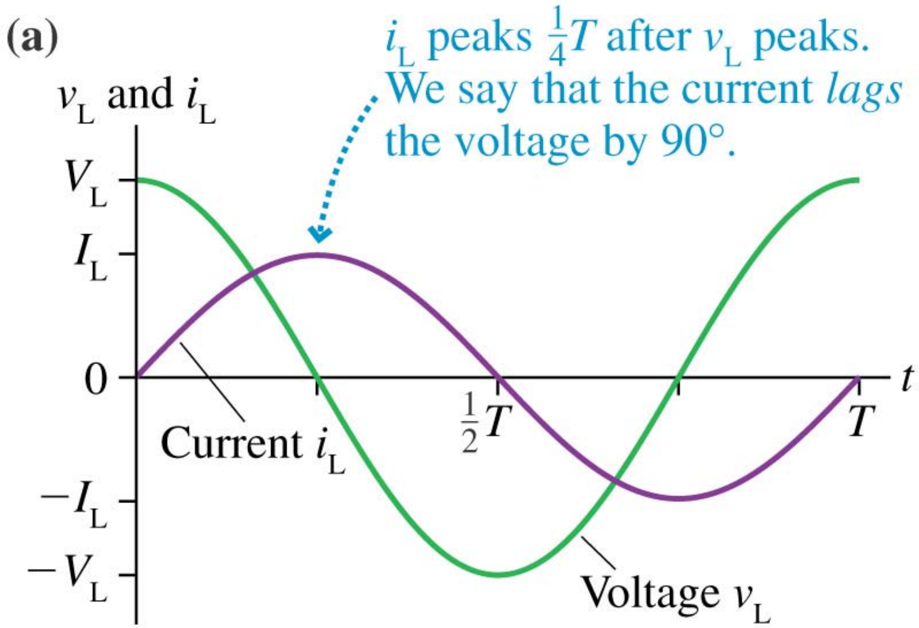
$$I(t) = \frac{1}{\omega L} V_0 \sin \omega t$$

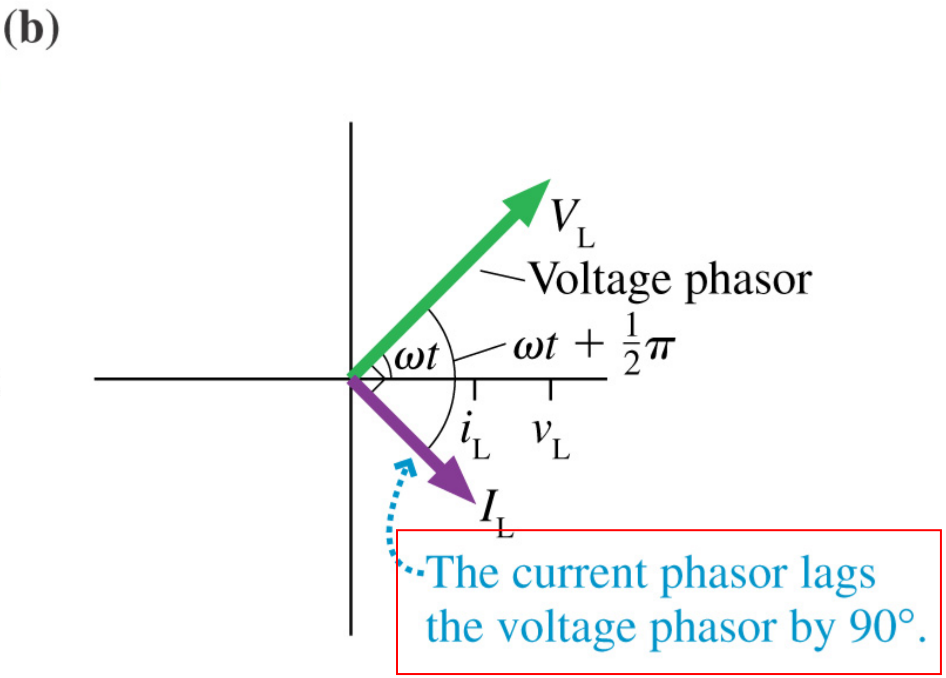
$$I(t) = \frac{V_0}{\omega L} \cdot \cos(\omega t - \tfrac{\pi}{2})$$

Energy is stored, NOT dissipated!

Phasors for Inductive Circuit







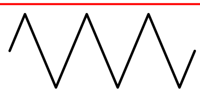
Reactance

All 3 elements provide an "impedance" (Z) to the flow of current, but one has to specify a phase difference between current and voltage as well as a change in amplitude.

Assume that the voltage is given by $V(t) = V_0 \cos \omega t$

$$I(t) = I_{max} \cos(\omega t + \phi)$$

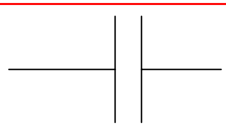
I and V in phase



$$I(t) = \frac{V_0}{R} \cos \omega t$$

$$Z = R \text{ and } \phi = 0$$

I leads V by 90°

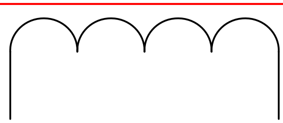


$$I(t) = \frac{V_0}{1/\omega C} \cos(\omega t + \tfrac{\pi}{2})$$

Reactance:

$$Z = X_C = \frac{1}{\omega C} \text{ and } \phi = +\tfrac{\pi}{2}$$

I lags V by 90°



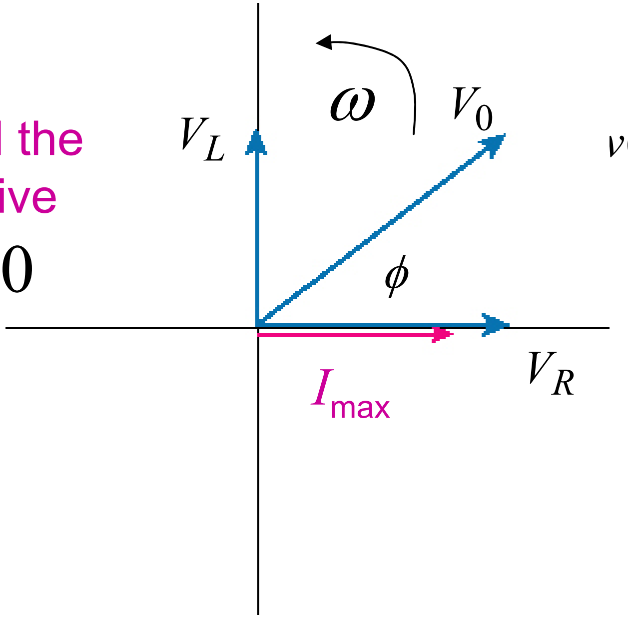
$$I(t) = \frac{V_0}{\omega L} \cos(\omega t - \tfrac{\pi}{2})$$

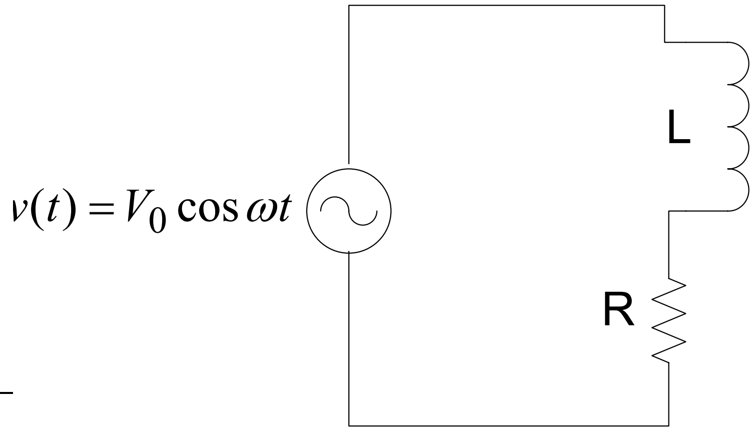
$$Z = X_L = \omega L \text{ and } \phi = -\tfrac{\pi}{2}$$

RL Circuit with AC Source

- Problem 35-47.

The current is lagging behind the voltage (negative phase). $\phi < 0$





$$i(t) = I_{\max} \cos(\omega t + \phi)$$

$$V_0 = \sqrt{V_R^2 + V_L^2} = I_{\max} \sqrt{R^2 + (\omega L)^2}$$

$$\phi = -\tan^{-1} \frac{V_L}{V_R} = -\tan^{-1} \frac{\omega L}{R}$$

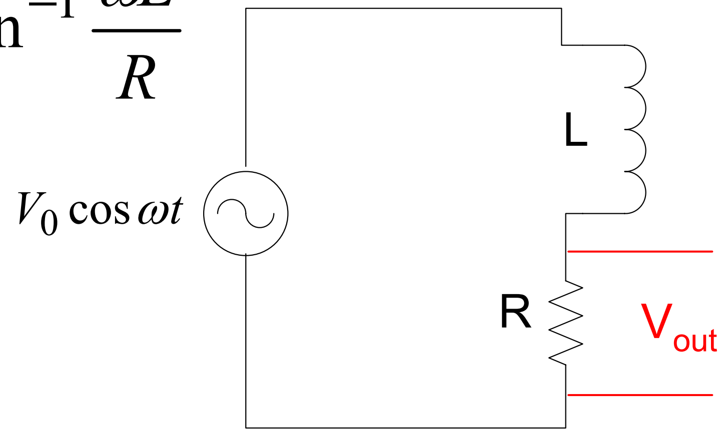
$$\text{Note}: \; Z = \sqrt{R^2 + X_L^2}$$

Problem 35-47, continued

$$i(t) = \frac{V_0}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t + \phi) \qquad \phi = -\tan^{-1}\frac{\omega L}{R}$$

$$v_R(t) = \frac{V_0 \cdot R}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t + \phi)$$

$$v_L(t) = \frac{V_0 \cdot \omega L}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t + \phi + \tfrac{\pi}{2})$$



Part b) V_R goes to zero as the frequency goes to infinity.

Part c) If V_{out}=V_R, then this is a low pass filter

Part d) The cross-over frequency occurs when R=X_L, or equivalently V_R=V_L. At that point, the output voltage is down by 1/sqrt(2) and the phase angle is –45°.

$$\omega_c = \frac{1}{R/L}$$