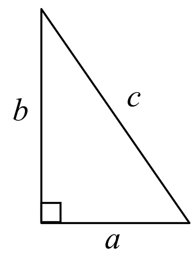
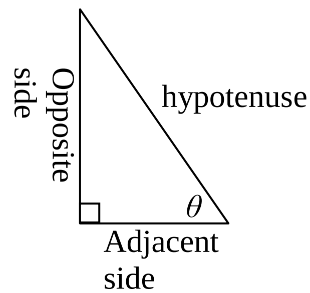
7.1 Basic Trigonometric Identities

Now that you know the definitions of the six trig functions, we can now apply them to the Pythagorean Theorem. Recall that the Pythagorean Theorem is $a^2 + b^2 = c^2$. With this we can correlate the sides of a right triangle with the relationships of the definitions of the six trig functions.





Let's do some substitution:

$a^2 + b^2 = c^2$

$(\text{adj.})^2 + (\text{opp.})^2 = (\text{hyp.})^2$ Let's divide both sides by $(\text{hyp.})^2$

$\left(\frac{adj.}{hyp.}\right)^2 + \left(\frac{opp.}{hyp.}\right)^2 = 1$ Let's substitute parentheses with trig functions

$(\cos\theta)^2 + (\sin\theta)^2 = 1$ This is a Pythagorean Identity

There are two other Pythagorean Identities that involve dividing by the other two parts.

$(\text{adj.})^2 + (\text{opp.})^2 = (\text{hyp.})^2$ Let's divide both sides by $(\text{opp.})^2$

$\left(\frac{adj.}{opp.}\right)^2 + \left(\frac{opp.}{opp.}\right)^2 = \left(\frac{hyp.}{opp.}\right)^2$ Let's substitute parentheses with trig functions

$(\cot\theta)^2 + 1 = (\csc\theta)^2$ This is a Pythagorean Identity

$(\text{adj.})^2 + (\text{opp.})^2 = (\text{hyp.})^2$ Let's divide both sides by $(\text{adj.})^2$

$\left(\frac{adj.}{adj.}\right)^2 + \left(\frac{opp.}{adj.}\right)^2 = \left(\frac{hyp.}{adj.}\right)^2$ Let's substitute parentheses with trig functions

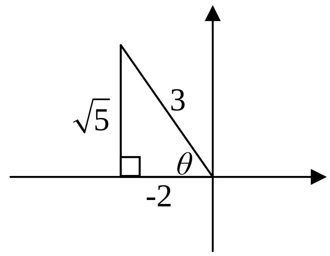
$1 + (\tan\theta)^2 = (\sec\theta)^2$ This is a Pythagorean Identity

These three identities are **very** commonly used. Remember these.

In 6-3 we discussed how to find the other trig functions given one and its location. Let's make sure we remember how to do this.

Example Given cos θ = -2/3 and $\pi/2 < \theta < \pi$. Find tan θ and sec θ.

First you want to draw a right triangle on a Cartesian graph representing the given information.



Applying the Pythagorean Theorem we can find the length of the side opposite θ.

Now we are ready to find tan θ and sec θ.

tan θ = opp./adj. = $\sqrt{5}/-2 = -\sqrt{5}/2$

sec θ = hyp./adj. = 3/-2 = -3/2

Try the following:
Given tan θ = -4/5 and $3\pi/2 < \theta < 2\pi$. Find the values of the other five trig functions.

Answers:
sin $\theta = -4\sqrt{29}/29$; cos $\theta = 5\sqrt{29}/29$; cot θ = -5/4 ; sec $\theta = \sqrt{29}/5$; csc $\theta = -\sqrt{29}/4$