# Minimum Shift Keying

Mattias Svanström

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**Abstract**

We investigate two possibilities of detecting a minimum-shift keying signal: either as a BFSK signal or as two BPSK signals. We also consider how to allocate signals to information bits in order to avoid error propagation.

## 1 Continuous-Phase FSK

In traditional FSK we use signals of two different frequencies $f_0$ and $f_1$ to transmit a message $m = 0$ or $m = 1$ over a time of $T_b$ seconds,

$$s_0(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_0 t, \qquad 0 \leq t < T_b;$$

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t, \qquad 0 \leq t < T_b.$$

We assume that $f_0 > f_1 > 0$. If we choose the frequencies so that in each time interval $T_b$ there is an integer number of periods,

$$f_0 = \frac{k_0}{T_b}; \qquad f_1 = \frac{k_1}{T_b},$$

with $k_0$ and $k_1$ integers, the signal is guaranteed to have continuous phase. Figure 1 shows an example of a signal that is discontinuous, a signal with discontinuous phase and a signal with continuous phase. As phase-continuous signals in general have better spectral properties than signals that are not phase-continuous, we prefer to transmit signals that have this property.

If either $f_0$ or $f_1$ are chosen such that there is a non-integer number of periods the traditional FSK modulator will output a signal with discontinuities in the phase. In order to maintain phase continuity, we can let the transmitter have memory. We choose the signals for a general CPFSK transmitter to be

$$s_0(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_0 t + \theta(0)), \qquad 0 \leq t < T_b;$$

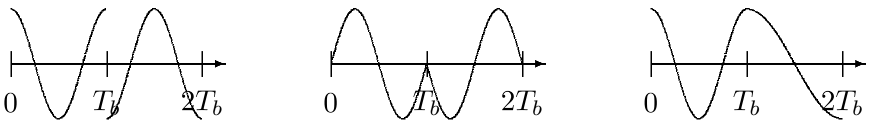


Figure 1: SIGNALS WITH DIFFERENT DEGREES OF DISCONTINUITY.

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t + \theta(0)), \qquad 0 \leq t < T_b.$$

We keep the phase continuous by letting $\theta(0)$ be equal to the argument of the cosine pulse for the previous bit interval. For the signals over an arbitrary bit interval, $kT_b \leq t < (k+1)T_b$, the general phase memory term is $\theta(kT_b)$.

As we are using two different frequencies, we can express the CPFSK signal as a baseband signal with respect to an arbitrary center frequency $f_c$. We choose $f_c$ to be the arithmetic mean of $f_0$ and $f_1$. Define

$$f_c = \frac{f_0 + f_1}{2}; \qquad f_d = \frac{f_0 - f_1}{2}.$$

We have

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta(0) \pm 2\pi f_d t), \qquad 0 \leq t < T_b, \tag{1}$$

where $+$ corresponds to the transmission of $f_0$ and $-$ corresponds to the transmission of $f_1$. We expand equation (1) in its baseband quadrature components,

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(\theta(0) \pm 2\pi f_d t) \cos 2\pi f_c t - \sqrt{\frac{2E_b}{T_b}} \sin(\theta(0) \pm 2\pi f_d t) \sin 2\pi f_c t =$$

$$= s_I(t) \cos 2\pi f_c t - s_Q(t) \sin 2\pi f_c t, \qquad 0 \leq t < T_b. \tag{2}$$

We define the *deviation ratio* $h$ of this system as

$$h = k_0 - k_1 = T_b(f_0 - f_1),$$

so that we have an alternative way of expressing the frequency difference

$$2\pi f_d t = \frac{\pi h t}{T_b}.$$

With these definitions we have a phase difference over one bit interval, with respect to the phase of a signal with frequency $f_c$,

$$\theta(T_b) - \theta(0) = \begin{cases} +\pi h, & \text{when transmitting } f_0; \\ -\pi h, & \text{when transmitting } f_1. \end{cases} \tag{3}$$

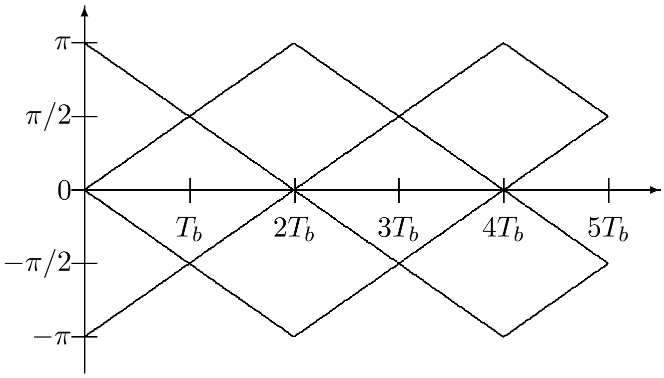


Figure 2: Phase trellis for $h = 1/2$.

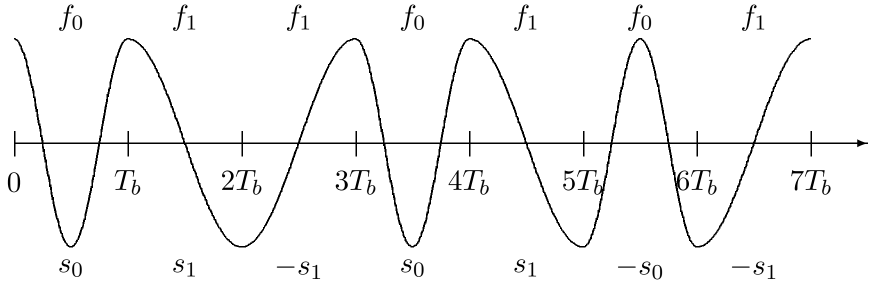


Figure 3: Example of an MSK signal.

If we use (3) and count the phase modulo $2\pi$, we can depict the phase variation over time in a *phase trellis*, see Figure 2. In Figure 2 we have assumed $h = 1/2$ and $\theta(0) = 0$ or $\theta(0) = \pi$. We see that for every multiple of the bit time the phase can only take on one of two values, the values being 0 and $\pi$ for $t = 2kT_b$, and $\pm\pi/2$ for $t = (2k+1)T_b$).

Continuous-phase FSK with deviation ratio $h = 1/2$ is called *minimum shift keying,* MSK. The frequency difference

$$f_0 - f_1 = \frac{1}{2T_b}$$

that results from choosing $h = 1/2$ is the smallest possible difference if the signals of the two frequencies are to be orthogonal over one bit interval. An example of an MSK signal with $k_0 = 1$ and $k_1 = 1/2$ is given in Figure 3.

## 2 Detecting MSK as BFSK

We note that $h = 1/2$ implies that if

$$f_0 = \frac{k_0}{T_b},$$

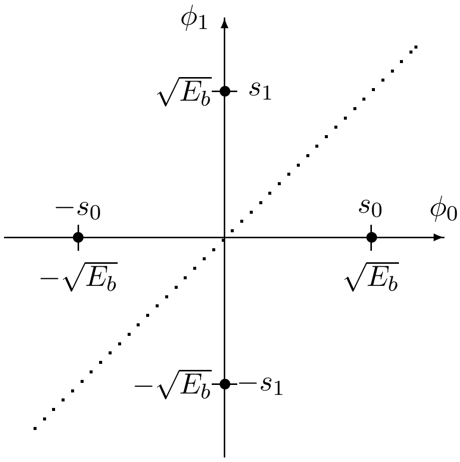


Figure 4: Signal space diagram when decoding MSK as BFSK.

we have

$$f_1 = \frac{k_0 - 1/2}{T_b}.$$

Thus, if one of the signals is an integer number of periods, the other is an integer number of periods plus one half period. From the phase-continuity requirement we conclude that the system uses four signals, but the transmitter is restricted to choosing one out of two signals in each bit interval. This is also indicated by the phase trellis, as there are two possible phase values at the beginning of each bit interval and from each of these there are two possible choices for the phase: increasing or decreasing.

Assume that $\theta(0) \in \{0, \pi\}$ and assume for simplicity that $k_0 = 1$. For the analysis of BFSK detection of MSK, we define the signals

$$s_0(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_0 t, \qquad 0 \le t < T_b;$$

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t, \qquad 0 \le t < T_b.$$

The transmitter uses one of the four signals $\pm s_0(t - kT_b)$ and $\pm s_1(t - kT_b)$ in bit interval $k$, as indicated in Figure 3. We have the signal space diagram of Figure 4, and we can choose decision areas as indicated, since we in each interval either decide between $s_0(t - kT_b)$ and $s_1(t - kT_b)$ or between $-s_0(t - kT_b)$ and $-s_1(t - kT_b)$ depending on the previously transmitted signal. For the details, see Figure 4 and Table 1.

## 3 Detecting MSK as BPSK

BFSK detection of MSK is perhaps the most natural first choice for a detector principle. However, we can do even better than that by using BPSK detection.

| Previous signal | Frequency $f_0$ | Frequency $f_1$ |
|---|---|---|
| $s_0(t-(k-1)T_b)$ | $s_0(t-kT_b)$ | $s_1(t-kT_b)$ |
| $-s_0(t-(k-1)T_b)$ | $-s_0(t-kT_b)$ | $-s_1(t-kT_b)$ |
| $s_1(t-(k-1)T_b)$ | $-s_0(t-kT_b)$ | $-s_1(t-kT_b)$ |
| $-s_1(t-(k-1)T_b)$ | $s_0(t-kT_b)$ | $s_1(t-kT_b)$ |

Table 1: Active signal set depending on the previous signal.

We choose to study the signal from equation (2) over *two* consecutive bitintervals. Let $\pm_{(k)}$ denote $+$ if $f_0$ is transmitted for $kT_b \leq t < (k+1)T_b$ and $-$ if $f_1$ is transmitted in the same interval.

For $-T_b \leq t < T_b$ we have

$$s_I(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(\theta(0) \pm_{(-1)} 2\pi f_d t), & -T_b \leq t < 0; \\ \sqrt{\dfrac{2E_b}{T_b}} \cos(\theta(0) \pm_{(0)} 2\pi f_d t), & 0 \leq t < T_b. \end{cases}$$

Thus, regardless of the specific signs chosen for $\pm_{(-1)}$ and $\pm_{(0)}$, we have

$$s_I(t) = \sqrt{\frac{2E_b}{T_b}} \cos\theta(0) \cos 2\pi f_d t, \qquad -T_b \leq t < T_b,$$

where $\cos\theta(0) = \pm 1$ by assumption.

Similarly, for $0 \leq t < 2T_b$ we have

$$s_Q(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \sin(\theta(T_b) \pm_{(0)} 2\pi f_d (t-T_b)), & 0 \leq t < T_b; \\ \sqrt{\dfrac{2E_b}{T_b}} \sin(\theta(T_b) \pm_{(1)} 2\pi f_d (t-T_b)), & T_b \leq t < 2T_b. \end{cases}$$

Regardless of the specific signs chosen for $\pm_{(0)}$ and $\pm_{(1)}$, we have

$$s_Q(t) = \sqrt{\frac{2E_b}{T_b}} \sin\theta(T_b) \cos 2\pi f_d t, \qquad 0 \leq t < 2T_b,$$

where $\sin\theta(T_b) = \pm 1$ by assumption.

These results hold (with obvious modifications) for all intervals $(2k-1)T_b \leq t < (2k+1)T_b$ and $2kT_b \leq t < (2k+2)T_b$. This indicates that we can view MSK as a way of signalling on the in-phase and quadrature components with half-cycle sine pulses of length $2T_b$, offset in time by $T_b$. See Figure 5.

Moreover, this signalling format keeps the in-phase and quadrature channels orthogonal over any multiple of the bit time, since

$$\int_0^{T_b} \cos 2\pi f_d t \; \cos 2\pi f_c t \; \sin 2\pi f_d t \; \sin 2\pi f_c t \; dt =$$

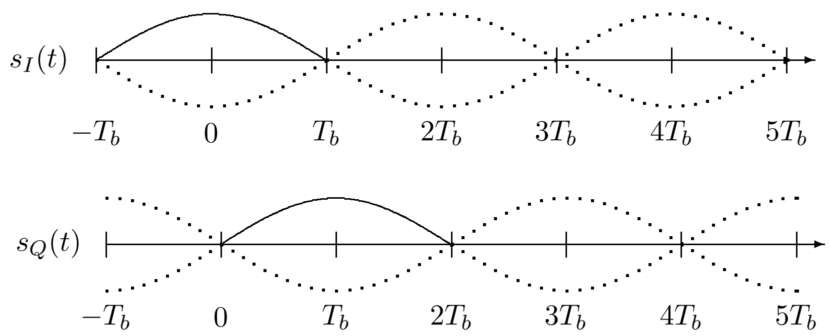


Figure 5: In-phase and quadrature components.

$$= \frac{1}{4}\int_0^{T_b} \sin 4\pi f_d t \ \sin 4\pi f_c t \ dt = \frac{1}{8}\int_0^{T_b} \cos 4\pi f_1 t - \cos 4\pi f_0 t \ dt = 0.$$

The pulse transmitted on the in-phase channel does not affect the pulse on the quadrature channel and vice versa. On each of the channels we send a positive or negative half-sine pulse with energy $E_b$ and thus we can detect them as BPSK pulses independently for the in-phase and quadrature components.

# 4 Bit allocation and error probabilities

When transmitting traditional FSK signals we customarily use the signal $s_0(t)$ for transmitting the message $m = 0$ and the signal $s_1(t)$ for transmitting the message $m = 1$. Duplicating this, we might assign the signals $\pm s_0(t)$ to $m = 0$ and assign $\pm s_1(t)$ to $m = 1$. However, when detecting MSK as FSK, this leads to problems with error propagation. From Figure 4 and Table 1 we see that if the signal $s_0$ is mistakenly detected as $s_1$, this affects future decisions as well. Assume that the last transmitted signal was $s_0$ and assume that we want to transmitt the two message bits $m = (0, 0)$. This would correspond to frequencies $(f_0, f_0)$ and, by Table 1, to signals $(s_0, s_0)$. If the first signal is mistakenly detected as $s_1$, a correct decision on the second signal will be interpreted as $-s_1$ instead of $s_0$. The message estimate is $(1, 1)$, and the error in the first bit has propagated to the second. This error propagation will continue until the next detection error.

A solution to this problem is to let $s_0$ and $-s_1$ correspond to $m = 0$ and $s_1$ and $-s_0$ correspond to $m = 1$. This way, detection of a signal to the lower right of the decision boundary in Figure 4 is always detected as $m = 1$, irrespective of what has been transmitted previously. With this latter choice of bit encoding, FSK detection of MSK has the same bit error probability as ordinary FSK,

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right).$$

| $-T_b \le t < 0$ | $0 \le t < T_b$ | $\theta(-T_b)$ | $\theta(0)$ | $\theta(T_b)$ | $\cos\theta(0)$ | $\sin\theta(T_b)$ |
|---|---|---|---|---|---|---|
| $s_0$ | $s_0$ | $-\pi/2$ | $0$ | $+\pi/2$ | $+1$ | $+1$ |
| $s_0$ | $s_1$ | $-\pi/2$ | $0$ | $-\pi/2$ | $+1$ | $-1$ |
| $s_1$ | $-s_0$ | $-\pi/2$ | $\pi$ | $-\pi/2$ | $-1$ | $-1$ |
| $s_1$ | $-s_1$ | $-\pi/2$ | $\pi$ | $+\pi/2$ | $-1$ | $+1$ |
| $-s_0$ | $-s_0$ | $+\pi/2$ | $\pi$ | $-\pi/2$ | $-1$ | $-1$ |
| $-s_0$ | $-s_1$ | $+\pi/2$ | $\pi$ | $+\pi/2$ | $-1$ | $+1$ |
| $-s_1$ | $s_0$ | $+\pi/2$ | $0$ | $+\pi/2$ | $+1$ | $+1$ |
| $-s_1$ | $s_1$ | $+\pi/2$ | $0$ | $-\pi/2$ | $+1$ | $-1$ |

Table 2: BPSK detection result depending on transmitted signal.

For BPSK detection of MSK, we consult Table 2.

If we assign signals to bits according to the traditional FSK assignment, with $\pm s_0$ assigned to $m = 0$ and $\pm s_1$ assigned to $m = 1$, we see that the demodulation mapping of $\cos\theta(0)$ into $\hat{m} = 0$ or $\hat{m} = 1$ depends on the value of $\theta(-T_b)$. If $\theta(-T_b)$ was incorrectly estimated, we will get error propagation into the estimate of $\cos\theta(0)$. Similarly the demodulation mapping of $\sin\theta(T_b)$ into $\hat{m} = 0$ or $\hat{m} = 1$ depends on a correct estimate of $\theta(0)$ and so on. With this signal assignment we have the same problem with error propagation as in BFSK detection.

With the suggested mapping of using $s_0$ and $-s_1$ for $m = 0$ and using $s_1$ and $-s_0$ for $m = 1$, we find in Table 2 that the we have dependence only on the current symbol. Furthermore, the demodulation mapping of $\theta$ values into $\hat{m}$ becomes very simple.

With this latter choice of bit encoding, BPSK detection of MSK has the same bit error probability as ordinary BPSK,

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right).$$