Chapitre 2 : Electrostatique

Physique :

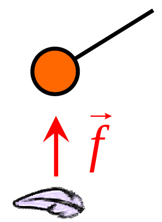
" étude des composants du monde matériel et de leurs interactions mutuelles "

Les interactions fondamentales :

- gravitation — mécanique
- électricité } électromagnétisme
- magnétisme } électromagnétisme
- interaction faible } physique nucléaire (→ électromagnétisme)
- interaction forte } physique nucléaire

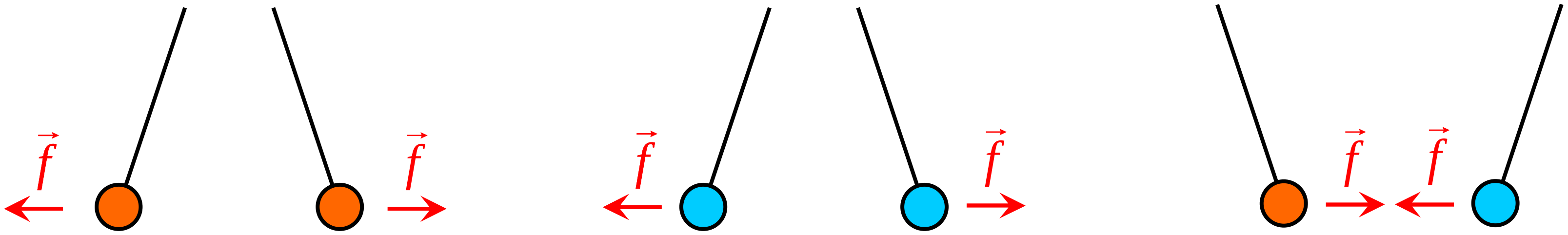
électrostatique : distribution des charges "figée"

ambre : elektron ⇨ électricité (1646)



?

Charles-François Dufay (1733) : deux "sortes" d'électricité



électricité "résineuse"

électricité "vitrée"

Benjamin Franklin (1750) : "algèbre" de l'électricité

$+ \times + = +$: répulsion

$- \times - = +$: répulsion

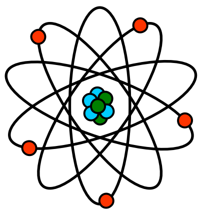
$+ \times - = -$: attraction

signe des charges électriques

$$q_1 \oplus q_2 = q_1 - q_2$$

corps neutre : $q \oplus q = 0$

Connaissances modernes :



électron : $q = -q_e$

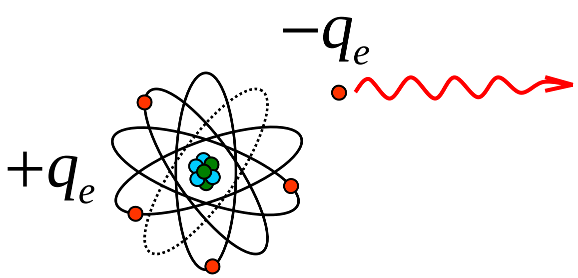
proton : $q = +q_e$

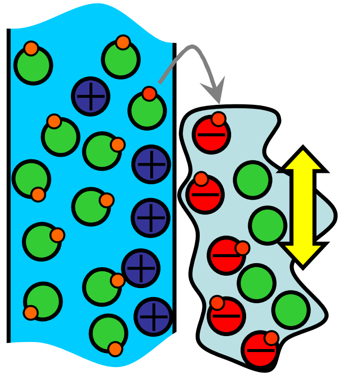
$$q_e = 1,6\,10^{-19}\ \mathrm{C}$$

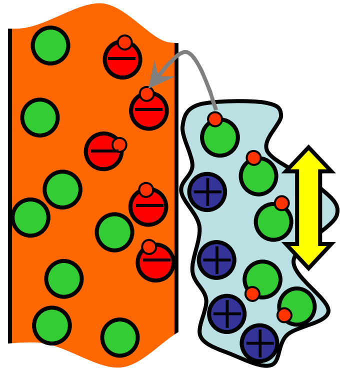
la charge est conservée

(remarque : $E = mc^2$)

Electrisation par frottement







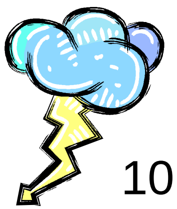
remarque : ions positifs = charges positives libres NaCl (+ H_2O) ⟶ $Na^+ + Cl^-$

ion = "voyageur"

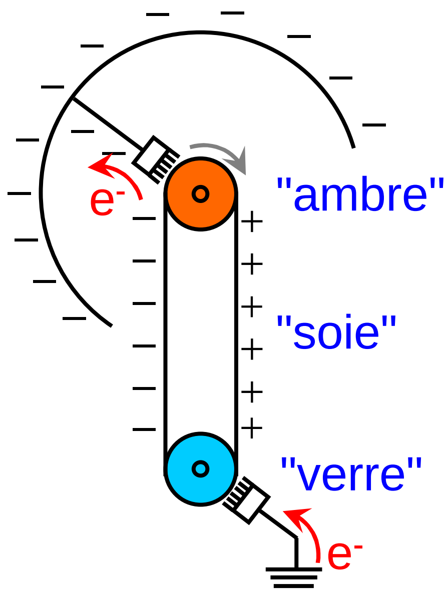
Générateur électrostatique de Van de Graaff (1930)

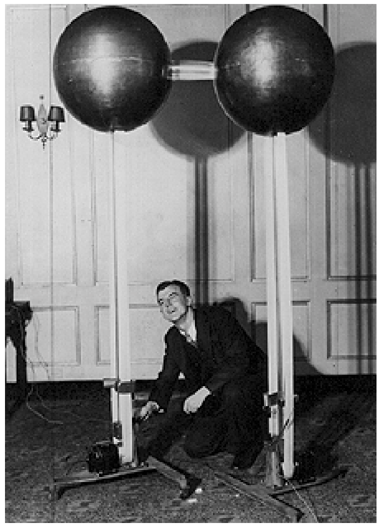
Frottement atmosphère-terre

1500 C/s



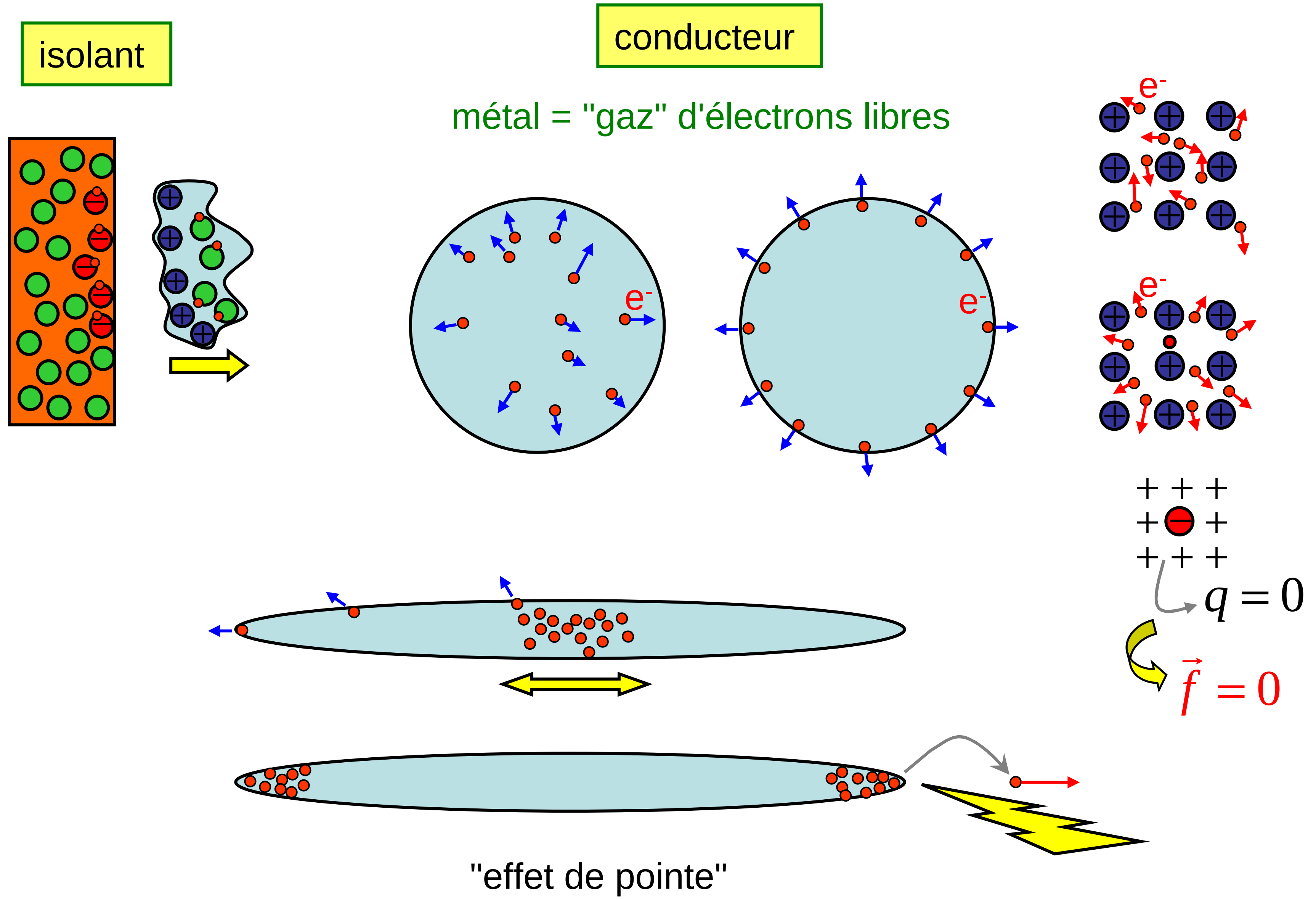
10 C





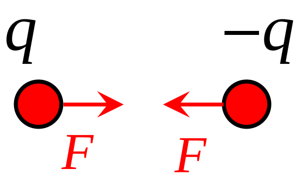
~1930

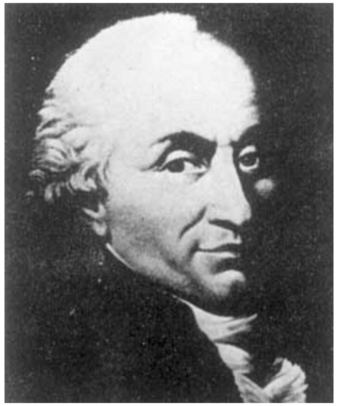
Transfert de charges et conductivité



Force électrique

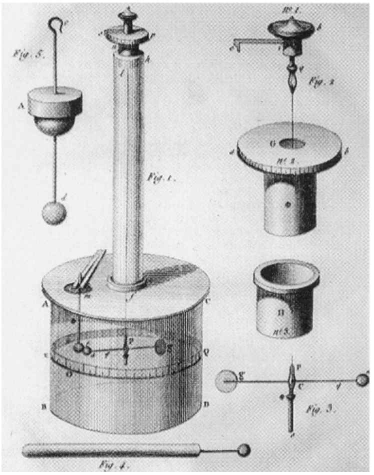
Loi de Coulomb



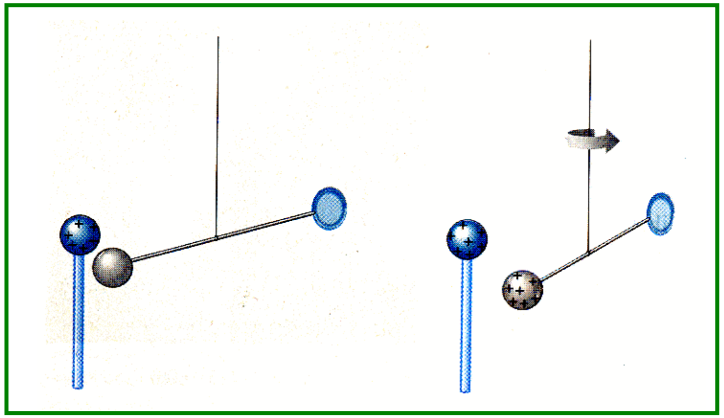


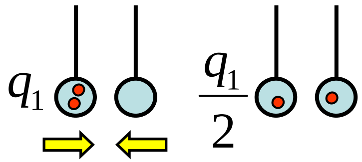
Charles-Augustin Coulomb
1736-1806

analogie à la force gravitationnelle : $F \propto \dfrac{1}{r^2}$

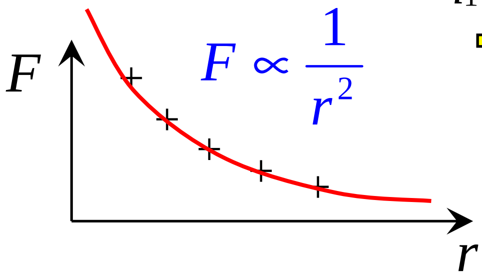


Balance à torsion





$F \propto q_1$



$$F \propto \frac{q_1 q_2}{r^2}$$

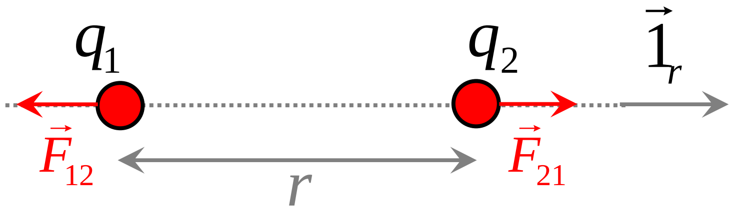
$$F = k_0 \frac{q_1 q_2}{r^2}$$

$$k_0 = 8{,}987\ 10^9\ \frac{\mathrm{Nm}^2}{\mathrm{C}^2}$$

$$q_1 = q_2 = 1\ \mathrm{C},\ \ r = 1\ \mathrm{m}$$

$$F \approx 9\ 10^9\ \mathrm{N}$$

Principe de superposition

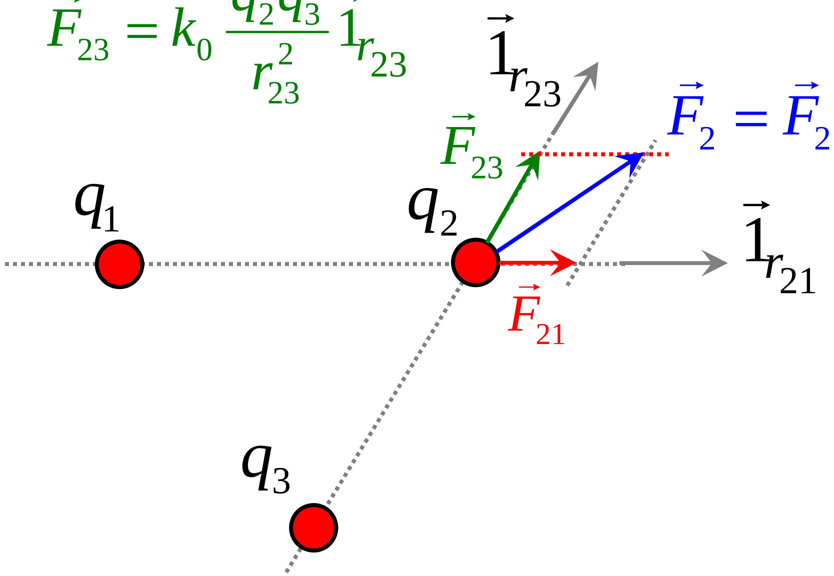


$$\vec{F}_{21} = k_0 \frac{q_1 q_2}{r^2} \vec{1}_r \qquad \vec{F}_{12} = -\vec{F}_{21}$$

$$\begin{cases} q_1 > 0,\ q_2 > 0 \\ q_1 < 0,\ q_2 < 0 \\ \Rightarrow \vec{F}_{21} = k_0 \dfrac{|q_1||q_2|}{r^2} \vec{1}_r \end{cases} \qquad \begin{cases} q_1 > 0,\ q_2 < 0 \\ q_1 < 0,\ q_2 > 0 \\ \Rightarrow \vec{F}_{21} = -k_0 \dfrac{|q_1||q_2|}{r^2} \vec{1}_r \end{cases}$$

$$\vec{F}_{23} = k_0 \frac{q_2 q_3}{r_{23}^2} \vec{1}_{r_{23}}$$

$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23}$$

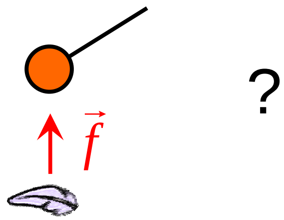


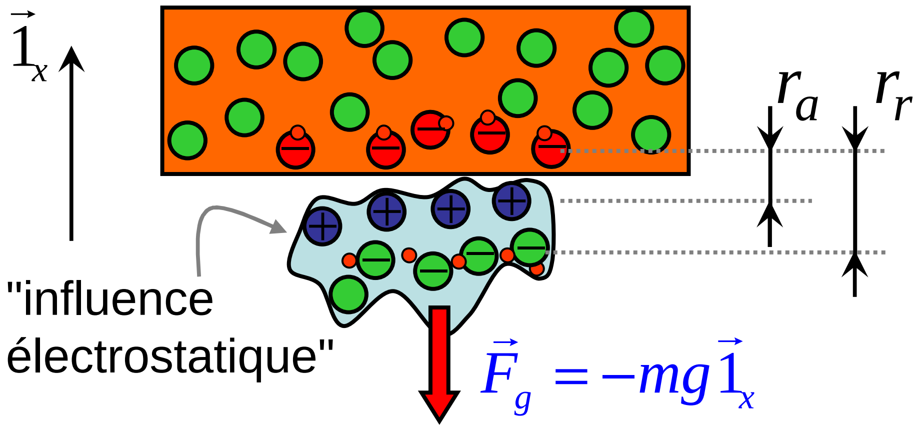
principe de superposition

$$\vec{F}_m = \sum_n \vec{F}_{mn}$$

$$\vec{F}_m = k_0 \sum_n \frac{q_n q_m}{r_{mn}^2} \vec{1}_{r_{mn}}$$

<u>Illustration</u> :





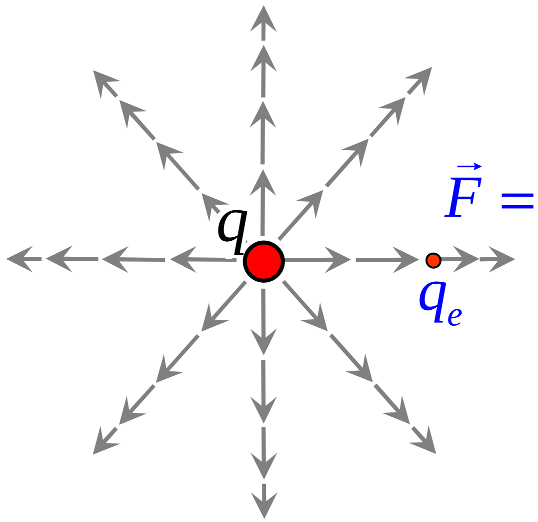
$$\vec{F}_E = \underbrace{k_0 \frac{q_1 q_2}{r_a} \vec{1}_x}_{\text{attraction}} - \underbrace{k_0 \frac{q_1 q_2}{r_r} \vec{1}_x}_{\text{répulsion}}$$

$$\vec{F}_E = k_0 q_1 q_2 \underbrace{\left[\frac{1}{r_a} - \frac{1}{r_r} \right]}_{>0} \vec{1}_x$$

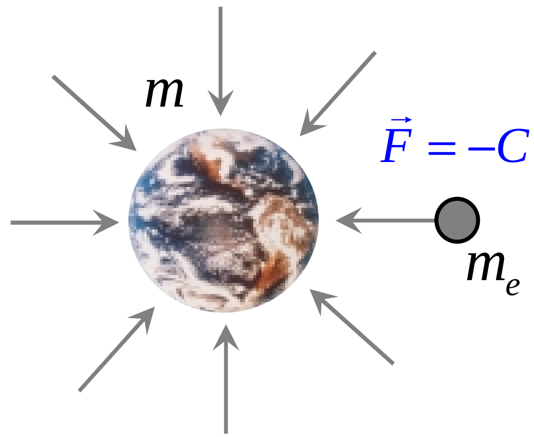
$$r_a \to 0 : \; \left| \vec{F}_E \right| > \left| \vec{F}_g \right|$$

Le champ électrique

Champ de force



$$\vec{F} = k_0 \frac{q q_e}{r^2} \vec{1}_r$$

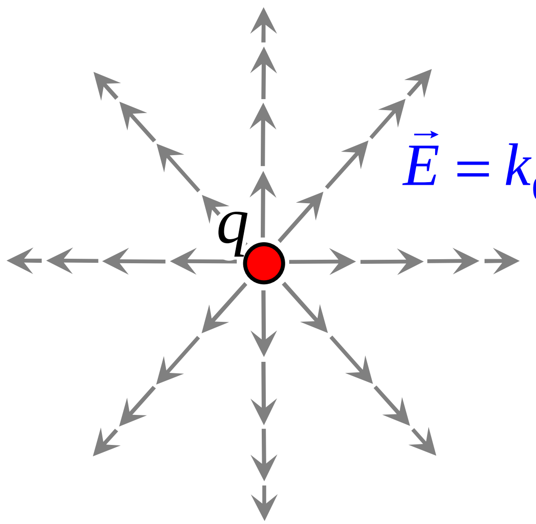


$$\vec{F} = -C \frac{m m_e}{r^2} \vec{1}_r$$

champ gravitationnel

Champ électrique : $\vec{E} \equiv \dfrac{\vec{F}}{q_e} = k_0 \dfrac{q}{r^2} \vec{1}_r$

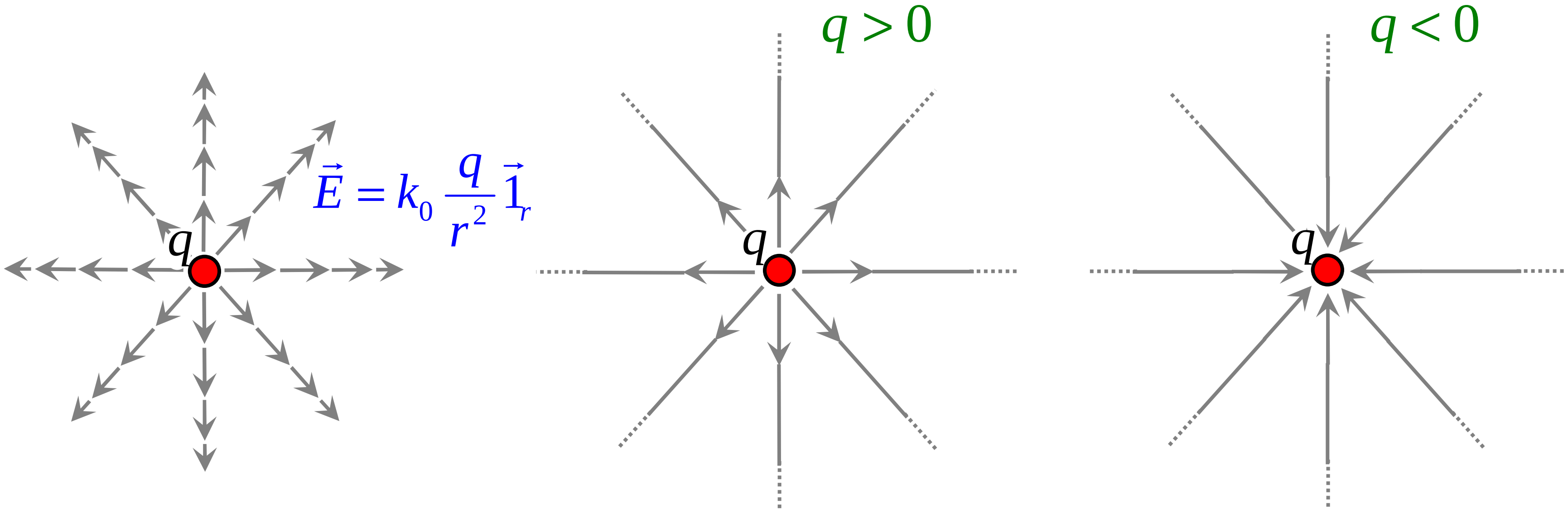
$\vec{F} = q_e \vec{E}$ unités : $\left[\vec{E}\right] = \dfrac{N}{C}$



$$\vec{E} = k_0 \frac{q}{r^2} \vec{1}_r$$

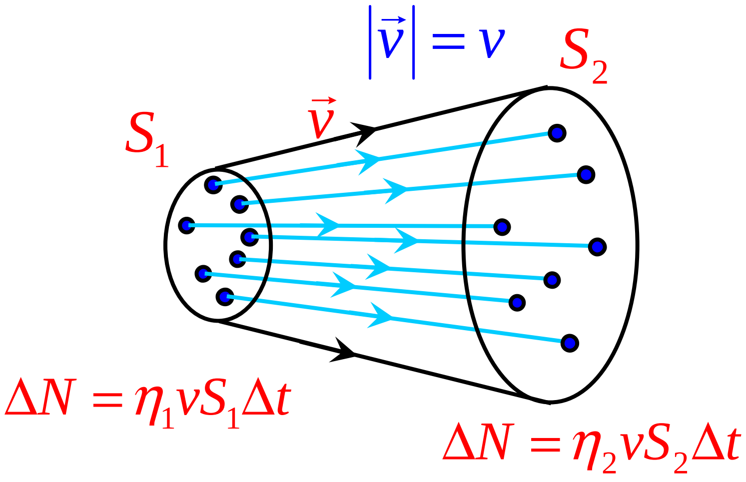
champ électrique

Michael Faraday (~1820) : Lignes de champ

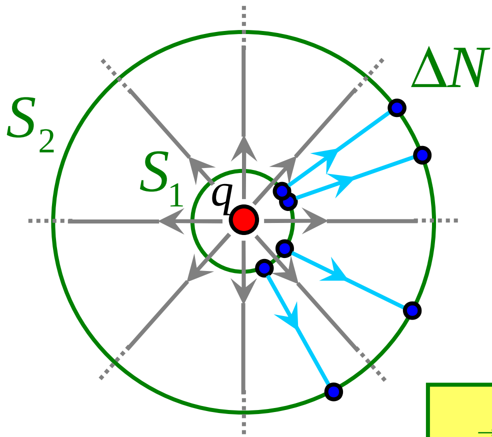


$$\vec{E} = k_0 \frac{q}{r^2} \vec{1}_r$$

Analogie avec les lignes de courant : flux de particules de vitesse constante



$\left(\text{remarque : } S_{1,2} \perp \vec{v}\right)$



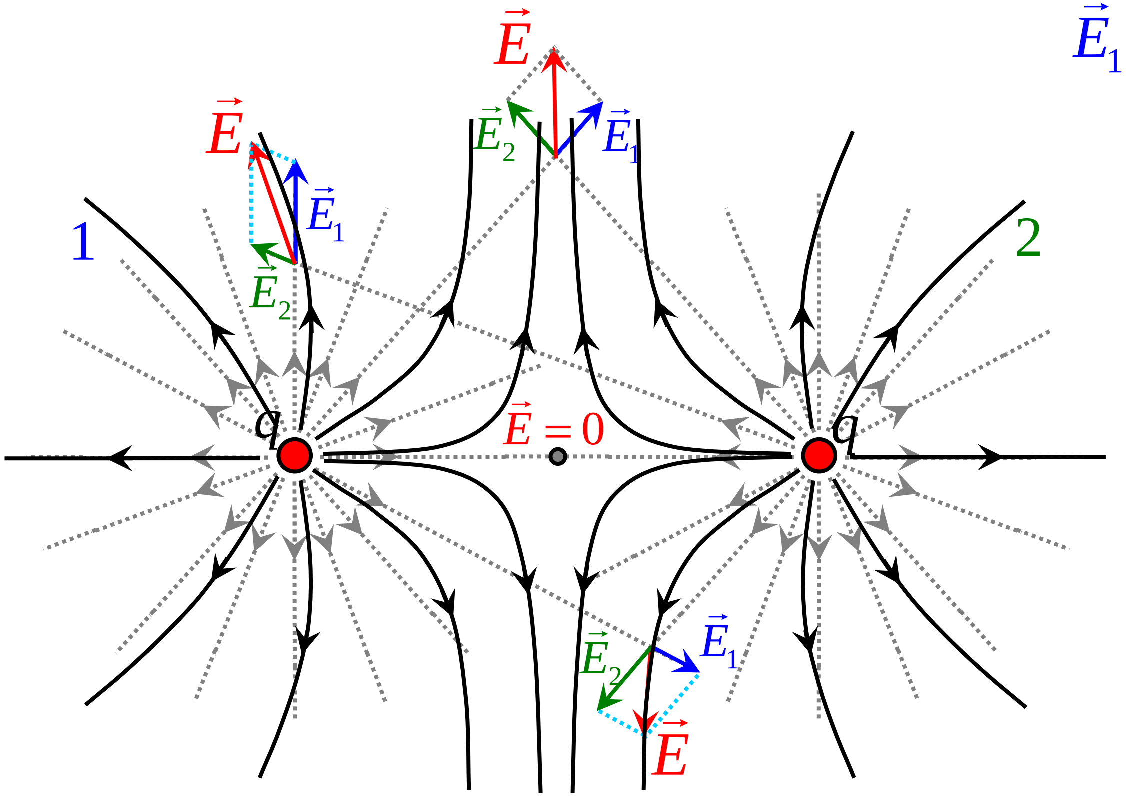
$$\Phi = \frac{\Delta N}{\Delta t} = \eta_1 v S_1 = \eta_2 v S_2$$

$$\Phi = \eta_1 v 4\pi r_1^2 = \eta_2 v 4\pi r_2^2$$

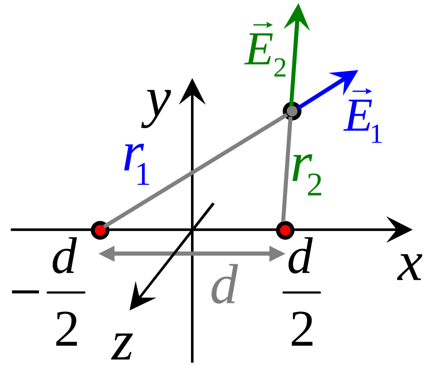
$$\eta_1 v = \frac{\Phi}{4\pi r_1^2}, \quad \eta_2 v = \frac{\Phi}{4\pi r_2^2}$$

$$\eta\vec{v} \rightarrow \vec{E}, \Phi \rightarrow 4\pi k_0 q : \vec{E} = k_0 \frac{q}{r^2} \vec{1}_r$$

exemples de lignes de champ



$$\vec{E}_1 = k_0 \frac{q}{r_1^{\,2}} \vec{1}_{r1} \qquad \vec{E}_2 = k_0 \frac{q}{r_2^{\,2}} \vec{1}_{r2}$$



$$r_1 = \sqrt{\left(x + \frac{d}{2}\right)^2 + y^2 + z^2}$$

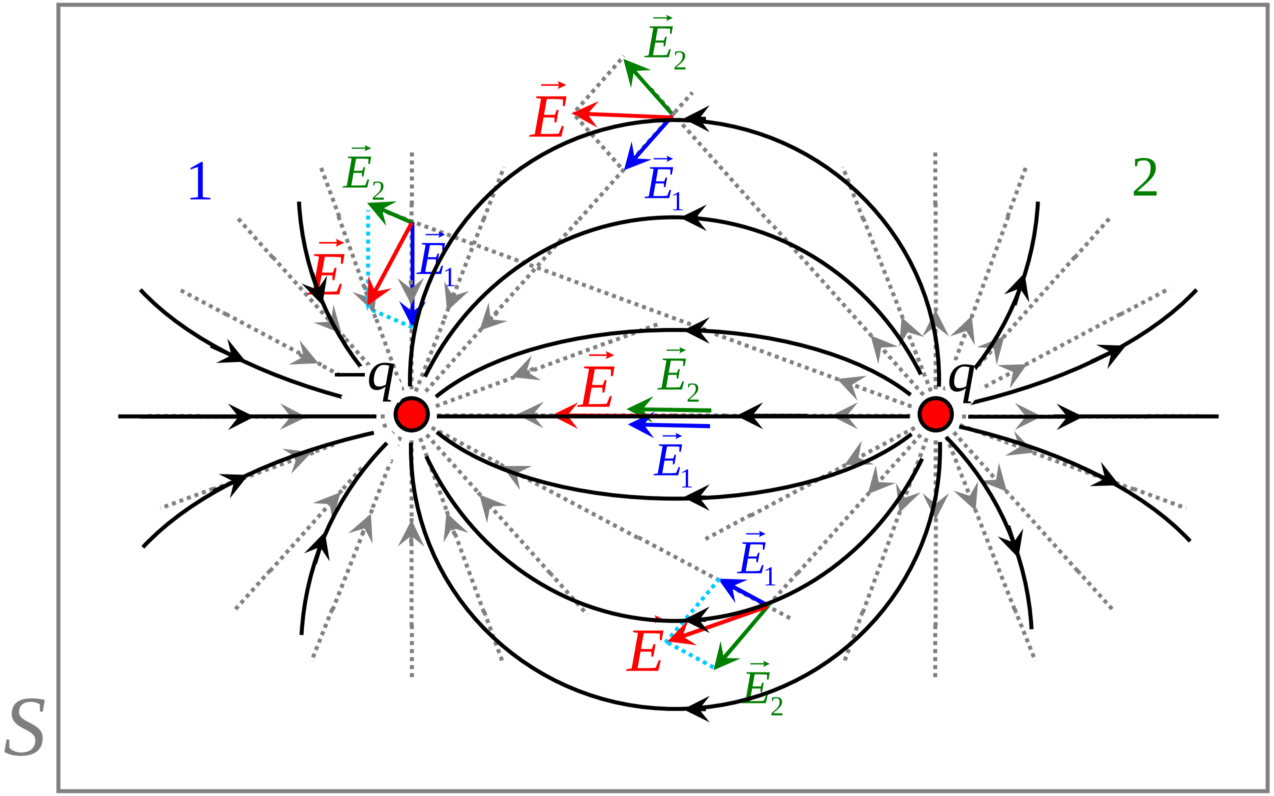
$$r_2 = \sqrt{\left(x - \frac{d}{2}\right)^2 + y^2 + z^2}$$

$$\vec{1}_{r1} = \left[(x + \frac{d}{2})\vec{1}_x + y\vec{1}_y + z\vec{1}_z \right] / r_1$$

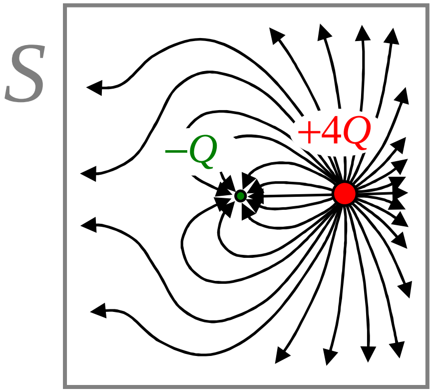
$$\vec{1}_{r2} = \left[(x - \frac{d}{2})\vec{1}_x + y\vec{1}_y + z\vec{1}_z \right] / r_2$$

$$\Rightarrow \quad \vec{E} = k_0 \frac{q}{r_1^{\,2}} \vec{1}_{r1} + k_0 \frac{q}{r_2^{\,2}} \vec{1}_{r2}$$

Remarque : les lignes de champ ne se croisent jamais



flux de particules au travers de la surface S : $\Phi = \frac{\Delta N}{\Delta t} = 0$ ⇨ $q_{tot} = 0$

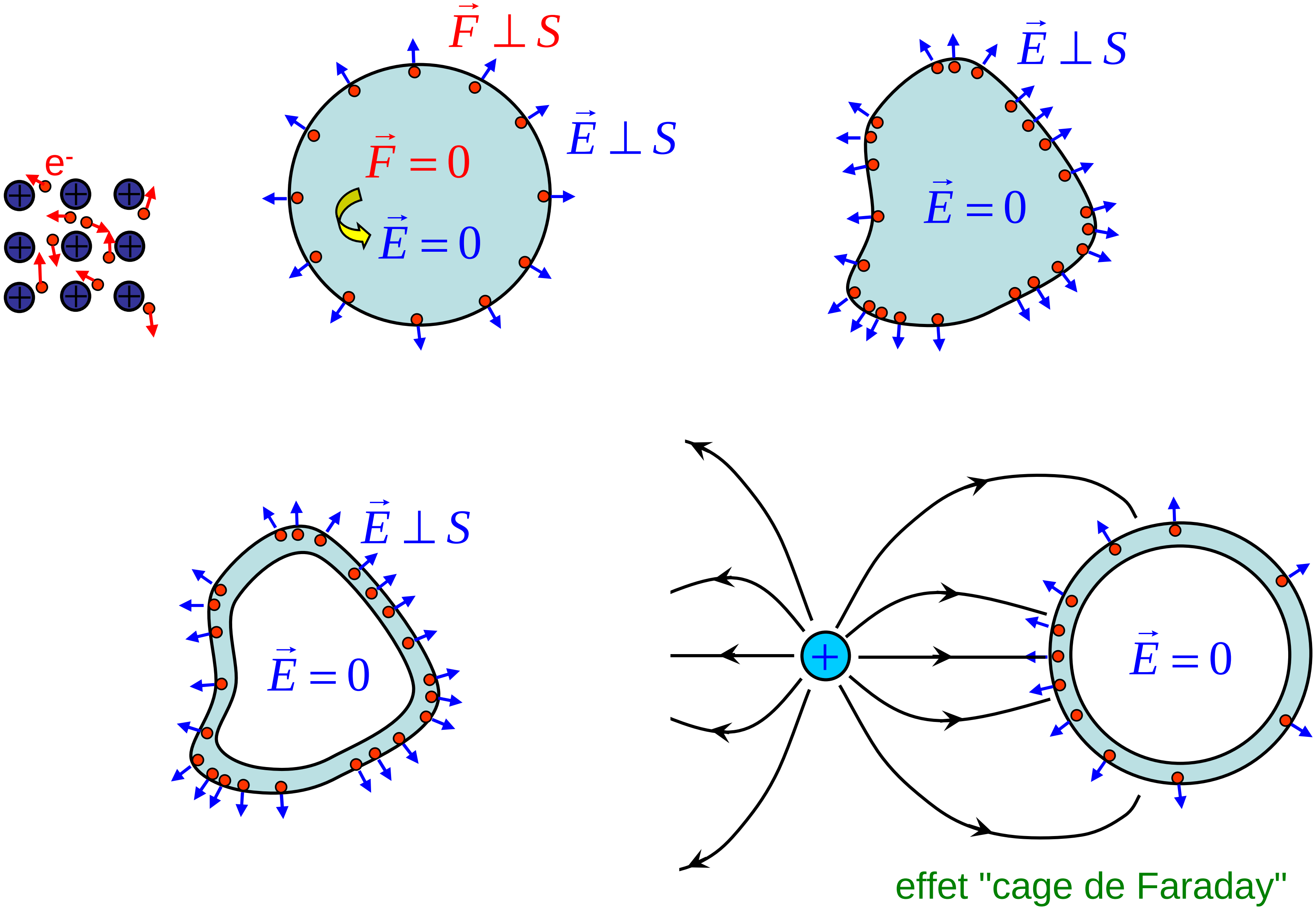


$q_{tot} = 3Q$

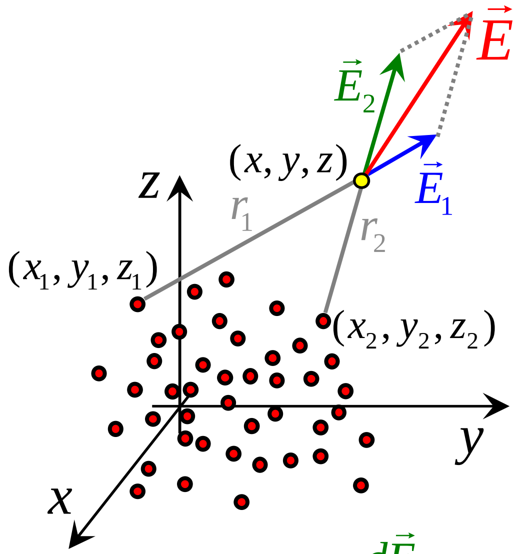
$\eta\vec{v} \to \vec{E},\ \Phi \to 4\pi k_0 q :\ \vec{E} = k_0 \frac{q}{r^2}\vec{1}_r$

densité des lignes de champs $\propto \left|\vec{E}\right|$

Champ électrique dans les conducteurs



Distributions de charge continues

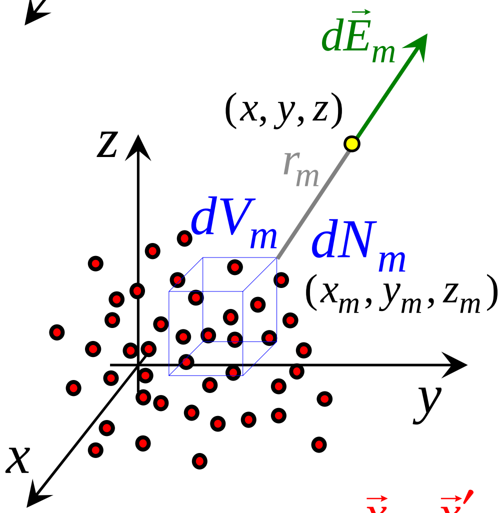


$$\vec{E} = \sum_{n=1}^{N_e} \vec{E}_n = k_0 \sum_{n=1}^{N_e} \frac{q_e}{r_n^{\ 2}} \ \vec{1}_{rn}$$

où $r_n = |\vec{x} - \vec{x}_n| = \sqrt{(x - x_n)^2 + (y - y_n)^2 + (z - z_n)^2}$

$1 \text{ cm}^3 \Longrightarrow N_e \approx 10^{20}$

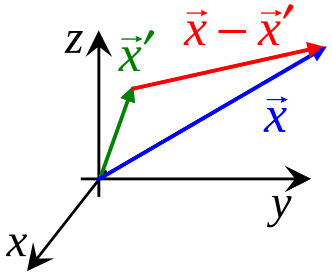
densité d'électrons : $\frac{N_e}{V} \approx 10^{20} \text{cm}^{-3}$



$$\vec{E} = \sum_{m=1}^{N_b} d\vec{E}_m = k_0 \sum_{m=1}^{N_b} \frac{dq_m}{r_m^{\ 2}} \vec{1}_{rm}$$ où $dq_m = dN_m q_e$

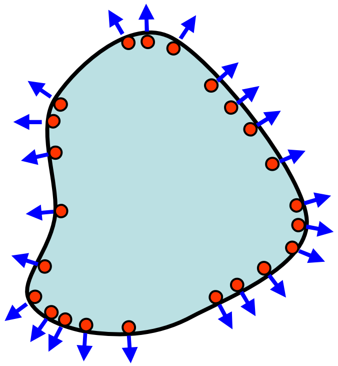
densité de charge : $\rho(\vec{x}_m) \equiv \frac{dq_m}{dV_m}$ $[\text{C/m}^3]$

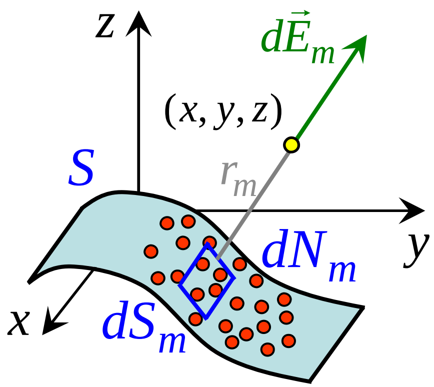
$dq_m = \rho(\vec{x}_m) dV_m \Longrightarrow \vec{E} = k_0 \sum_{m=1}^{N_b} \frac{\rho(\vec{x}_m) dV_m}{r_m^{\ 2}} \vec{1}_{rm}$



$$\vec{E}(\vec{x}) = k_0 \int \frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|^2} \vec{1}_{(\vec{x}-\vec{x}')} dV'$$

Charges de surface





$$\vec{E} = \sum_{m=1}^{N_b} d\vec{E}_m = k_0 \sum_{m=1}^{N_b} \frac{dq_m}{r_m^{\ 2}} \vec{1}_{rm}$$

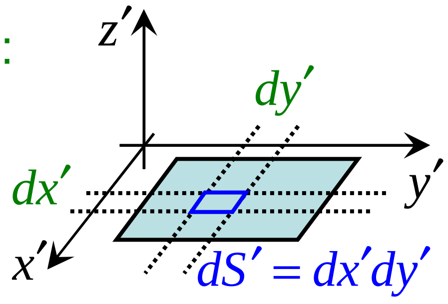
$$dq_m = dN_m q_e$$

densité "surfacique" de charge : $\sigma(\vec{x}_m) \equiv \dfrac{dq_m}{dS_m}$ $\quad [\mathrm{C/m^2}]$

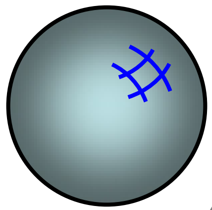
$$\vec{E} = k_0 \sum_{m=1}^{N_b} \frac{\sigma(\vec{x}_m) dS_m}{r_m^{\ 2}} \vec{1}_{rm} \Rightarrow \boxed{\vec{E}(\vec{x}) = k_0 \int \frac{\sigma(\vec{x}')}{|\vec{x} - \vec{x}'|^2} \vec{1}_{(\vec{x}-\vec{x}')} dS'}$$

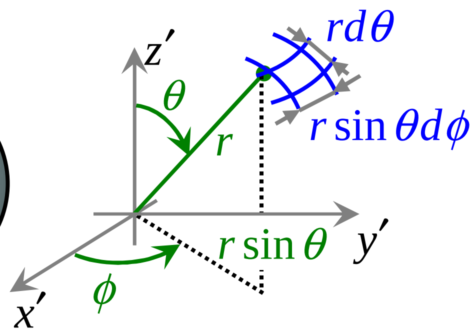
exemples :

- le plan



- la sphère

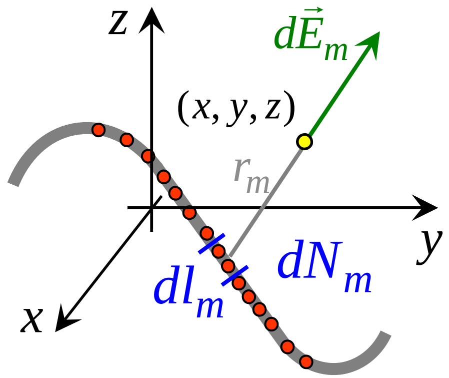




$$\vec{E}(\vec{x}) = k_0 \int_{x'} \int_{y'} \frac{\sigma_0}{|\vec{x} - \vec{x}'|^2} \vec{1}_{(\vec{x}-\vec{x}')} dx'dy'$$

$$\vec{E}(\vec{x}) = k_0 \int_{\theta} \int_{\phi} \frac{\sigma_0}{|\vec{x} - \vec{x}'|^2} \vec{1}_{(\vec{x}-\vec{x}')} r^2 \sin\theta d\theta d\phi$$

Charge de ligne



$$\vec{E} = \sum_{m=1}^{N_b} d\vec{E}_m = k_0 \sum_{m=1}^{N_b} \frac{dq_m}{r_m^{\ 2}} \vec{1}_{rm}$$

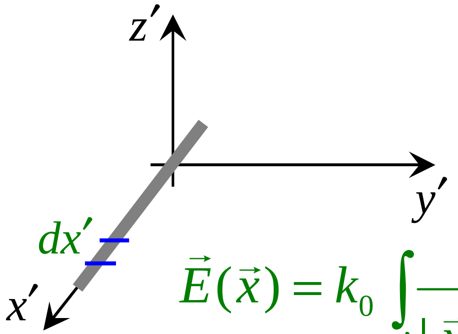
$$dq_m = dN_m q_e$$

densité "linéique" de charge : $\lambda(\vec{x}_m) \equiv \dfrac{dq_m}{dl_m}$ $\quad$ [C/m]

$$\vec{E} = k_0 \sum_{m=1}^{N_b} \frac{\lambda(\vec{x}_m) dl_m}{r_m^{\ 2}} \vec{1}_{rm} \Rightarrow \vec{E}(\vec{x}) = k_0 \int \frac{\lambda(\vec{x}')}{|\vec{x}-\vec{x}'|^2} \vec{1}_{(\vec{x}-\vec{x}')} dl'$$

exemples :

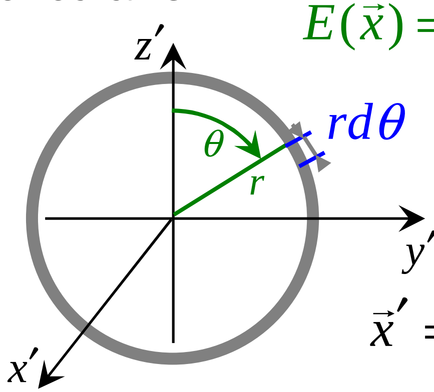
- le fil rectiligne



$$\vec{E}(\vec{x}) = k_0 \int_{x'} \frac{\lambda_0}{|\vec{x}-\vec{x}'|^2} \vec{1}_{(\vec{x}-\vec{x}')} dx'$$

- le fil circulaire

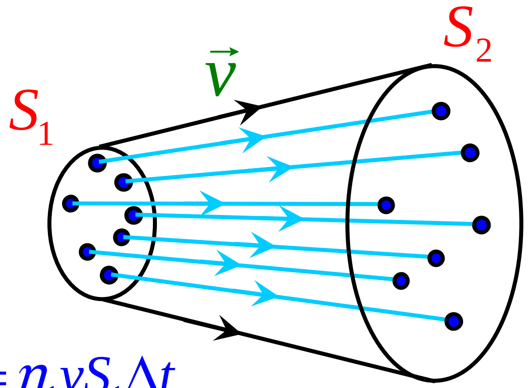
$$\vec{E}(\vec{x}) = k_0 \int_{\theta} \frac{\lambda_0}{|\vec{x}-\vec{x}'|^2} \vec{1}_{(\vec{x}-\vec{x}')} r d\theta$$



$$\vec{x}' = r\cos\theta \ \vec{1}_{z'} + r\sin\theta \ \vec{1}_{y'}$$

Théorème de Gauss

Flux du champ électrique



$$\Delta N = \eta_1 v S_1 \Delta t$$

$$\Delta N = \eta_2 v S_2 \Delta t$$

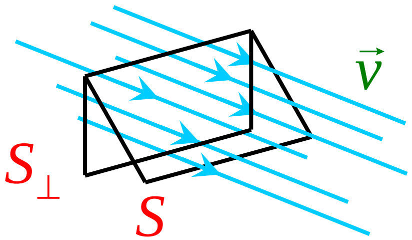
$$\Phi = \frac{\Delta N}{\Delta t} = \eta_1 v S_1 = \eta_2 v S_2 \qquad \eta\vec{v} = \frac{\Phi}{4\pi r^2}\vec{1}_r$$

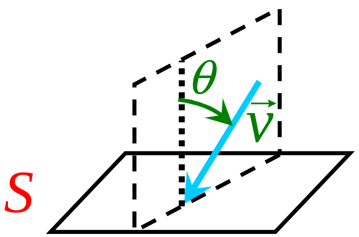
$$\eta\vec{v} \rightarrow \vec{E}\ ,\ \Phi \rightarrow 4\pi k_0 q$$

$$\vec{E} = k_0 \frac{q}{r^2}\vec{1}_r$$

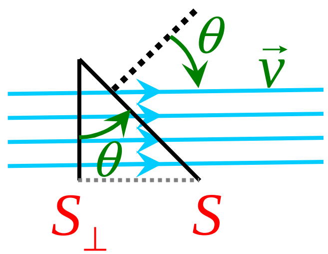
remarque : $S_{1,2} \perp \vec{v}$

Surface d'angle quelconque, champ uniforme :



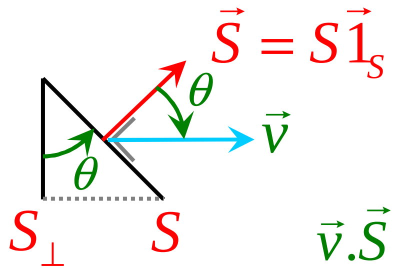


$$\Phi = \frac{\Delta N}{\Delta t} = \eta v S_\perp = \eta v S \cos\theta$$



$$S_\perp = S\cos\theta$$

vecteur de "surface"

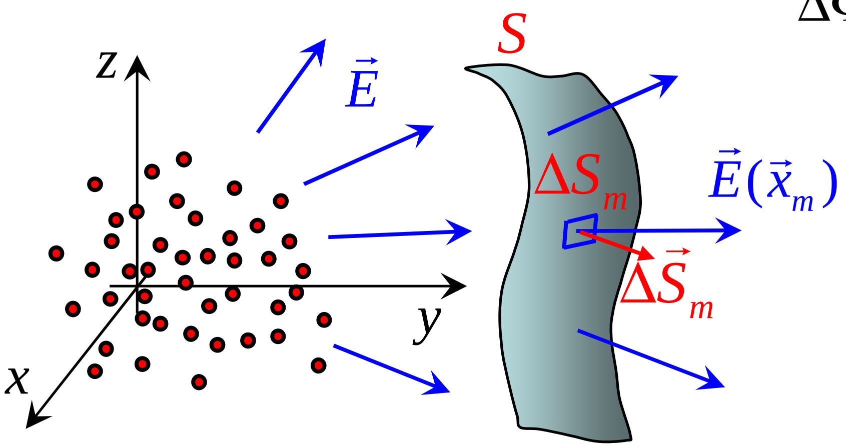


$$\vec{v}.\vec{S} = vS\cos\theta$$

$$\Phi = \eta v S\cos\theta = \eta\vec{v}.\vec{S}$$

$$\boxed{\Phi_E = \vec{E}.\vec{S}}$$

Surface et champ quelconques



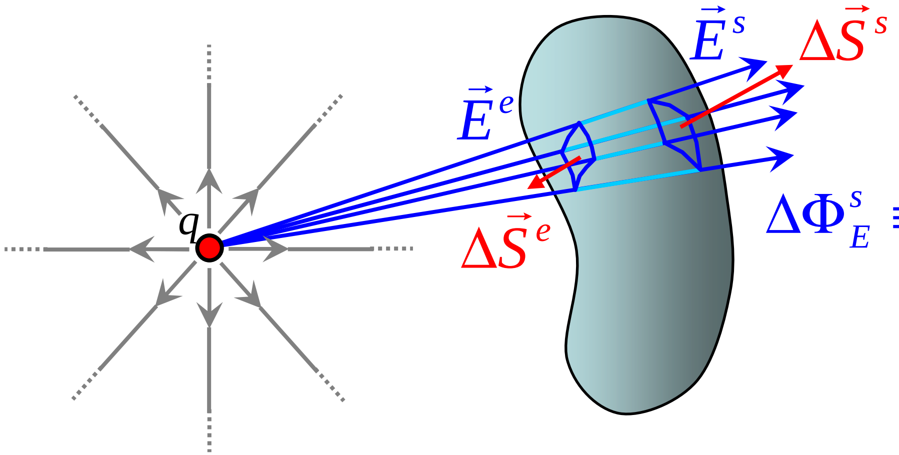
$$\Delta\Phi_{Em} = \vec{E}(\vec{x}_m).\Delta\vec{S}_m$$

$$\Phi_E = \sum_m \Delta\Phi_{Em} = \sum_m \vec{E}(\vec{x}_m).\Delta\vec{S}_m$$

$$\Phi_E = \int_S \vec{E}(\vec{x}).d\vec{S}$$

Charge ponctuelle et surface fermée

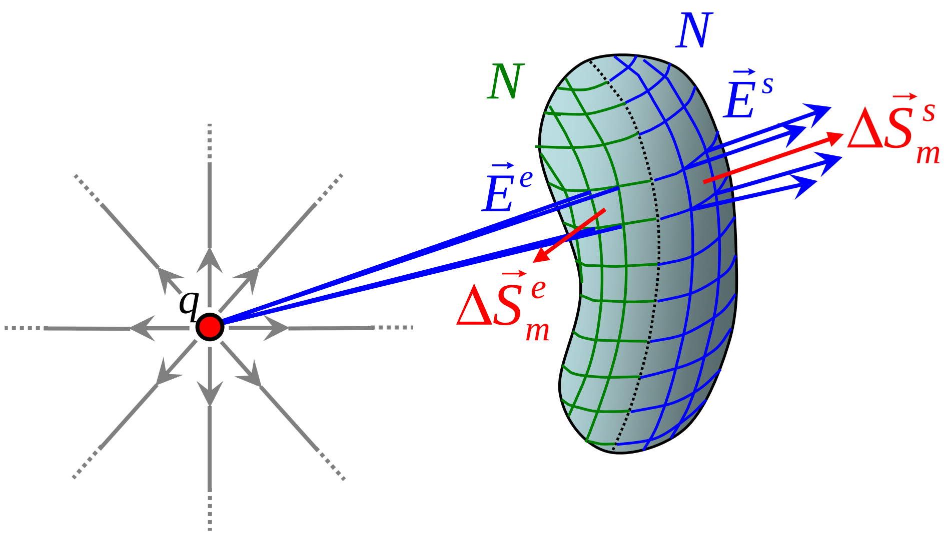
convention : $\Delta\vec{S}$ extérieur au volume



$$\Delta\Phi_E^s \equiv \Delta\Phi_E^e$$

$$\begin{cases} \Delta\Phi_E^s = \vec{E}(\vec{x}^s).\Delta\vec{S}^s > 0 \\ \Delta\Phi_E^e = \vec{E}(\vec{x}^e).\Delta\vec{S}^e < 0 \end{cases}$$

$$\begin{cases} \Delta\Phi_E^s \equiv \left|\Delta\Phi_E^e\right| \\ \Delta\Phi_E^s + \Delta\Phi_E^e = 0 \end{cases}$$



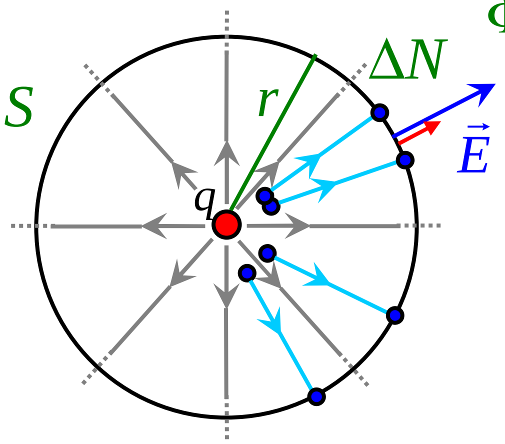
$$\Phi_E = \sum_{m=1}^{N} \Delta\Phi_{Em}^s + \sum_{m=1}^{N} \Delta\Phi_{Em}^e = 0$$

$$\begin{cases} \Delta\Phi_{Em}^s = \vec{E}(\vec{x}_m^s).\Delta\vec{S}_m^s > 0 \\ \Delta\Phi_{Em}^e = \vec{E}(\vec{x}_m^e).\Delta\vec{S}_m^e < 0 \end{cases}$$

$$\Delta\Phi_{En} = \vec{E}(\vec{x}_n).\Delta\vec{S}_n \qquad n = 1, 2, ..., 2N$$

$$\Phi_E = \sum_{n=1}^{2N} \Delta\Phi_{En} = 0 \;\Rightarrow\; \Phi_E = \sum_{n=1}^{2N} \vec{E}(\vec{x}_n).\Delta\vec{S}_n = 0 \;\Rightarrow\; \Phi_E = \oint_S \vec{E}(\vec{x}).d\vec{S} = 0$$

Charge ponctuelle interne

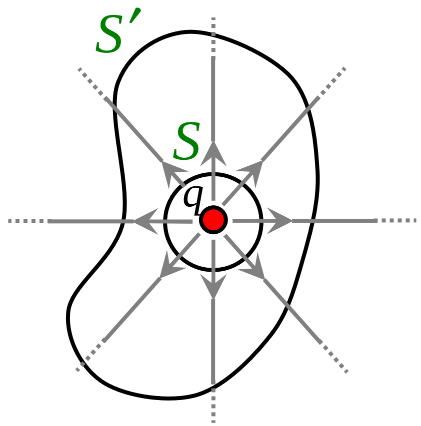


$$\Phi = \frac{\Delta N}{\Delta t} = \eta v S = \eta v 4\pi r^2$$

$$\vec{E} = k_0 \frac{q}{r^2} \vec{1}_r \qquad d\vec{S} = dS\,\vec{1}_r$$

$$\Phi_E = |\vec{E}|\, 4\pi r^2 = k_0 \frac{q}{r^2} 4\pi r^2$$

$$\boxed{\Phi_E = 4\pi k_0 q}$$



$$\Phi_E^{S'} = \oint_{S'} \vec{E}(\vec{x}).d\vec{S}' = 4\pi k_0 q$$

Changement de notation : la "permittivité"

soit $k_0 = \frac{1}{4\pi\varepsilon_0}$ ⟹ $\varepsilon_0 = 8,854\ 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N.m}^2}$

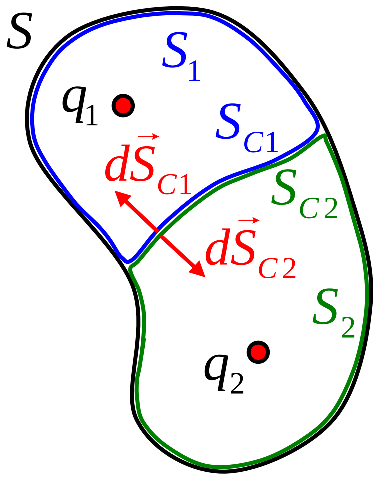
loi de Coulomb : $F = k_0 \frac{q_1 q_2}{r^2}$

$k_0 = 8,987\ 10^9\ \frac{\mathrm{Nm}^2}{\mathrm{C}^2}$

$$\boxed{\Phi_E = \frac{1}{\varepsilon_0} q \quad , \quad \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{1}_r}$$

(Heaviside, 1892)

Plusieurs charges ponctuelles internes



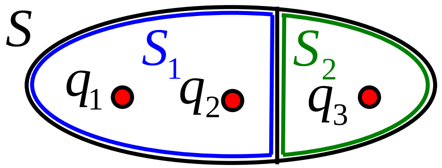
$$\begin{cases} \Phi_E^{S_1} = \oint_{S_1} \vec{E}(\vec{x}).d\vec{S}_1 = 4\pi k_0 q_1 = \dfrac{1}{\varepsilon_0} q_1 \\ \Phi_E^{S_2} = \oint_{S_2} \vec{E}(\vec{x}).d\vec{S}_2 = 4\pi k_0 q_2 = \dfrac{1}{\varepsilon_0} q_2 \end{cases}$$

($\vec{E}$ = champ dû aux deux charges)

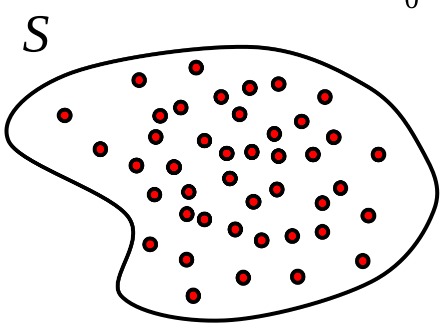
$$\Phi_E^S = \oint_{S_1} \vec{E}.d\vec{S}_1 - \int_{S_{C1}} \vec{E}.d\vec{S}_{C1} + \oint_{S_2} \vec{E}.d\vec{S}_2 - \int_{S_{C2}} \vec{E}.d\vec{S}_{C2} = \frac{1}{\varepsilon_0} q_1 + \frac{1}{\varepsilon_0} q_2$$

$$d\vec{S}_{C1} = -d\vec{S}_{C2} \quad \Rightarrow \quad \Phi_E^S = \frac{1}{\varepsilon_0}(q_1 + q_2)$$

Généralisation :

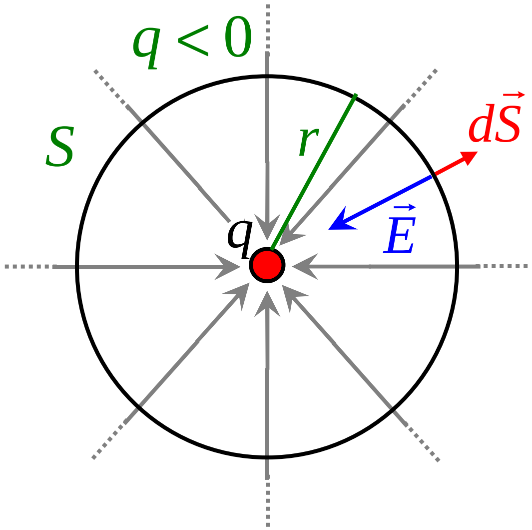


$$\left.\begin{aligned} \Phi_E^{S_1} &= \frac{1}{\varepsilon_0}(q_1 + q_2) \\ \Phi_E^{S_2} &= \frac{1}{\varepsilon_0} q_3 \end{aligned}\right\} \Rightarrow \Phi_E^S = \frac{1}{\varepsilon_0}(q_1 + q_2 + q_3)$$



$$\boxed{\Phi_E^S = \oint_S \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0} \sum_{m=1}^{N} q_m}$$

Charges négatives



$$\vec{E} = \frac{1}{\varepsilon_0}\frac{q}{r^2}\vec{1}_r = -\frac{1}{\varepsilon_0}\frac{|q|}{r^2}\vec{1}_r$$

$$d\vec{S} = dS\,\vec{1}_r$$

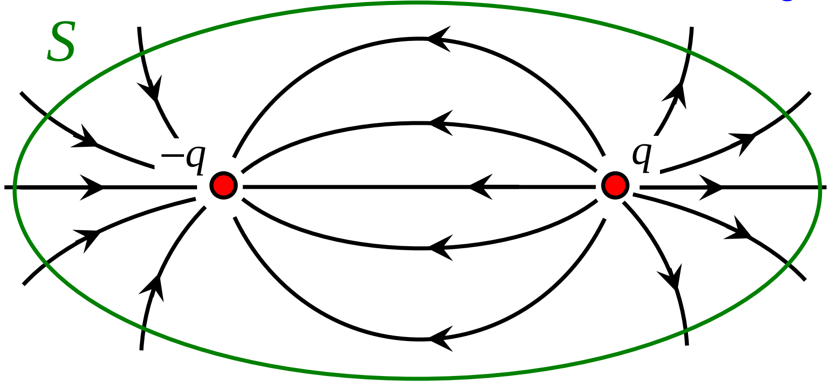
$$\Rightarrow \vec{E}.d\vec{S} < 0$$

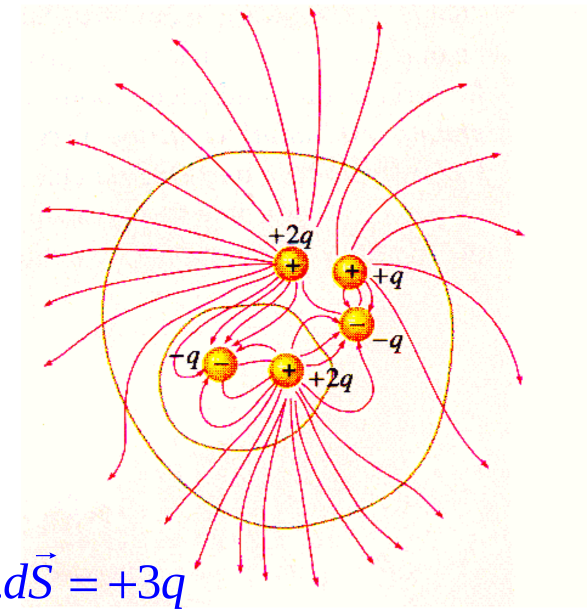
$$\begin{cases} \Phi_E^S = \oint_S \vec{E}.d\vec{S} < 0 \\ \Phi_E^S = -|\vec{E}|\,4\pi r^2 = \frac{1}{\varepsilon_0} q \end{cases}$$

$$\boxed{\Phi_E^S = \oint_S \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0}\sum_{m=1}^{N} q_m}$$

Illustration : le dipôle

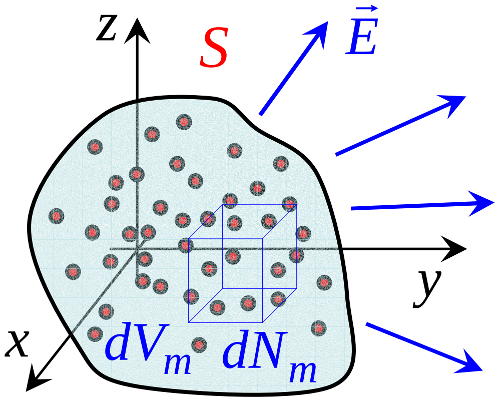
$$\Phi_E^S = \oint_S \vec{E}.d\vec{S} = 0$$





$$\Phi_E^S = \oint_S \vec{E}.d\vec{S} = +3q$$

Distributions de charge continues



$q_n = \pm q_e$

$$\Phi_E^S = \oint_S \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0}\sum_{n=1}^{N} q_n = \frac{1}{\varepsilon_0}\sum_{m=1}^{N_b} dq_m = \frac{1}{\varepsilon_0}\int \rho(\vec{x})dV$$

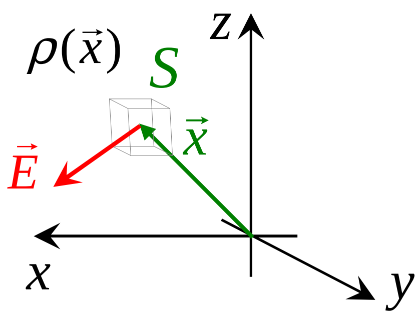
$$dq_m = dN_m q_e$$

$$\rho(\vec{x}_m) \equiv \frac{dq_m}{dV_m}$$

$$\Phi_E^S = \oint_S \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0}\int \rho(\vec{x})dV$$

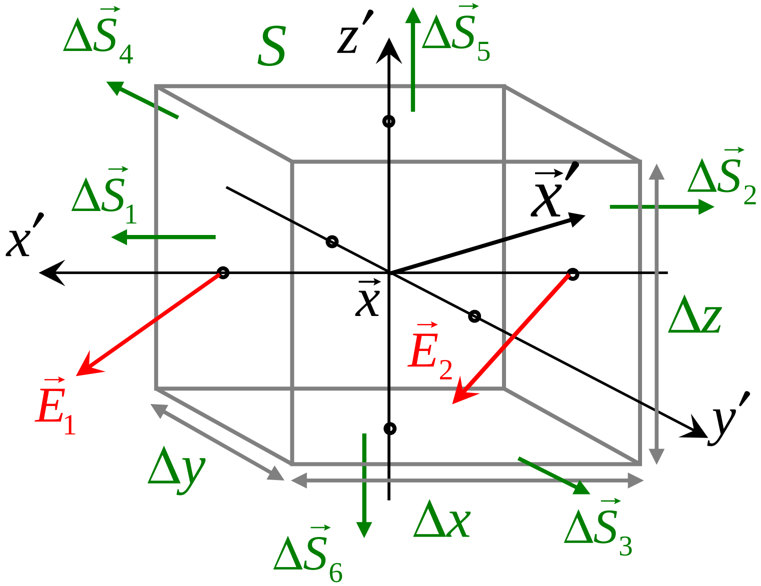
Théorème de Gauss

Forme locale du théorème de Gauss



$$\Phi_E^S = \oint_S \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0}\int \rho(\vec{x})dV$$

$$\oint_S \vec{E}.d\vec{S} = \sum_{n=1}^{6} \int_{\Delta\vec{S}_n} \vec{E}_n.d\vec{S}_n = \sum_{n=1}^{6} \Phi_E^{\Delta\vec{S}_n}$$



$$\begin{cases} d\vec{S}_1 = dy'dz'\vec{1}_x \\ d\vec{S}_2 = -dy'dz'\vec{1}_x \end{cases} \qquad \begin{cases} \vec{E}_1 = \vec{E}(x+\Delta x/2, y+y', z+z') \\ \vec{E}_2 = \vec{E}(x-\Delta x/2, y+y', z+z') \end{cases}$$

$$\begin{cases} d\vec{S}_3 = dx'dz'\vec{1}_y \\ d\vec{S}_4 = -dx'dz'\vec{1}_y \end{cases} \qquad \begin{cases} \vec{E}_3 = \vec{E}(x+x', y+\Delta y/2, z+z') \\ \vec{E}_4 = \vec{E}(x+x', y-\Delta y/2, z+z') \end{cases}$$

$$\begin{cases} d\vec{S}_5 = dx'dy'\vec{1}_z \\ d\vec{S}_6 = -dx'dy'\vec{1}_z \end{cases} \qquad \begin{cases} \vec{E}_5 = \vec{E}(x+x', y+y', z+\Delta z/2) \\ \vec{E}_6 = \vec{E}(x+x', y+y', z-\Delta z/2) \end{cases}$$

$$\Phi_E^{\Delta\vec{S}_1} = \int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} \vec{E}(x+\Delta x/2, y+y', z+z').\vec{1}_x\, dy'dz'$$

$$\Phi_E^{\Delta\vec{S}_1} = \int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} E_x(x+\Delta x/2, y+y', z+z')dy'dz'$$

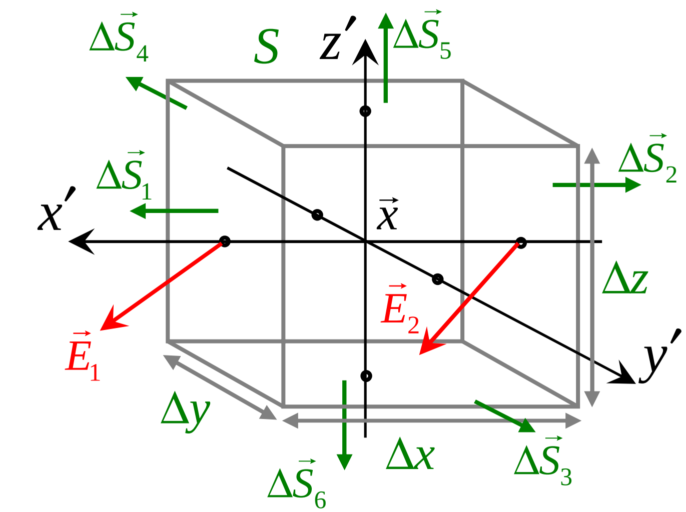
$$\Phi_E^{\Delta\vec{S}_1} = \int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} E_x(x+\Delta x/2, y+y', z+z')\,dy'dz'$$

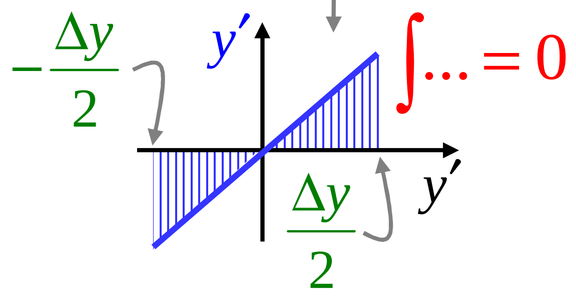
$$f(x+\Delta x) = f(x) + \Delta x \frac{df(x)}{dx} \qquad \text{et} \qquad f(x+\Delta x, y, z) = f(x,y,z) + \Delta x \frac{\partial f(x,y,z)}{\partial x}$$

$$f(x+\Delta x, y+y', z+z') = f(x,y,z) + \Delta x \frac{\partial f(x,y,z)}{\partial x} + y' \frac{\partial f(x,y,z)}{\partial y} + z' \frac{\partial f(x,y,z)}{\partial z}$$

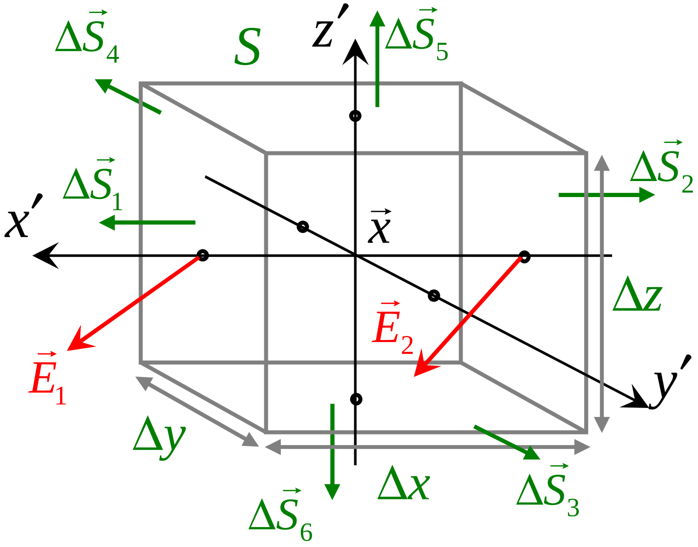
$$\Phi_E^{\Delta\vec{S}_1} = \int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} \left[E_x(x,y,z) + \Delta x/2 \frac{\partial E_x}{\partial x} + y' \frac{\partial E_x}{\partial y} + z' \frac{\partial E_x}{\partial z} \right] dy'dz'$$

$$\Phi_E^{\Delta\vec{S}_1} = \left[E_x(x,y,z) + \Delta x/2 \frac{\partial E_x}{\partial x} \right] \int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} dy'dz' + \frac{\partial E_x}{\partial y} \int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} y'dy'dz' + \frac{\partial E_x}{\partial z} \int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} z'dy'dz'$$





$$\Phi_E^{\Delta\vec{S}_1} = \left[E_x(x,y,z) + \Delta x/2 \frac{\partial E_x}{\partial x} \right] \Delta y \Delta z$$



$$\Phi_E^{\Delta\vec{S}_1} = \left[E_x + \frac{\Delta x}{2}\frac{\partial E_x}{\partial x} \right]\Delta y \Delta z$$

$$\Delta\vec{S}_2 : \Delta x \to -\Delta x \ \text{ et } \ \Delta\vec{S}_1 = \Delta y \Delta z \overline{1}_x \to \Delta\vec{S}_2 = -\Delta y \Delta z \overline{1}_x$$

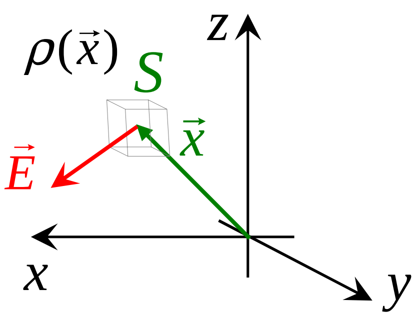
$$\Phi_E^{\Delta\vec{S}_2} = \left[-E_x - (-\Delta x/2)\frac{\partial E_x}{\partial x} \right]\Delta y \Delta z$$

$$\Phi_E^{\Delta\vec{S}_2} = \left[-E_x + \frac{\Delta x}{2}\frac{\partial E_x}{\partial x} \right]\Delta y \Delta z$$

symétrie :

$$\begin{cases} \Phi_E^{\Delta\vec{S}_1} + \Phi_E^{\Delta\vec{S}_2} = \dfrac{\partial E_x}{\partial x}\Delta x \Delta y \Delta z \\ \Phi_E^{\Delta\vec{S}_3} + \Phi_E^{\Delta\vec{S}_4} = \dfrac{\partial E_y}{\partial y}\Delta x \Delta y \Delta z \\ \Phi_E^{\Delta\vec{S}_5} + \Phi_E^{\Delta\vec{S}_6} = \dfrac{\partial E_z}{\partial z}\Delta x \Delta y \Delta z \end{cases}$$

$$\Rightarrow \oint_S \vec{E}.d\vec{S} = \sum_{n=1}^{6} \Phi_E^{\Delta\vec{S}_n} = \left[\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right]\Delta x \Delta y \Delta z$$



$$\oint_S \vec{E}.d\vec{S} = \left[\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right]\Delta x \Delta y \Delta z$$

Gauss : $\Phi_E^S = \oint_S \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0}\int \rho(\vec{x})dV = \frac{1}{\varepsilon_0}\rho(\vec{x})\Delta x \Delta y \Delta z$

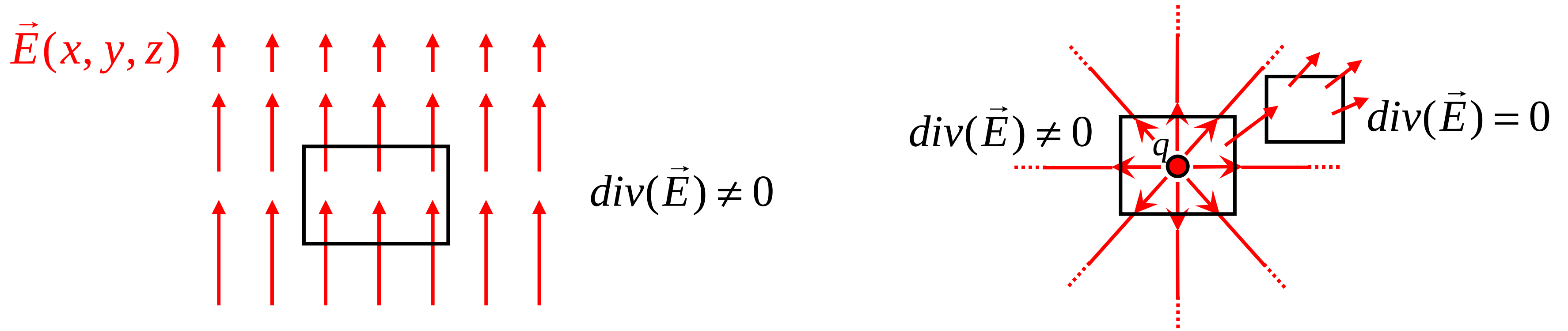
$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{1}{\varepsilon_0}\rho(\vec{x})$$

"divergence" d'une fonction vectorielle

$$div[\vec{f}(x,y,z)] \equiv \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

$$div(\vec{E}) = \frac{1}{\varepsilon_0}\rho(\vec{x})$$

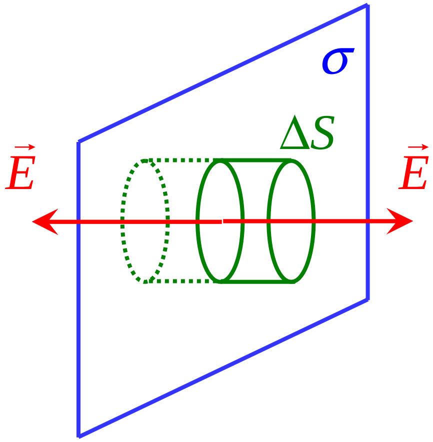
Théorème de Gauss (forme locale)



Application du théorème de Gauss :

$$\oint_S \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0}\int \rho(\vec{x})dV$$

- calcul du champ dû à un plan chargé



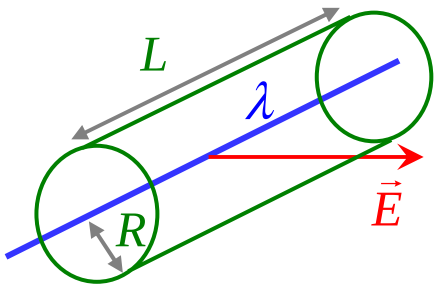
$$\Phi_E^S = \oint_S \vec{E}.d\vec{S} = \left|\vec{E}\right| \; \Delta S + \left|\vec{E}\right| \; \Delta S$$

$$\Phi_E^S = \frac{1}{\varepsilon_0}\int \rho(\vec{x})dV = \frac{1}{\varepsilon_0}\sigma \; \Delta S$$

$$\Rightarrow \left|\vec{E}\right| = \frac{1}{2\varepsilon_0}\sigma$$

$$\vec{E} = \frac{1}{2\varepsilon_0}\sigma \; \vec{1}_S$$

- calcul du champ dû à un fil rectiligne chargé



$$\Phi_E^S = \oint_S \vec{E}.d\vec{S} = \left|\vec{E}\right| \; 2\pi RL$$

$$\Phi_E^S = \frac{1}{\varepsilon_0}\int \rho(\vec{x})dV = \frac{1}{\varepsilon_0}\lambda \; L$$

$$\Rightarrow \left|\vec{E}\right| = \frac{1}{\varepsilon_0}\frac{\lambda}{2\pi R}$$

$$\vec{E} = \frac{1}{\varepsilon_0}\frac{\lambda}{2\pi R} \; \vec{1}_R$$