Notes on the Demodulation of AM Signals

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A DSBC (double side band plus carrier) signal may be expressed (Fig.1) as:

$$f(t) = A\,[1 + m.g(t)\,]\cos\omega_c t \qquad \ldots(1)$$

where:

$g(t)$ = modulating signal
m = modulation index $(0 < m \le 1)$
$\omega_c = 2\pi f_c$
f_c = carrier frequency
A = amplitude of unmodulated carrier

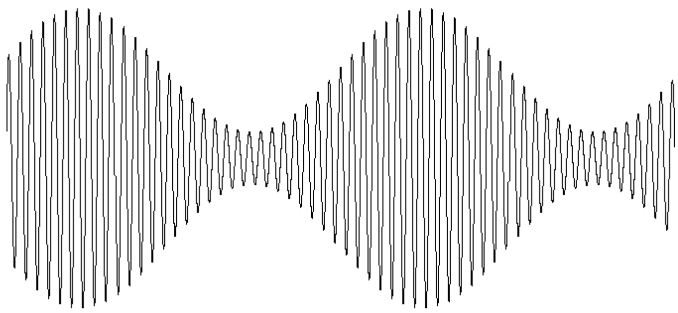

Fig.1 DSBC AM signal

Let:

$g(t) = \cos\omega_m t$
$\omega_m = 2\pi f_m$
f_m = frequency of the modulating signal

Then:

$$f(t) = A\,[1 + m.\cos\omega_m t\,]\cos\omega_c t \qquad \ldots(2)$$

which may be written as:

$$f(t) = A\cos\omega_c t + \frac{mA}{2}\cos(\omega_c + \omega_m)t + \frac{mA}{2}\cos(\omega_c - \omega_m)t \qquad \ldots(3)$$

1. Square-law detection of DSBC signals

The output of a square-law detector is of the form:

$$y = x^2 \qquad \ldots(4)$$

where x = input signal.

Vaccum-tube diodes and semiconductor (solid state) diodes have this type of response for small inputs.

Substituting eq. (3) in (4) yields:

$$f^2(t) = A^2\cos^2\omega_c t + \frac{m^2A^2}{4}\cos^2(\omega_c+\omega_m)t + \frac{m^2A^2}{4}\cos^2(\omega_c-\omega_m)t$$
$$+ mA^2\cos\omega_c t\cdot\cos(\omega_c+\omega_m)t + mA^2\cos\omega_c t\cdot\cos(\omega_c-\omega_m)t$$
$$+\frac{m^2A^2}{2}\cos(\omega_c+\omega_m)t\cdot\cos(\omega_c-\omega_m)t$$

which may be arranged as:

$$f^2(t) = \frac{A^2}{2}(1+\cos2\omega_c t) + \frac{m^2A^2}{8}[1+\cos2(\omega_c+\omega_m)t] + \frac{m^2A^2}{8}[1+\cos2(\omega_c-\omega_m)t]$$
$$+\frac{mA^2}{2}[\cos(2\omega_c+\omega_m)t+\cos\omega_m t] + \frac{mA^2}{2}[\cos(2\omega_c-\omega_m)t+\cos\omega_m t] \qquad \ldots(5)$$
$$+\frac{m^2A^2}{4}[\cos2\omega_c t+\cos2\omega_m t]$$

The output of the square-law detector contains AF and RF components. After filtering out the latter we are left with:

$$D(t) = mA^2\cos\omega_m t + \frac{m^2A^2}{4}\cos2\omega_m t \qquad \ldots(6)$$

The first term resembles the modulation. The second term constitutes a distortion component.

2. Product (synchronous) detection

If we multiply eq. (3) by the term s(t) = B cosω_ct, we obtain an expression for the output of a product or synchronous detector, whose schematic representation is given in Fig.2.

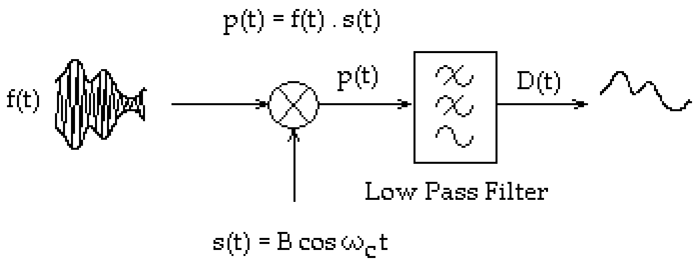

Fig.2 Product (synchronous) detector

The output of this type of detector is given by p(t) = f(t) . s(t), this is,

$$f(t)\cdot s(t) = AB\cos^2\omega_c t + \frac{mAB}{2}\cos\omega_c t\cdot\cos(\omega_c+\omega_m)t + \frac{mAB}{2}\cos\omega_c t\cdot\cos(\omega_c-\omega_m)t$$

which may be written as:

$$\begin{aligned} f(t)\cdot s(t) &= \frac{AB}{2}(1+\cos 2\omega_c t) + \frac{mAB}{4}[\cos(2\omega_c+\omega_m)t + \cos\omega_m t] \\ &+ \frac{mAB}{4}[\cos(2\omega_c-\omega_m)t + \cos\omega_m t] \end{aligned} \quad \ldots(7)$$

After filtering out RF components we obtain:

$$D(t) = \frac{mAB}{2}\cos\omega_m t \quad \ldots(8)$$

and the distortion component of frequency $2\omega_m$ does not exist.

3. Single side band (SSB) detection

An SSB signal may be expressed by:

$$f(t) = A\cos(\omega_c + \omega_m)t$$

or …(9)

$$f(t) = A\cos(\omega_c - \omega_m)t$$

the plus sign used for an upper side band (USB) signal and the minus sign for a lower side band (LSB) signal. Product and mixer-type demodulators are used for SSB detection.

3.1 Product detection of SSB signals

Consider a product detector with inputs:

$$f(t) = A\cos(\omega_c + \omega_m)t$$

and

$$s(t) = B\cos\omega_c t$$

The detector performs the mathematical function:

$$p(t) = f(t)\cdot s(t) = AB\cos\omega_c t.\cos(\omega_c+\omega_m)t$$

which is identical to:

$$p(t) = \frac{AB}{2}\left[\cos(2\omega_c + \omega_m)t + \cos\omega_m t\right] \qquad \ldots(10)$$

Clearly, if we filter out RF components from p(t) we get:

$$D(t) = \frac{AB}{2}\cos\omega_m t \qquad \ldots(11)$$

which is the desired modulation signal.

3.2 SSB detection using mixer-type demodulators

Here, a SSB signal at IF frequencies is mixed with the output of a beat frequency oscillator (BFO). This permits modulation retrieval.

Let $f(t) = A\cos(\omega_c + \omega_m)t$ be the SSB signal and $s(t) = B\cos(\omega_c + \Delta\omega)t$ the output from the BFO. The mixer's output is:

$$p(t) = [f(t) + s(t)]^2 \qquad \ldots(12)$$

Then:

$$p(t) = \left[A\cos(\omega_c + \omega_m)t + B\cos(\omega_c + \Delta\omega)t\right]^2$$

or equivalently,

$$\begin{aligned} p(t) = {} & \frac{A^2}{2}\left[1 + \cos 2(\omega_c + \omega_m)t\right] + \frac{B^2}{2}\left[1 + \cos 2(\omega_c + \Delta\omega)t\right] \\ & + AB\left[\cos(2\omega_c + \omega_m + \Delta\omega)t + \cos(\omega_m - \Delta\omega)t\right] \end{aligned} \qquad \ldots(13)$$

After removal of the RF frequency components from the output of the mixer we are left with:

$$D(t) = AB\cos(\omega_m - \Delta\omega)t \qquad \ldots(14)$$

We observe that the spectrum of the modulating signal has been recovered shifted down in frequency by an amount $\Delta\omega/2\pi$ Hertz. There the importance of the correct adjustment of the BFO.

4. Detection of DSBC signals using a regenerative receiver in the oscillating mode

Usually DSBC signals are detected in regenerative receivers adjusting the regenerative gain slightly below the oscillation point, the nonlinearities of the active device being responsible for the demodulation process, usually of a square-law type.

However, in the oscillating mode, DSBC signal detection is also possible. When tuned to the same frequency, a gently oscillating regenerative receiver will lock onto an incoming carrier. Both signals will be present across the tank circuit and hence will be mixed by the active device.

If A is the amplitude of the unmodulated carrier and B is that of the oscillation across the tuned circuit (having the same frequency as the carrier), then, using eq.(2) we get:

$$f(t) = A\,(1 + m.\cos\omega_m t)\cos\omega_c t + B\cos\omega_c t \qquad ...(15)$$

for the signals across the tank circuit.

Eq.(15) may be written as:

$$f(t) = (A + B)\cos\omega_c t + \frac{mA}{2}\cos(\omega_c + \omega_m)t + \frac{mA}{2}\cos(\omega_c - \omega_m)t \qquad ...(16)$$

After square-law detection and filtering we are left with the recovered modulation and an unwanted distortion term:

$$D(t) = mA(A + B)\cos\omega_m t + \frac{m^2 A^2}{4}\cos 2\omega_m t \qquad ...(17)$$

Usually, harmonic distortion levels are tolerable.

5. Demodulation of SSB signals in a regenerative receiver

Detection of SSB signals requires the receiver to be working in the oscillating mode. Strong oscillations are needed so the receiver doesn´t lock to the incoming signal. Otherwise, the recovered audio will have the known “quack-quack” type of sound. Alternatively, input signals can be attenuated by the operator.

Detection is, again, of the square-law type, after a mixing process carried out by the nonlinearities of the active device.

If **A $\cos(\omega_c + \omega_m)t$** is the SSB signal and **B $\cos(\omega_c + \Delta\omega)t$** the receiver’s oscillation, both across the tank circuit, the mixer’s output will be:

$$p(t) = [A\cos(\omega_c + \omega_m)t + B\cos(\omega_c + \Delta\omega)t]^2 \qquad ...(18)$$

yielding after filtering out RF frequency components:

$$D(t) = AB\cos(\omega_m - \Delta\omega)t \qquad ...(19)$$

Again, careful tuning of the receiver will be necessary so that $\Delta\omega \rightarrow 0$. Also, the oscillation’s frequency should be very stable.

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