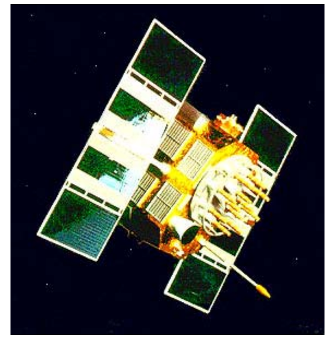
Navigating at the Speed of Satellites

Unit Topic: Navigation
Grade Level: 7th grade (with suggestions to scale for grades 6 to 8)
Lesson No. 8 of 10
Lesson Subject(s): The Global Positioning System (GPS), time, position.
Key Words: GPS, Satellite, Receiver, Speed of Light, Trilateration, Triangulation



Lesson Abstract —

Navigators have looked to the sky for direction for thousands of years. Today, celestial navigation has simply switched from natural objects to artificial satellites. A constellation of satellites, called the *Global Positioning System (GPS),* and hand-held receivers allow very accurate navigation. How do they work? The basic concepts of the system — trilateration and using the speed of light to calculate distances — will be investigated in this lesson. Lesson activities are:

- *State your Position* – students discover how several GPS satellites are used to find a position.
- *It's About Time* – students act out the part of the GPS signal traveling to the receiver to learn how travel time is converted to distance.

Lesson Opening Topics / Motivation —

The idea of using satellites for navigation began with the launch of Sputnik 1 on October 4, 1957. Scientists at Johns Hopkins University's Applied Physics Laboratory monitored the satellite. They noticed that when the transmitted radio frequency was plotted on a graph, a characteristic curve of a Doppler shift appeared. By studying the change in radio frequency as the satellite passed overhead, they were able to figure out the orbit of Sputnik. It turns out that you can use this same concept in reverse. If the satellite orbit is known, measurements of frequency shift can be used to find a location on the earth. Knowing the orbits of four satellites, as well as their distances away, a Global Positioning System — or GPS — unit can trilaterate a location. Activity 1, *State Your Position* shows how to do trilateration.

Navigation satellites are like orbiting landmarks. Rather than seeing these landmarks with our eyes, we "hear" them using radio signals. The Global Positioning System is a constellation (or set) of at least 24 satellites that continuously transmit faint radio signals toward the earth. These radio signals carry information about the location of the satellite and special codes that allow someone with a GPS receiver to measure distance to the satellite. Combining the distances and satellite locations, the receiver can find its latitude, longitude, and height. GPS satellite signals are free and available for anyone to use. GPS receivers are decreasing in cost every year and can be found in sporting good stores, are embedded in cell phones and even in watches.

How does the receiver know how far away the satellites are? Early on, scientists recognized the principle that, given velocity and the time required for a radio signal to be transmitted between two points, the distance between the two points can be computed. In order to do this calculation, a precise, synchronized time of departure and measured time of arrival of the radio signal must be obtained. By synchronizing the signal transmission time to two precise clocks, one in a satellite and one at a ground-based receiver, the transit time could be measured and then multiplied by the exact speed of light to obtain the distance between the two positions. This concept will be looked at in Activity 2, *It's About Time.*

Lesson Desired Student Outcomes —

Students will understand the basic concepts that make GPS work. They should understand that at least four satellites are needed to find a spot on the Earth and that knowing the distance to *more than* four satellites increases the accuracy of the position. Students will also learn how distance is determined by knowing the time it takes the signal to travel from the satellite to the receiver. They should learn that a delay in the signal's travel would make the receiver think the satellite is farther away.

Science: Students should be able to:

- Predict (hypothesize). (1)
- Use metric units. (1)
- Show the advantages and disadvantages of using a GPS unit. (5)
- Describe how using a GPS can determine our location. (5)
- Describe the effects of the atmosphere on GPS. (4)

Math: Students should be able to:

- Use numbers to count, label, and indicate distances on a map. (1)
- Measure distances and construct arcs. (5)
- Analyze error and its effect on real-world problems. (5)
- Graph a discrete linear function. (2)
- Fit data points to a line of best-fit. (3)

Colorado State Standards Met

http://www.mcrel.org/compendium/search.asp

- Science Standard 1, 4, 5

- Math Standard 1, 2, 3, 5

Lesson Background & Concepts for Teachers —

GPS – The Global Positioning System

GPS is based on satellite *ranging.* Our position on earth is calculated by measuring our distance from a group of satellites in space. This is done by timing how long it takes a radio signal to reach us from a satellite. The signal travels at the speed of light (186,000 miles per second), so we are able to calculate the distance (Velocity x Time = Distance. Refer to *Dead Reckoning* in Lesson 2).

GPS satellite ranging allows a receiver to figure out its 3-dimensional position: latitude, longitude, and height. Because the ranging measurements are based on timing, the times in the satellite transmitter and the user's receiver have to be coordinated. A GPS receiver measures range to four satellites to determine latitude, longitude, height and this timing correction.

Let's take this one step at a time. For now, assume that the satellite and receiver clocks are already coordinated, and the positions of the satellites are known. If we measure distance to one satellite, we know that we are located on a sphere of that radius, centered on the satellite. With two satellite range measurements, our location is limited to a circle and with three satellites to one of two points. A fourth satellite can be used to find the correct point and to take care of the time coordination.

If it has extra information, a receiver can figure out its position with fewer satellites. For example, if you know that you are on the ocean surface, you can use this piece of information and only three satellites to find your latitude, longitude, and timing. In this case, height is not needed because you already know it.

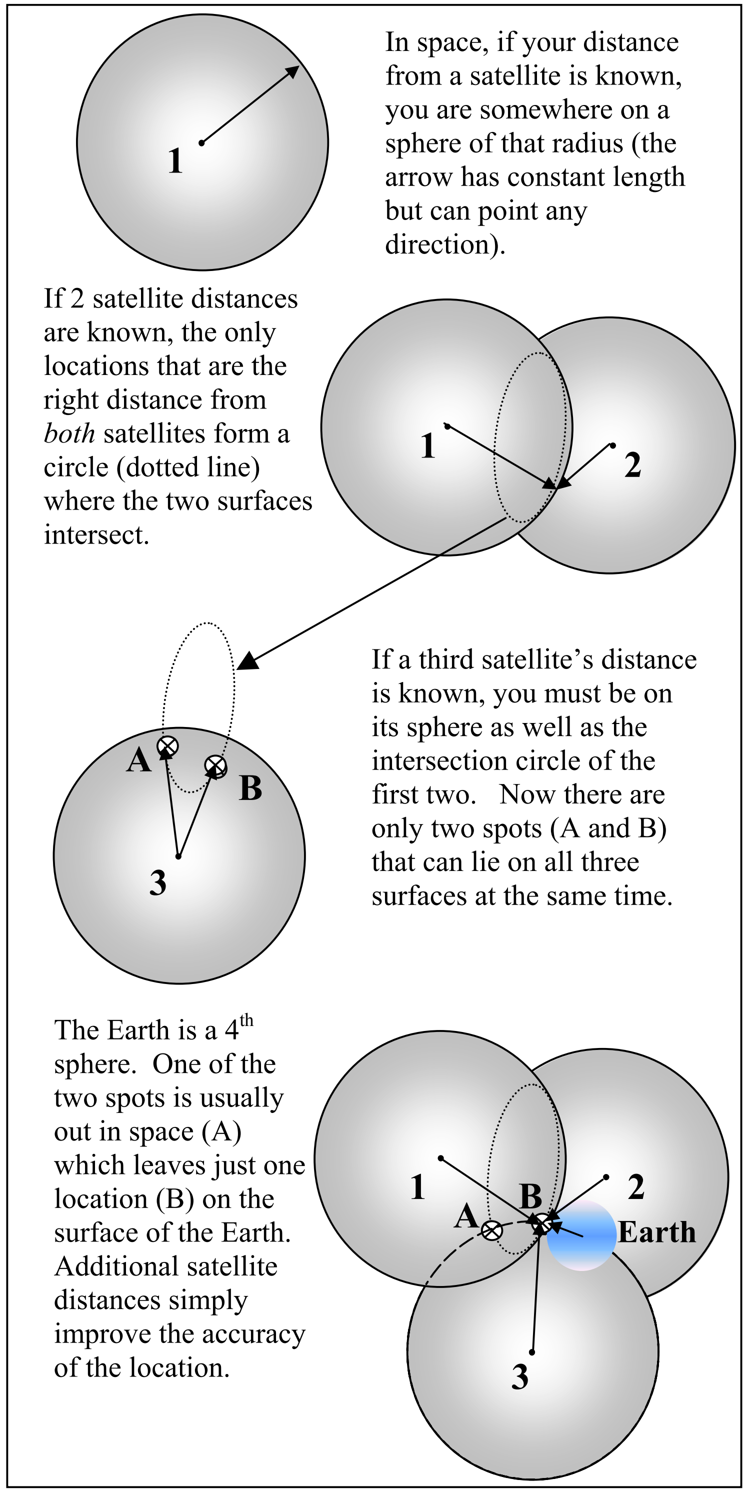


Diagram created by: Matt Lippis, University of Colorado, Boulder

So, how do we know where the satellites are? All satellites are constantly monitored. They have a 12-hour orbit and the DoD (Department of Defense) is able to monitor the satellites from ground stations around the world. The satellites are checked for errors in their position, height, and speed. These minor errors are caused by gravitational pulls from the moon, sun, and even pressure from solar radiation on the satellite. The satellites transmit special codes for timing purposes, and these codes carry a data message about their exact location. This helps to locate the satellite precisely.

Lesson Vocabulary List —

- **GPS** – The Global Positioning System.
- **Satellite –** An object launched specifically to orbit the Earth.
- **Receiver–** A device that receives incoming signals and converts them to a usable form.
- **Orbit** – The path an object in space follows as it circles the Earth.
- **Trilateration –** Position determined by intersecting distances.
- **Triangulation** – The location of an unknown point by the formation of a triangle.

Activity Attachments —

Activity 1: State Your Position – students see how several GPS satellites are used to find a position.

Activity 2: It's About Time – students act out the part of the GPS signal traveling to the receiver to see how travel time is converted to distance.

Lesson Closure and Follow-up —

These are the basic concepts of how a GPS receiver determines a location to the highest accuracy possible. In addition, the signal carries information about the best estimate of the satellite's orbit. The satellite does not know its own orbit; people on the ground have to track the satellite to see how the orbit changes over time. The orbit is predicted for several weeks ahead and this information is then sent to the satellite. If the orbits were not updated, the accuracy of the GPS system would deteriorate. Additionally, the GPS satellites have a certain lifetime on orbit, as their fuel and mechanical parts can only last so long. Satellites can last 10 to 20 years, and sometime longer, but they must be replaced eventually.

Should we throw away all the old navigation equipment? Does anyone need to learn all those complicated methods of celestial navigation? If you were traveling 100 years into the future, would you take a sextant or a GPS receiver? The GPS system is an easy and accurate navigation tool, but it should not be taken for granted.

Lesson Extension Activities —

- A demonstration could be done in the classroom (with three very long pieces of string) to find a hidden item in the class. The instructor must measure and hide everything ahead of time.

- Research at home or in a library how the GPS system has changed an aspect of navigation. Present results in class.
- Have students come up with their own trilateration measurements on a map or in the classroom.
- Set a globe on top of the map. Have students notice that in 3-dimensions, three satellite distances give one possible solution on the globe and one out in space! (Note: you can demonstrate this without numbers and adjust it as you proceed, or you must measure the three distances ahead of time if you want to give the students numbers that work.)

Lesson Assessment and Evaluation —

Pre-Lesson Assessment

- Discussion Question: Solicit, integrate, and summarize student responses
  - Why are satellites a good tool for navigation? (Answer: They are "visible" for thousands of miles. Their orbits, and therefore positions, can be tracked to a high degree of accuracy. They can send information as well as simple location data in their signals.)

Post-Introduction Assessment

- Question/Answer:
  - Is it hard to understand how satellites work? Why or why not? (Answer: Have several students answer and discuss that the satellite is a tool, and while its inner workings may be complicated, its basic actions and uses are not hard to understand. Tell them they will learn more about the workings of satellites in this lesson.)

Post-Lesson Assessment

- Discussion Question:
  - If you were traveling 100 years into the future, would you take a sextant or a GPS receiver? Discuss. (Answer: Open question. Possible reasons for not taking a GPS receiver would be: GPS system was not kept up or fails in the future; a new better system has been put in place, and the old receiver is not compatible; if the GPS receiver breaks, the system is useless. Reasons to take a sextant: no batteries required, sun moon and stars will always be there, much easier to fix if something on it breaks.)

Homework

- Internet Search: Instruct students to look up some of the concepts from the lesson on the Internet. Lead a brief discussion of findings in the next class.

Lesson References —

http://rst.gsfc.nasa.gov/Sect16/ Sect16_11.html

http://www.nps.gov/ncrc/portals/rivers/hydro_proj_map.html

Other interesting sites to learn about GPS:

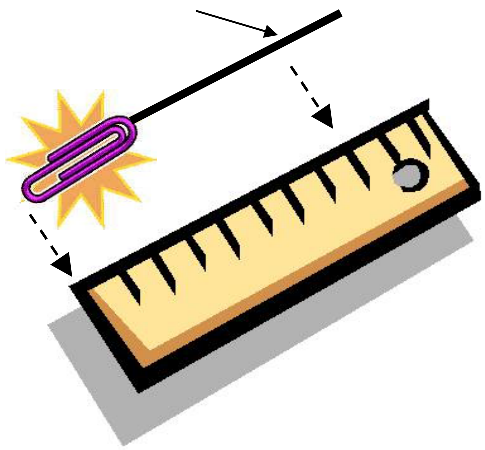
GPS News
http://www.igeb.gov/

About GPS
http://www.aero.org/publications/GPSPRIMER/index.html
http://www.colorado.edu/geography/gcraft/notes/gps/gps_f.html

GPS Applications Information
http://www.nasm.si.edu/galleries/gps/work.html
http://gps.losangeles.af.mil/

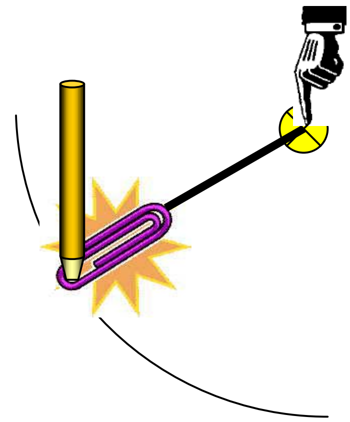
Where am I?

You've been dropped off at mystery spot somewhere in the United States! Your GPS unit has malfunctioned: it gives you distances from the satellites and where the satellites are, but won't do the calculation to find your position (and today is the one day you left your sextant at home). Fortunately, you have a U.S. map and quickly program the unit to give you the satellite distances relative to your map (nice work!). Time for some old fashioned Triangulation…



Arc Instructions: When you get the distance data from a satellite (see below) have one person in the group measure and mark that distance on your string tied to the paper clip (including the paper clip).

The second group member will hold the marked spot on the string to the circled "x" by the correct satellite. Now your paper clip should swing in an arc across your map. Have the third group member insert a pencil into the end of the paper clip and draw the first arc holding the string tight. (Then switch with a partner so they can make the next arc.)



Turn on the GPS receiver and let's get started:

1. Your receiver has picked up data from Satellite 1. You are approximately 12 cm from Satellite 1… make an arc using the method above and then list all the States that you may be in (every state the arc crosses). ____

   ____

2. Ah – you've locked onto Satellite 4. It is 15 cm from your position. Make another arc. Which States might you be in now (where do the two arcs cross)? Remember – these distances may be off by 2-3 mm so if you are near a border you should include both States as possible locations!

   ____

3. Finally – Satellite 2 data! 18 cm away… Do that arc thing again. Why don't the three arcs cross at exactly the same point?

   ____

   What if one of the signals had a large error and was off by 5-6 mm? Which States are still a possibility for your location? ____

4. Sometimes it is good to have 4 satellites locked in – you can get a much more accurate position. Satellite 3 distance pops up: 9 cm. Can you confidently name your location now?

   ____

Use the table on the back of this sheet to help locate some friends with the GPS data they have supplied!

Where Are They?

Note: Distances may be off by +/- 0.5 cm to make the triangulation slightly more challenging.

Name	Distance to Satellite 1 (cm)	Distance to Satellite 2 (cm)	Distance to Satellite 3 (cm)	Distance to Satellite 4 (cm)	Which State?
YOU!	12	18	9	15	

George	5	20	13	22	
Patricia	23	15	16	3	
Shawn	12	14	13	15	

Isaac	17	9	17	11	
Sarah	21	7	20	11	
Carla	14	15	11	12	

Oscar	17	12	14	10	
Olivia	19	11	15	8	
Lin	9	23	9	21	

BONUS CONVERSION: The actual accuracy of typical commercial GPS receivers (with 4 satellites locked) is roughly 5 meters. On the scale of this map, 5 meters is what fraction of a cm?

Hint: 5 meters should be equal to a VERY small fraction of a centimeter and don't forget to convert km to meters!

ANSWER KEY

Note: Distances may be off by +/- 0.5 cm to make the triangulation slightly more challenging.

Name	**Distance to Satellite 1 (cm)**	**Distance to Satellite 2 (cm)**	**Distance to Satellite 3 (cm)**	**Distance to Satellite 4 (cm)**	**Where am I?**
YOU!	12	18	9	15	Colorado
George	5	20	13	22	Washington
Patricia	23	15	16	3	Florida
Shawn	12	14	13	15	South Dakota
Isaac	17	9	17	11	Michigan
Sarah	21	7	20	11	New York
Carla	14	15	11	12	Kansas
Oscar	17	12	14	10	Illinois
Olivia	19	11	15	8	Kentucky
Lin	9	23	9	21	California
Add more states (or other specific locations on the map) to the list:					
	17	18	8	11	Texas
	17.8	18.1	8.4	9.7	Austin

You can have students find any point on the map – just print out a map, measure, and record the distances ahead of time.

BONUS CONVERSION: The actual accuracy of typical commercial GPS receivers (with 4 satellites locked) is roughly 5 meters. On the scale of this map, that accuracy would correspond to 0.000025 cm!
~ 2.5 cm = 500,000 m and X cm = 5 m. X = 2.5*5/500,000 or 2.5/100,000 = 0.000025 ~
To visualize this, look at a 1-millimeter division on your ruler and imagine that it is divided into 4000 more divisions! Wow!

Courtesy of The General Libraries, The University of Texas at Austin.

Activity: It's About Time!

This activity is planned for 28 students working in groups of 2.

Activity Materials List —

- 14 stopwatches or timekeeping devices that can count seconds.
- 28 Time Worksheets

Activity Equipment and Tools List —

- Each student needs a pencil

Activity Cost Estimate —

$0

Activity Attachments —

Time Worksheets

Activity Time Estimate —

40-50 min.

Activity Procedure —

A. Background:

GPS – The Global Positioning System

In the old days, ocean navigators used to throw a piece of wood over the side of the ship and count the time it took the wood to be passed by the ship. They would then use this time and the length of the ship to calculate the speed of the ship get an estimate of how far they had traveled. Would this same basic idea work for the signals of a satellite navigation system? (Answer: Absolutely. In fact it works better because we already know the speed of the signal is the speed of light — a constant.)

GPS is based on satellite ranging. Our position on earth is calculated by measuring our distance from a group of satellites in space. This process, called trilateration, is done by timing how long it takes a radio signal to reach us from a satellite. The signal travels at the speed of light (186,000 miles per second), so we are able to calculate the distance (Velocity times Time equals Distance – remember Dead Reckoning?). If the signal is delayed for even a 1/1000 of a second, the distance will be off by 186 miles.

So how do we know where the satellites are? (Answer: All satellites are constantly monitored.) They have a 12-hour orbit and the DoD (Department of Defense) is able to monitor the satellites from ground stations around the world. The satellites are checked for errors in their position, altitude, and speed. These minor errors are caused by gravitational pulls from the moon, sun, and even pressure from solar radiation on the satellite. The satellites transmit special codes for timing purposes and these codes carry a data message about their exact location. This helps the DoD to locate the satellite precisely.

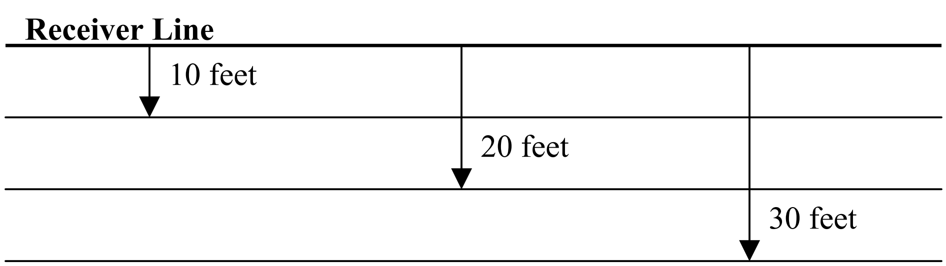
There can be errors in reading a GPS location because of various reasons. The Earth's ionosphere and atmosphere can cause delays in the signal, which make the distances seem longer than they should be. The ionosphere is a blanket of electrically charged particles 80 to 120 miles above the earth and the atmosphere can have varying amounts of dust, water vapor (clouds), and other particles floating around.

How else is the signal slowed down? Picture the signal coming into a raindrop at an angle; it travels slightly slower through water than air. The left edge of the signal wave encounters the raindrop first, and slows down. The point of the signal just to the right travels a bit farther at the higher speed, but then it encounters the material also, and slows down. This happens all along the wave. The signal can be refracted and reflected as it goes though a raindrop that scatters parts of the wave and weakens the signal.

Satellite clock errors, receiver errors on earth, and signal reflection can also cause errors in final position. In 2002, a typical receiver could tell you your latitude and longitude to within 20 ft.

B. Before the activity:

1. Print out worksheets for each student.
2. In an area like a playground, field, or gym where the students can run (or make them speed walk if you prefer), mark off four parallel lines. The first line is where the "receivers" will sit facing away from the other lines. Each receiver should have enough space that they can reach out toward their neighbors and not touch. The first line behind the receiver line should be 10 feet away. The next, 20 feet from the receiver line. And the last line should be 30 feet away from the receiver line.



C. With the Students:

1. Run voting assessment as directed in the assessment section below.
2. Give each student a Time Worksheet.
3. When the students have decided whom is the first "receiver" (or decide for them) give that student a stopwatch (or other time device that counts seconds).
4. Take the students to the lined area and have them follow and record their data on the worksheets.

5. Have students plot the data with the X-axis being time and the Y-axis being feet. (These three data points may not be linear. If not, have students estimate a line of best fit.)
6. Have students make predictions on how much time it would take for the "signal" to travel 15, 25, 50, and 100 feet.
7. Ask Question/Answer assessment as directed in assessment section below.

Math Skills Reinforced —

6th, 7th and 8th: Students will graph data points and apply estimation and prediction to real-life situations. Students will also examine error propagation (time delay).

Activity Troubleshooting Tips —

The graph that students make will probably be quasi-linear (it will not make a straight line). Have students make a line that comes closest to all three of them (a line of best fit).

Activity Desired Student Outcomes —

After this activity, students should understand that GPS distance is determined by knowing the time it takes a signal to travel from a satellite to the receiver and multiplying that time by the speed of light. They should learn that a delay in the signals travel will increase the time and make the receiver think the satellite is farther away.

Activity Assessment & Evaluation —

Pre-Activity Assessment

- Voting: Ask a true/false question and have students vote by holding thumbs up for true and thumbs down for false. Count the number of true and false and write the number on the board. Give the right answer.
    - In the old days, ocean navigators used to throw a piece of wood over the side of the ship and count off the time it took the wood to be passed by the ship. They would then use this time and the length of the ship to calculate the speed of the ship to get an estimate of how far they had traveled. Would this same basic idea work for the signals of a satellite navigation system? (Answer: Absolutely. In fact, it works better because we already know the speed of the signal is the speed of light — a constant.)

Activity Embedded Assessment

- Calculations: Students follow and complete the worksheet.

Post-Activity Assessment

- Question/Answer: Have student raise their hands with the correct response.
    - Is it possible for the signal to arrive earlier than expected? (Answer: Not if the satellite is in the orbit you think it is in: the speed of light is absolute and cannot be exceeded. However, if the satellite's orbit were closer than you think it is, the signal would *seem* to arrive early. Hopefully you are locked into enough other satellites' signals to know that this one has an error.

Suggestions to Scale Activity for Grades 6 to 8 —

6th Grade: Do as is.

7th Grade: Have students figure out what the average travel time is for a GPS signal if the orbit is 20,000 km and the Earth's radius is about 7,000 km. (Answer: 13,000 km at 300,000,000 km/sec = 0.000043 seconds.)

8th Grade: After students have gotten the hang of the activity, have them all sit in a very large circle facing in, with a wall or obvious landmark in one or more directions. Have one student volunteer to be the "receiver" and then pick three students spaced around the circle to stand up and be "signals." The receiver will be able to look at where the three signals are standing but will then be blindfolded. The receiver will then be taken in a round about way (be careful not to make them dizzy) to a random spot in the circle and sat down. When ready, the receiver yells "GO!" The signals all run to the receiver as directly and quickly as possible saying "Here!" when they arrive. The goal of the receiver is to determine from the arrival times of the signals, roughly where they are in the circle. Unlike a normal receiver, the student will have the advantage of hearing the *direction* of the signals arrival. Can they also figure out which way they are facing?

Its About Time

Name: ___________________ Date: ____________________

Take turns being the "Receiver" and the "Signal" as follows:

Receiver

When it is your turn to be the receiver, sit facing away from the signal lines and your partner. When ready with the timing device, without turning around or looking back, start timing and yell, "GO!" at the same time. When your partner arrives next to you and says, "Here!" – stop timing and record your data in the table below. The first three times, your partner will tell you where they are starting, and you can learn their pace. The last four times, your partner will randomly choose a line at which they started. You must try to guess how far away they started!

Signal Starting Point	Time To Arrival
10 feet	
20 feet	
30 feet	
?_______________	
?_______________	
?_______________	
?_______________	

Signal

When it is your turn to be the signal, start at the 10-foot line and tell your partner you are starting there. When they are ready and yell go, run directly and quickly up to them or past them. Yell "Here!" when you arrive or pass them. Repeat for the 20 and 30-foot lines again letting the receiver know where you are starting. On the fourth and fifth runs, you may start from any of the three lines. Do not tell the receiver where you are starting, and do not give away your position with noise either! When the receiver yells go, run to them at the same pace as you did for the first three times. (Remember the speed of light does not change.)

On the sixth and seventh times, drop your pencil flat on the ground half way between your start line and the receiver. This time when the receiver yells go, run quickly to the receiver as before. But this time, you must stop and pick up the pencil on the way. How did this affect the time and the distance estimate?

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