Physics 54

Lenses and Mirrors

And now for the sequence of events, in no particular order.
— Dan Rather

Overview

We will now study transmission of light energy in the **ray approximation**, which assumes that the energy travels in straight lines except when there is reflection, refraction., or interception by an obstacle. This approximation works well when the sizes of apertures and obstacles are large compared to the wavelength of the light.

Our main interest will be in formation of images by lenses and mirrors. We will also study the use of these in simple optical instruments.

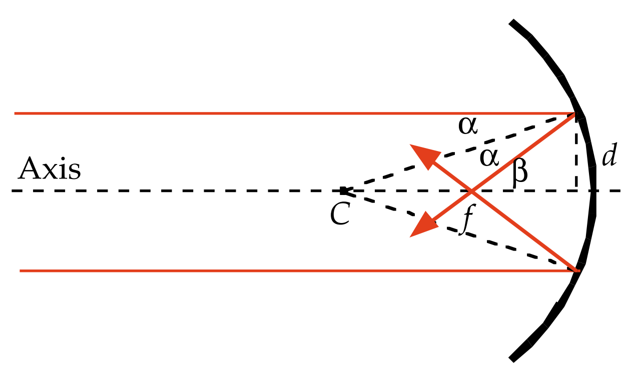
The lenses and mirrors will generally be assumed to have either plane or spherical surfaces. This simplifies the geometry. For the most part, we will further assume that the rays of light make small angles with the symmetry axis of the device. This **paraxial ray approximation** allows derivation of simple formulas for locating and describing images. Finally, we will assume that the indices of refraction of lenses are independent of the wavelength of light, ignoring the effects of dispersion.

Some "aberrations" that arise from violations of these approximations will be discussed briefly.

Mirrors

We consider mirrors made of a conducting material (so the reflection is essentially 100%) in the shape of part of a sphere. If the mirror surface is concave, the mirror is called "converging" or "positive" (for reasons to be made clear); if the surface is convex, the mirror is called "diverging" or "negative".

Consider first a concave mirror, shown in a side view. The line along the sphere's diameter is the symmetry axis. We consider two incident rays, parallel to the axis and close to it, so that the angles of incidence and reflection are small.



Point *C* is the center of the spherical surface, which has radius R. After reflection the two rays cross each other at a point on the axis. This is the **focal point** of the mirror. Its distance *f* from the mirror (the **focal length**) is obtained by some simple geometric arguments.

The two right triangles with opposite side *d* give (using the paraxial ray assumption that both angles are small)

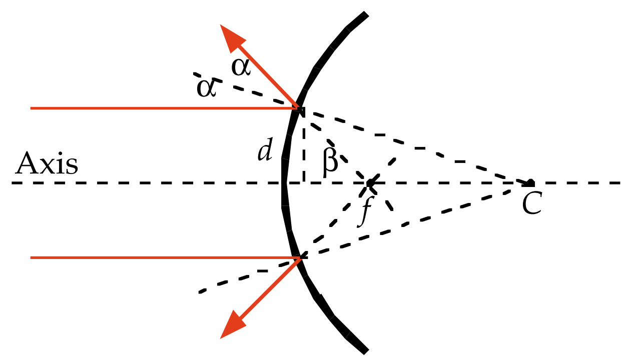
$$\alpha \approx \tan\alpha = d / R \,, \;\; \beta \approx \tan\beta = d / f$$

The angle of reflection is equal to the angle of incidence (α), so $\beta = 2\alpha$. Thus we find a simple formula for *f*:

Focal Length of a Mirror	$f = R / 2$

In this case the parallel rays converge to a focus, which is why the mirror is called "converging." Its radius *R* and its focal length *f* are assigned positive values in this case, which is why it is also called a "positive" mirror.

Next consider a convex mirror, as shown. The center of the sphere is on the side opposite to that where the light impinges and is reflected.



The reflected rays diverge *as though* they had come from a point behind the mirror. This focal point is **virtual**. Light does not actually come from this apparent source behind the mirror, but the brain of an observer viewing the reflected rays will interpret them as though they originated from that point.

Our sense of where an object is located comes from the capacity of our brain to project back diverging rays to their source, whether that source is "real" or "virtual".

The same argument used above leads to the same formula for *f* in terms of *R*. But because of the virtual nature of this focal point, *f* is defined to be negative, and *R* is correspondingly negative. This mirror is called "negative." Because it diverges parallel incident rays, it is also called "diverging."

In the sign convention widely used in geometric optics, positive distances represent "real" things, while negative distances represent "virtual" things. For mirrors, centers of curvature and focal points in front of the mirror are "real" and *R* and *f* are positive; centers of curvature and focal points behind the mirror are "virtual" and *R* and *f* are negative.

Image Formation by Mirrors

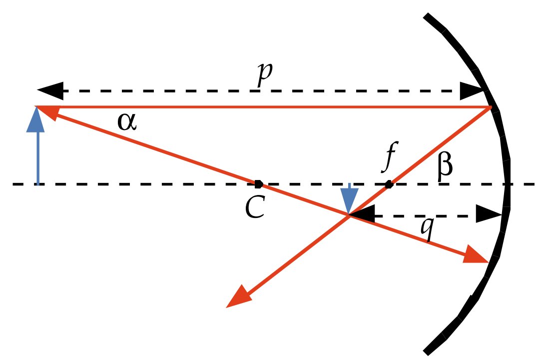
Images are of two types:

- **Real images**. Rays from a point in the object are converged by the optical system at a point in space, which is the corresponding point in the real image.
- **Virtual images**. Rays from a point in the object are diverged by the optical system as though they had emanated from a point in space, which is the corresponding point in the virtual image.

To locate the image point formed by a mirror, one uses two or three **principal rays**:

- A ray from the object point, passing through or toward the center of curvature. This ray strikes the mirror at normal incidence and is reflected straight back.
- A ray parallel to the symmetry axis. For a positive mirror, this ray is reflected through the real focal point. For a negative mirror it is reflected away from the virtual focal point.
- A third principal ray, passing through or toward the focal point, and emerging parallel to the axis, can also be used.

We consider a converging mirror, with an object (represented by the upright arrow) located at a distance from the mirror greater than the focal length. The two principal rays from the tip of the object converge to form the tip of the image (represented by the small inverted arrow). Since the rays do actually converge, this is a real image.



The distance of the object from the mirror (the **object distance**) is denoted by p; the distance of the image from the mirror (the **image distance**) is denoted by q.

Different textbook authors use different notations for these two distances. None are standard.

Using the right triangles formed with the two small angles shown, one finds after a short calculation that

Image Location Formula	$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

Let the height of the object be h and that of the image h'. Then one finds that $h'/h = q/p$. This ratio gives the "magnification" of the image relative to the object. It is customary to define the magnification with a negative sign to denote the fact that a real image is inverted relative to the object. Thus we have

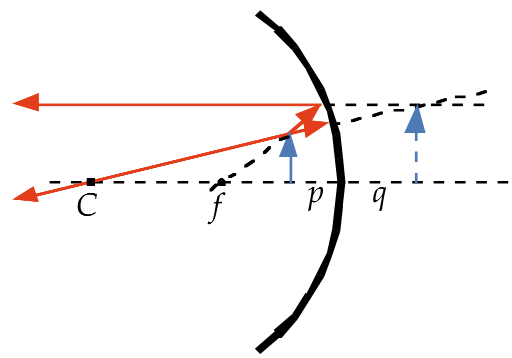
Lateral Magnification	$m = -\frac{q}{p}$

In the case shown, $q < p$ so the image is smaller than the object. (This is always the case when the image distance is greater than $2f$.)

Suppose the inverted arrow were the object. Then the rays would be the same, except reversed in direction. The upright arrow would then become the image. In this case, where the object distance is between f and $2f$, the image distance is greater than $2f$. The image is real, inverted (relative to the object) and enlarged.

This is an example of the usefulness of the "principle of reversibility", which says that reversing the directions of all rays gives another possible optical situation.

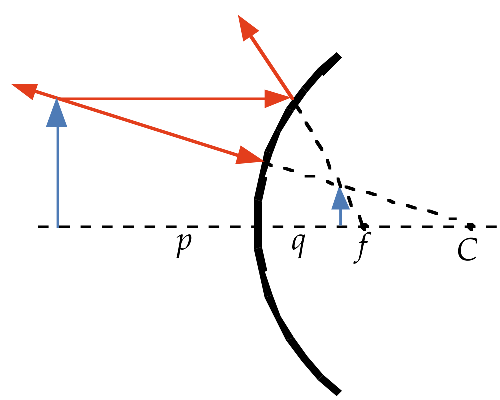
Things are different if the object is closer to the mirror than the focal point. Shown is such a case.



The rays diverge after reflection as though they had come from the tip of the dashed arrow. This is the *virtual* image. The image distance q is *negative*, because the image is behind the mirror. The image is erect (relative to the object) and enlarged. The details can be calculated using the formulas given above.

An object placed exactly at the focal point of a positive mirror results in reflected rays that are all parallel to the axis. The reflector mirror in a searchlight is an example of this. Conversely, an object at essentially infinite distance will produce an image at the focal point. The reflector mirrors in astronomical telescopes are examples.

A negative mirror always gives a virtual image of a real object. Shown is a case.



The virtual image is erect and reduced. The details can be calculated from the formulas, but one must remember that f is negative.

Surveillance mirrors in shops are of this type, as are the outside right mirrors in modern automobiles. Note that the image is always closer to the mirror than the object.

Thin Lenses

Lenses are systems made of a transparent material; their purpose is to manipulate light by refraction, usually to form images.

Our analysis will be restricted to lenses with spherical surfaces, and with thickness that are small compared to the radii of curvature of the surfaces. These are "thin" lenses.

One defines focal points for lenses in a way similar to that for mirrors. If, after passing through the lens, incident rays parallel to the axis are converged at a point, then that point is the focal point, and we have a converging or positive lens; its focal length (the distance from the lens to the focal point) is positive. If, after passing through the lens, the parallel rays diverge as though coming from a point on the same side of the lens as the incident light, then we have a diverging or negative lens; the focal length is negative.

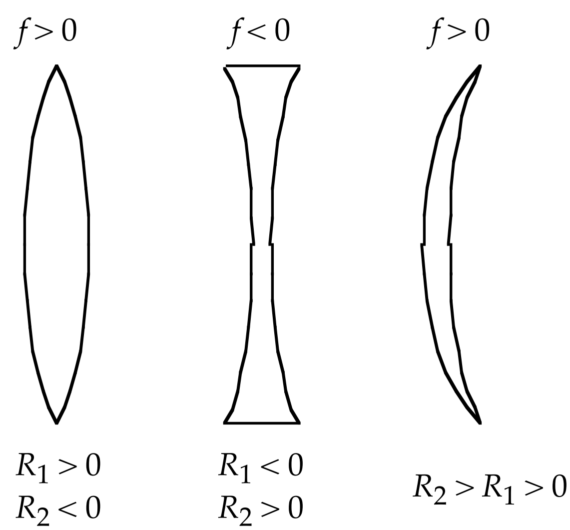
For paraxial rays one can show, using the law of refraction and small angle approximations, that the focal length is given by the following formula:

Lens Maker's Formula	$\frac{1}{f} = \left(\frac{n}{n_0} - 1\right) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Here n is the index of refraction of the substance from which the lens is made (usually glass or plastic), n_0 is the index of refraction of the transparent medium on either side of the lens (usually air, for which $n_0 = 1$). R_1 and R_2 are the radii of the two lens surfaces, for which there are sign conventions.

As one imagines light entering the lens, R_1 is the radius of the first surface encountered, while R_2 is the radius of the other surface. These numbers are positive if the surfaces are convex (as the light impinges on them); they are negative if the surfaces are concave.

For ordinary glass lenses in air, the following shows typical types



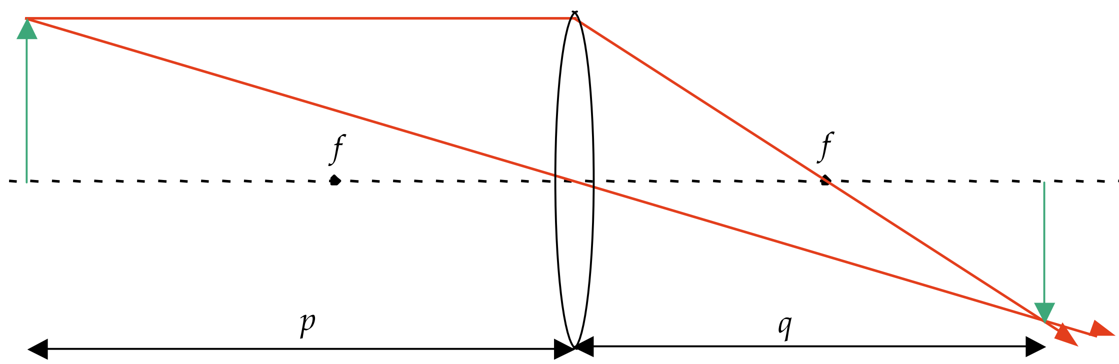
One sees a simple rule here: lenses that are thicker in the middle have positive focal length; those that are thinner in the middle have negative focal length.

Image Formation with Lenses

The procedure for locating images with lenses is similar to that for mirrors. The commonly used principal rays are:

- A ray from the object point to the center of the lens, where the two surfaces are parallel. This ray passes through essentially without deflection.
- A ray from the object point parallel to the axis. This is refracted through the focal point for a positive lens, or away from it for a negative lens.
- A ray passing through or toward a focal point emerges parallel to the axis. This is another principal ray.

Shown is a real image formed by a positive lens, with object beyond the focal point.

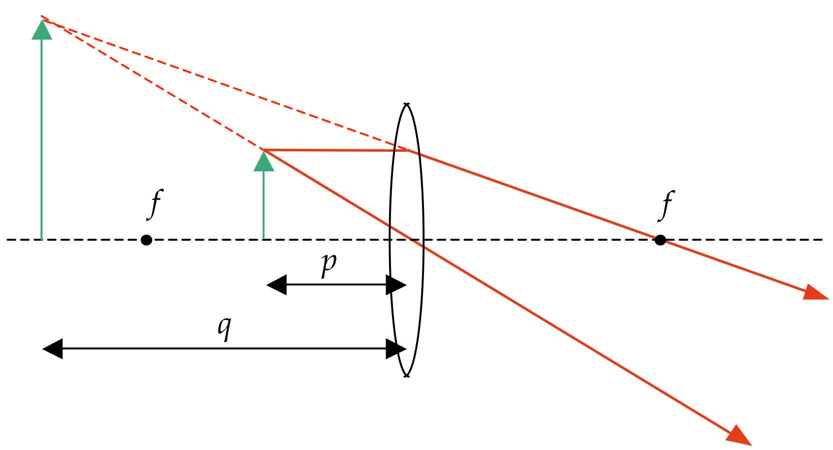


The analysis (in the paraxial ray approximation) gives the same formulas for location of the image (and for lateral magnification) as we had for mirrors.

In the case shown above, the object distance is greater than $2f$ so the image distance is less than $2f$, and the image is real, inverted and *reduced*. (To form a real and *enlarged* image, the object distance must be between f and $2f$.)

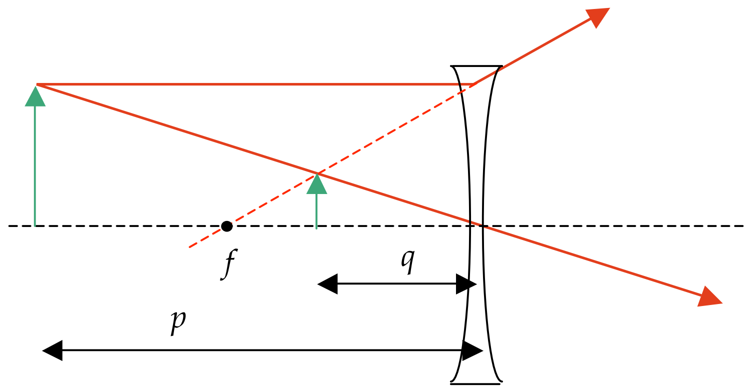
Most cameras form real, reduced images of objects more distant than $2f$. Slide projectors produce real and enlarged images on a distant screen, of objects between f and $2f$.

As with the positive mirror, an object placed closer to the lens than f will form a virtual image. The rays are as shown, forming an upright, enlarged virtual image.



The image is erect and enlarged. An example of this is production of an enlarged virtual image by a magnifying glass.

Like negative mirrors, negative lenses produce only virtual images of real objects. The images are erect and reduced, located between the object and the lens, as the following diagram shows.



In this case, both f and q are negative.

Aberrations

The formulas we have discussed are simple because of the approximations we made. Deviations from them are to be expected, and do occur. They are called **aberrations**.

Some of the aberrations have to do with inadequacy of the approximations used in deriving our simple formulas. Three of the most common problems are these:

- If the actual system has incident rays that are not paraxial, i.e., whose distance from the axis is not small compared to the radii of surfaces of lenses and mirrors, then our claim that all rays parallel to the axis will be brought to a single focal point is not valid. The resulting blurring of images is called *spherical aberration*. It can be reduced by putting a small aperture next to the lens, permitting only paraxial rays to enter. Of course this limits the amount of light admitted, and hence the brightness of the image. It also increases the effects of diffraction, as we will see later.
- If the optical system is not really axially symmetric, we have an aberration called *astigmatism*. This is a common defect of the eye.
- In the case of lenses, we have ignored the slight variation of the indices of refraction with wavelength (dispersion). As a consequence of dispersion, light waves of different wavelengths have different focal points. The resulting blurring of images is called *chromatic aberration*.

Professional optical systems often use multi-element systems to correct for aberrations.