Loop Antenna Basics and Regulatory Compliance for Short-Range Radio

Part 5 of this series addresses the basics of loop antenna design for Short-Range Radio using integrated PLL transmitters. The several basic matching methods are reviewed and contrasted for efficiency and harmonic performance, and a fundamentally sound and apparently new tapped loop antenna analysis based on the underlying electromagnetics is presented.

Jan Van Niekerk, RF Applications Engr Mgr, (480) 792-4150, jan.van.niekerk@microchip.com
Farron L. Dacus, RF Architecture Mgr, (480) 792-7017, farron.dacus@microchip.com
Steven Bible, Principal RF Applications Engr, (480)792.4298, steven.bible@microchip.com
Microchip Technology Inc.
2355 West Chandler Blvd
Chandler, AZ 85224-6199
www.microchip.com

Introduction. In this 6 part series on short-range radio we have covered one way short-range system design including link budgeting, regulatory issues, and some issues of silicon design at the transmit side. Parts 1, 2, 3, and 4 published in Sept. and Oct., 2001, and in Feb. and March, 2002. We close out this introductory series with practical antenna design and regulatory compliance using primarily single ended drivers, with an introduction to the understanding of differential drivers. There is some analysis available on matching these antennas, though this is often error prone due to lack of appreciation of the strong impact of the underlying electromagnetics. Since most of the available literature is aimed at the paging receiver application, there is little information on the harmonic performance needed in transmit mode to meet regulatory requirements. An apparently original method of fundamentally sound analysis of the matching of the tapped small loop antenna is given, one that is somewhat in disagreement with the established methods, but that we have confirmed experimentally and via electromagnetic simulation. Though the term "tapped loop" is common, we shall refer to this method as the "transformer" loop antenna in reference to what is actually its fundamental mode of operation. This method also leads directly to understanding the harmonic performance of the tapped/transformer loop antenna. Basic performance is contrasted with the other basic matching choices to help in understanding the risk/cost/performance trade-offs.

The Basic (unmatched) Loop Antenna. Most control and security type applications in the UHF range require antennas on the PCB, due to the combination of small size, high ruggedness, and low cost required of these designs. The frequency range involved is generally 285 to 470 MHz (see Part 2), where a full sized quarter wave whip would be from 6.28 to 10.4 inches, or 16.0 to 26.3 cm. This size generally eliminates full sized whips, leading to the printed loop as the most popular. This antenna generally only has 1%-20% radiation efficiency, but it is small, easy to design (with the exception of significant errors in some published matching methods), insensitive to minor design errors since it usually has to be tuned, provides a modest amount of harmonic suppression (improved by matching, as discussed later), and may actually have enhanced efficiency when near the human body. The low conductivity of the human body decreases electric field and increases magnetic field (Ref. 8, p. 295), and has led to the general view that electrically small "magnetic" loop antennas are the most efficient for miniature human worn equipment like pagers, RF tags, and controllers. The magnetic field intensification near (within one quarter wavelength of) the human body is about 4.5 dB at 285 MHz, dropping to about 2.8 dB at 470 MHz and 0 dB at 900 MHz. In use the loop inductance is usually considered to be parallel resonated with a variable tuning capacitor so that the driver sees a large real load which must be matched for optimum power delivery. Other options to manual tuning include resistor de-Queing to allow fixed capacitors and on-die automatic tuning, but unfortunately the losses imposed by these methods are sometimes unacceptable. In particular, when a low cost wideband receiver must be used that prevents setting the IF bandwidth to match the spectral occupancy of the transmitted signal, then "averaging" as described in Part 2 is often used to maintain link quality, which requires higher radiation efficiency and thus usually a well matched and individually tuned high Q loop antenna.

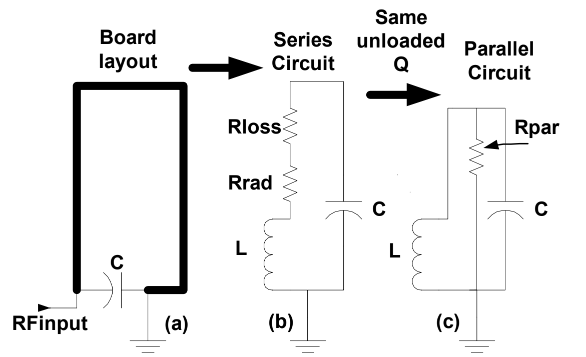

Figure 11: (a) Loop antenna physical implementation (b) The standard loop antenna model. (c) Transformed where all resistances are viewed in parallel.

Figure 11 shows the standard loop antenna model where series and loss resistances are moved over to give a total parallel equivalent resistance that sets the Q of the loop. A matched single ended driver would provide similar loading by driving into the non-grounded end of the capacitor, and Q will be cut approximately in half from the limit set by radiation and loss resistance. If the loop is directly driven by a lower impedance PA (unmatched) then Q will be lower still.

The radiation resistance of a loop, under the condition that it is electrically small (perimeter < 0.3 λ), is given (Ref. 9) as:

$$R_{rad} = 320\pi^4 \left(\frac{A^2}{\lambda^4} \right) \quad (45), \text{ where}$$

A = loop area (perimeter as center of width of trace) in square meters
λ = wavelength in meters

For the frequencies and sizes normally used, this equation generally holds out to about the 2nd to 4th harmonic and is adequate for predicting the lower order harmonic performance where regulatory compliance is more commonly an issue. At higher frequencies where the antenna is not electrically small, the current in the antenna varies as a function of position, and must be taken account of as outlined in ref. 13 or via simulation. For a rectangular antenna with sides L_1 and L_2 built in copper, given copper conductivity = 5.8E7, eq. 45 becomes:

$$R_{rad} = (3.84E - 30)(L_1 L_2)^2 f^4 \quad (46)$$

An expression for loss resistance derived from fundamental principles (skin depth based analysis), assuming that line width is much greater than line thickness, but thickness is also much greater than skin depth (true for practical boards), is given by:

$$R_{loss} = \frac{l\sqrt{\dfrac{\pi f \mu_0}{\sigma}}}{2w} \quad (47)$$

In eq. 47,

l = the total perimeter of the antenna in meters, measured at the center of the trace
w = the width of the trace in meters
σ = conductivity
μ = permeability

For the common rectangular antenna case with copper trace and with permeability of 1.256E-6, eq. 47 becomes:

$$R_{loss} = \frac{L_1 + L_2}{w}(2.61E-7)\sqrt{f} \quad (48)$$

The radiation efficiency of the loop is commonly given as:

$$\eta_r = \frac{R_{rad}}{R_{rad} + R_{lossL} + R_{lossC}} \quad (49)$$

For a given driving current to the loop this expression follows immediately from power being i^2R. The alert reader with RF design experience may immediately wonder about driving current changing with variation in loss and matching resistance if a perfect match is provided by other circuitry. A simple analysis can show that if match is maintained, this same expression results if efficiency is defined as the radiated power divided by the total driving power. Though often neglected, losses associated with the resonating capacitor are usually significant and are counted in the denominator of eq. 49 as another series resistance loss term. Good COG capacitors will typically have series loss resistances of 0.1 to 0.2 ohms, variable capacitors from 0.1 to 0.5 ohms, and X7R and Z5U dielectrics 0.5 and 1 ohms (see www.murata.com for an excellent database of these losses over capacitor construction, value, and frequency). These capacitor losses can dramatically affect both radiation efficiency and matching, and can have a moderate effect on harmonics.

It is often helpful in analysis to transform losses between series and parallel modes, which is valid around a narrow range of frequency. Using series losses as the base mode, we may define:

$Q_s = \frac{X_s}{R_s}$ (50), from which analysis gives the following highly useful set of basic relations.

$$R_p = R_s(Q_s^2 + 1) \quad (51)$$

$$X_p = X_s\left(\frac{Q_s^2 + 1}{Q_s^2}\right) \quad (52)$$

$$L_p = L_s\left(\frac{Q_s^2 + 1}{Q_s^2}\right) \approx L_s\left(for\,high\,Q\right) \quad (53)$$

$$C_p = C_s\left(\frac{Q_s^{\,2}}{Q_s^{\,2}+1}\right) \approx C_s\,(for\,high\,Q) \quad (54)$$

$$R_p = R_s(1+Q_s^2) = \frac{(\omega\,L_s)^2}{R_s} + R_s \approx \frac{(\omega\,L_s)^2}{R_s}(for\;high\;Q) \quad (55)$$

$$R_p = R_s(1+Q_s^2) = \frac{1}{(\omega C_s)^2\,R_s} + R_s \approx \frac{1}{(\omega C_s)^2\,R_s}\,(for\,high\,Q) \quad (56)$$

Of course, to resonate a loop we require an expression for loop inductance. A remarkably simple formula for inductance of a polygon of general shape that is usually good to within 5% is given by Ref. 10 as:

$$L = \frac{\mu}{2\pi}\,l\ln\left(\frac{8A}{lw}\right) \quad (57)$$

In eq. 57 *l* is perimeter as measured at the center of the trace, w is width, and A is area.

Let us consider an illustrative numerical example of a loop size we shall later match in several ways. Assume operation at 434 MHz (common European choice) with a rectangular antenna of 3.4 cm by 1.2 cm, with trace width of 2 mm, and with a capacitor with series loss at this frequency of 0.138 ohms. We may calculate loss resistance of 0.250 ohms, radiation resistance of 0.0227 ohm, total series resistance of 0.286 ohms, and resulting maximum efficiency of 7.95%. From eq. 57 inductance is 52.9 nH, and resonating capacitance is thus 2.54 pF. The unloaded Q is 505 and the equivalent parallel resistance is 72.9K.

The drivers on low power transmitters would normally have an output impedance of from 50 ohms to several KOhms, so a direct connection across this loop is obviously a bad mismatch that would not attain the maximum possible efficiency. The low impedance of the typical driver would also lower the Q drastically and reduce the harmonic rejection of the antenna. Despite these disadvantages an unmatched loop is occasionally used, so analysis is provided as follows. The total loss resistances of the loop antenna where losses are modeled as a resistance is series with the inductor are:

$$R_{LStot} = R_{rad} + R_{lossL} + (\omega_h^{\,2}\,LC)^2\,R_{lossC} \quad (58)$$

In eq. 58 R_{rad} is radiation resistance, R_{lossL} is ohmic loss resistance in the loop, R_{lossC} is capacitor series loss resistance, and all these are functions of frequency as described above. The coefficient of R_{lossC} is 1 at the fundamental, but greater than 1 at the harmonics. This coefficient results from moving capacitor series loss R_{cs} over to be in series with the inductor for modeling purposes. This sum may be represented in parallel form at the fundamental and harmonic frequencies by eq. 55, giving a quantity we shall call R_{PtotH}, where "H" represents the harmonic number and is 1 for the fundamental. Assuming the antenna still satisfies the constant spacial current approximation for the first few harmonics, we may write the radiation efficiency for the fundamental and first few harmonics as:

$\eta_H = \frac{R_{rad}}{R_{LStot}}$ (59), where it is understood that R_{rad} must be found from eq. 45 at the appropriate harmonic H. Converting impedances to admittances (and resistance to conductance), we may write a handy current divider function expressing the fraction of current at each harmonic H that flows in the equivalent parallel conductance at each harmonic. This conductance, G_{PtotH} in the equation below, also contains the loss resistances of the loop antenna and the capacitor. The desired divider function is given by:

$$D_{IH} = Mag\left(\frac{G_{PtotH}}{G_{PtotH} + G_{driver} + j(\omega_h C - \frac{1}{\omega_h L})} \right) \quad (60)$$

At the fundamental the circuit is resonant and the imaginary component is zero, but at the harmonics it is dominated by the capacitance and most of the driver current available at the harmonics is shunted to ground and does not radiate. The ratio of each harmonic current to driver fundamental current is needed to determine the harmonic rejection, but for approximation we may assume that the fundamental power is 10 dB over the first few harmonics (typical for a compressed class A single ended PA), but that the antenna is 5 dB more directional for the harmonics. The harmonic rejection (radiated harmonic to carrier power) in measured field strength for each harmonic H may thus be approximated to about +/- 5 dB accuracy as:

$$\frac{P_H}{P_1} \approx \frac{\eta_H}{\eta_1} \frac{0.316(D_{IH})^2 R_{PtotH}}{(D_{I1})^2 R_{Ptot1}} \quad (61)$$

The mismatch of the directly driven loop (power applied at the loop capacitor) is large. In general, for a source with impedance R_{driver} driving a load of parallel impedance Z_{in} (which at resonance should be real impedance R_{in}) , the "mismatch loss" (which does not include efficiency losses) may be determined by:

$$MisMatchLoss = \frac{4\left(\frac{Z_{in}}{R_{driver}}\right)}{\left(\frac{Z_{in}}{R_{driver}}\right)^2 + \frac{2Z_{in}}{R_{driver}} + 1} \quad (62)$$

See Table 7 for example performance numbers for the loop antenna numerical example given earlier (a 1.2 X 3.4 cm loop at 434 MHz) when directly driven by a source of 1.4KOhm impedance. The mismatch loss in this case is about 11 dB, the efficiency about 8% (resulting total efficiency less than 1%), and the harmonic rejection just over 20 dB. There is risk of failing harmonic regulatory requirements (see Part 2) in addition to a generally weak link performance, though the poor Q of the antenna is enabling towards not requiring tuning.

Tapped Capacitor Loop Antenna. The large mismatch and relatively poor harmonic suppression of the unmatched loop antenna may be much improved by the tapped capacitor matching method shown in single ended form in Fig. 12. Here the fundamental definition of "matching" is seen in elegant simplicity, where it means *viewing* the total set of loss impedances in the loop antenna as the single input parallel impedance $R_{par} = Z_{in}$ that gives the same unloaded Q. When R_{par} matches driver resistance, the maximum power transfer theorem is satisfied and the loaded Q will be half the unloaded Q. Intuitively, the tapping may be seen to give a down impedance transform through conservation of energy, with the voltage at the tap point lowered from the inductor voltage by the capacitive divider action, and thus requiring a lower impedance (if all loss resistance is modeled at that point to give the same Q) at the tap point to dissipate the same power. Pursuing this analytically will give the analysis equation 63 for parallel Z_{in} as a function of inductor parallel resistance R_p.

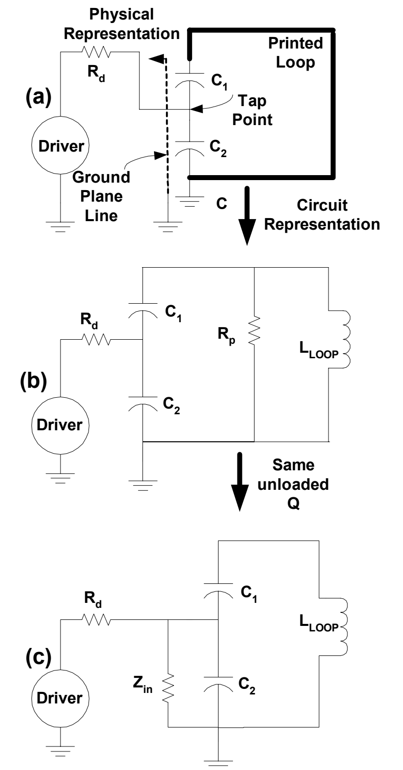

Figure 12: Single ended tapped capacitor antenna matching.

$$Z_{in} = \left(\frac{1}{1+\frac{C_2}{C_1}} \right)^2 R_p \quad \text{(63) (applies at resonance)}$$

To develop design equations for the tapped capacitor case and to understand its harmonic performance, we first write the broadband conductance looking into the capacitor tap as:

$$G_{in} = \frac{R_s}{R_s^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2} + j\left(\omega C_2 - \frac{\omega L - \frac{1}{\omega C_1}}{R_s^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2} \right)$$

(64)

In eq. 64 R_s is the resistance in series with the inductor that models all losses. We desire to solve this equation for the C_1 and C_2 force the desired Z_{in} and resonant frequency. The reciprocal of the real part of eq. 64 gives the input impedance at resonance and provides one equation. Setting the imaginary part equal to zero at the desired resonant frequency gives the other. The results are:

$$C_1 = \frac{1}{\omega_0 \left(\omega_0 L - \sqrt{Z_{in} R_s - R_s^2}\right)} \quad (65)$$

$$C_2 = \frac{L - \frac{1}{\omega_0^2 C_1}}{R_s^2 + \left(\omega_0 L - \frac{1}{\omega_0 C_1}\right)^2} \quad (66)$$

Eq. 64 also provides the way to understand the harmonic performance of the tapped capacitor loop antenna. For the large impedance transform from parallel resistance across the inductor to parallel Z_{in} at the tap point C_2 will normally be much larger than C_1, and much larger than the C of the unmatched loop. Thus, C_2 dominates the input conductance at the tap and shunts most current to ground, greatly improving harmonic rejection. This may be quantified in a manner similar to the unmatched loop, where we find the "current divider function" for harmonic current that flows in the real part of the input impedance (where it must flow to be radiated) as:

$$D_{IH} = Mag\left(\frac{\mathrm{Re}(G_{inH})}{G_{inH} + G_{driver}}\right) \quad (67)$$

Despite this divider function, some current still flows in the real part of loop radiation resistance that is transformed to the input, and it is this current that radiates power. The radiated power at the fundamental (H = 1) and at each harmonic (where the loop is still "small") is:

$$P_{radH} = \frac{\eta_H \left(i_{rmsH} D_{IH}\right)^2}{\mathrm{Re}(G_{inH})} \quad (68)$$

In eq. 68, i_{rmsH} is the rms current available from the source at harmonic frequency H, and is the fundamental current when H = 1 (where D_{IH} = 0.5 due to the match condition). The harmonic rejection relative to the carrier is given by the ratio of harmonic power in eq. 68 to the radiated carrier power also from eq. 68 (H=1), degraded by the extra directivity of the antenna at the harmonic frequency. Assuming the applied harmonics are 10 dB down from the carrier and that the antenna is no more than 5 dB more directive for the harmonics gives the approximation:

$$\frac{P_H}{P_1} \approx \frac{\eta_H}{\eta_1} \frac{1.26(D_{IH})^2}{\mathrm{Re}(G_{inH}) Z_{in1}} \quad (69)$$

At the harmonics we may for hand calculations simplify eq. 64 to be:

$$G_{inH} \approx \frac{R_{sH}}{(\omega_H L)^2} + j\omega_H C_2 \quad (70)$$

Technically both C_1 and C_2 must be well controlled to meet both desired resonance and input impedance conditions. In practice, with a variable capacitor for C_1 or C_2, the tapped capacitor method can yield a good match, near perfect resonance, and harmonic rejection over 40 dB. For the loop antenna discussed earlier at 434 MHz, with driver impedance of 1.4 KOhm, we find L = 52.9 nH, C1 = 2.95 pF, C2 = 18.3 pF, predicted second harmonic of -50.6 dBc, and predicted 3rd harmonic of -52 dBc. There is little mismatch loss, so the total efficiency is the loop and capacitor efficiency of about 8%. These harmonics will normally pass all regulatory requirements, but to achieve such low loop harmonic levels the board designer must beware of parasitic radiation from traces and bond wires that may actually dominate measured performance.

Transformer Approach to Loop Matching. We may also take a transformer approach to the matching of small loop antennas, and this approach is common due to its minimum parts count for inductively loaded configurations that maximize output power. As shown in Fig. 13, a small loop is placed near (usually actually sharing a side with) the radiating loop antenna. The radiating loop still contains a tuning capacitor C. The two loops actually form a loosely coupled transformer, though there is a strong tendency among circuit designers to want to view this structure as a tapped inductor (no mutual coupling) or autotransformer (tapped inductor with mutual coupling). The transformer model seems counterintuitive even to experienced RF designers, since they are trained to think in lumped component terms and not in the underlying electromagnetic terms upon which lumped models are based. Thus they normally conceive of a segment of trace as having complete inductance all by itself in the absence of a return path, which leads them to misinterpret Fig. 13 as a tapped inductor or autotransformer. No less an authority than Fujimoto (ref. 9) in his well respected work on small antennas mistakenly analyzes the loop antenna matching as an autotransformer, and this common error incorrectly influences the design of loop antennas to this day. The mistaken mental model has at its root the failure to understand that only closed current loops have inductance or mutual inductance. It is exacerbated by the fact that the form of transformer exhibited by Fig. 13 is not one the engineer has encountered in his basic training--no class we ever took showed a separated transformer model for a situation where primary and secondary currents actually share a path segment.

An open mind and a review of the underlying electromagnetics shall allow the short-range radio designer to add this important form of transformer antenna to his tool kit. To set about developing the correct first order understanding of this structure we shall state the basic electromagnetics upon which our transformer model argument is based with minimal explanation, leaving the reader to review his basic undergraduate e-mag text for verification. But, we shall interpret this electromagnetics with respect to this new situation, the loop antenna of Figure 13, in some detail to make the model fully clear and generate the correct mental model in the reader's mind.

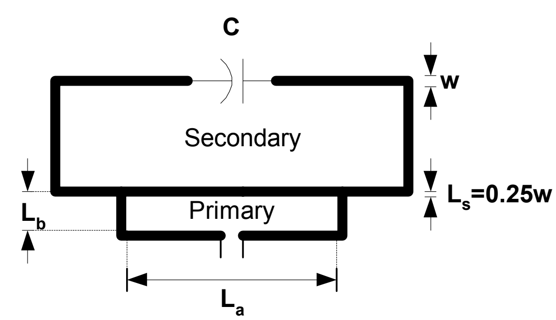

Figure 13: Transformer matched loop antenna geometric structure.

We first consider the definition of a voltage (electromotive force emf) as the closed line integral of electric field, which is the field form of Kirchoff's Voltage Law:

$$emf = \oint \vec{E} \bullet \overrightarrow{dL} \quad (71)$$

Next consider the fact that electric flux through a surface is given as the surface integral of flux density over that surface:

$$\Phi = \int_S \vec{B} \bullet \overrightarrow{dS} \quad (72)$$

Faraday's Law giving voltage (emf) as a function of flux is given as:

$$emf = -\frac{d\Phi}{dt} \quad (73)$$

Ampere's Law giving current as the closed line integral of magnetic field is:

$$I = \oint \vec{H} \bullet \overrightarrow{dL} \quad (74)$$

where magnetic field H is related to flux density B by:

$$\vec{B} = \mu_0 \vec{H} \quad (75)$$

When we can calculate terminal voltage and current, and can take impedance as their ratio, we have a circuit model captured. The electromagnetic equations above provide the means to get current and voltage relationships in terms of the physical geometry. Ampere's Law relates flux and current, over a closed line integral giving current contained within the closed path. From Ampere's Law, current can be found from H or B, or H and B can be found from current. When B is known, total flux can be found from eq. 72, and then with flux known, voltage can be found from eq. 73. Conceptually we have thus developed the full information needed for the circuit model, and from eqs 71-74 we see that this always relies upon closed paths around current or field, and not upon a line segment. Alternately, we can use the definition of inductance and mutual inductance given below to make this conceptual process a bit shorter:

$$L = \frac{N\Phi}{I} \quad (76)$$

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1} \quad (77)$$

N is the number of filamentary loops of current (one in Fig. 13) and I is the current "linked" by the flux, meaning the current that surrounds the area the flux density is integrated over to get the total flux. In eq. 77, M_{12} is the mutual inductance where flux produced by closed (or infinite) path I_1 links current in closed or infinite path I_2. It is also true that $M_{12} = M_{21}$. L and M result in circuit equations of the form:

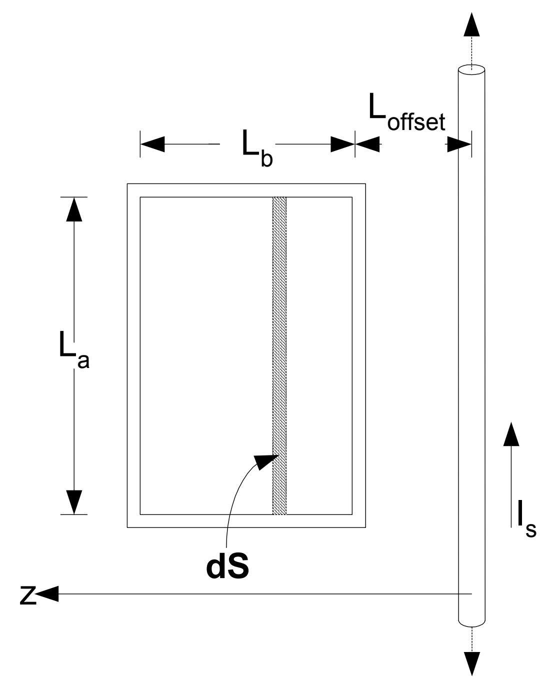

Figure 14: The mutual inductance between an infinite wire and small nearby loop provides a useful approximation of the mutual inductance between the primary and secondary of a tapped loop antenna.

$$V_1 = L\frac{dI_1}{dT} + M\frac{dI_2}{dT} \quad (78)$$

where V_1 is the total voltage through a self inductance with current I_1 that is also linked to a second current I_2 sharing mutual inductance M with the current path described by I_1. Note that in 76 and 77 inductance cannot be calculated for a segment of line. It requires a closed path around a surface to get the total flux quantities as the surface integral of flux density. This is why a tapped inductor or autotransformer model of Fig. 13 is simply wrong--it does not satisfy the definition of inductance. But an integration over a closed surface, such as the primary and secondary shown in Fig. 13, gives total flux linking a closed current path, which then by 76 and 77 allows calculation of self and mutual inductance that allows writing circuit equations.

With our normal circuit design mental model altered to take these fundamentals into account we may now start sneaking up on a correct (transformer based) circuit model for Fig. 13. Referring to Fig. 14, we have a loop intended as the primary of inner dimension L_a and L_b linked by the flux generated by an infinitely long thin round wire. The loop is here considered to be made of thin round wire also and its inner dimension is separated from the center of the infinite wire by distance L_{offset}. Of course, most antennas will be made on circuit board and thus made of flat trace, but the round wire model is simpler analytically and is a good approximation to an antenna made of trace, and so is used here. Most basic e-mag texts will go through the small exercise needed to use Ampere's Law (eq. 74) to get radial H and B fields around the infinite round wire induced by current I_s in the wire. This gives:

$$\vec{B} = \frac{\mu_0 I_s}{2\pi\ z} a_\phi \quad (79)$$

Using eq. 72 and the differential area element **dS** shown to integrate over the area of the primary (L_a X L_b), a few lines will yield the flux, and then dividing by I_s as per eq. 77 gives the mutual inductance:

$$M = \frac{\Phi}{I_s} = \frac{\mu_0\ L_a}{2\pi} \ln\left(1 + \frac{L_b}{L_{offset}}\right) \quad (80)$$

Eq. 80 is a usefully accurate approximation (slightly large) to the mutual inductance between a small primary and large secondary, as the other sides of the secondary are much farther away from the primary. The separated form as shown may be used if the maximum possible mutual inductance is not needed (it will be shortly shown how impedance is controlled by mutual inductance). If maximum mutual inductance is desired, the two loops may be brought into actual contact, at which point L_{offset} will be equal to the radius of the secondary wire plus the diameter of the primary wire (not zero, which would be unacceptable in the denominator in eq. 80). When the two loops are brought into contact there will be no drastic change in the circuit model, which is the tricky point for most circuit designers to accept. The only effect contact has on the model form is to force the primary and secondary current to mix in the shared segment, but this does not change the fundamental nature of the structure giving the mutual inductance which dominates the behavior. When the currents are shared in the segment, there is a small voltage induced in both the primary and secondary due to resistance in the shared segment, not only from each on its respective side, but also from the other. This leads to the technical need for the model to have either a single resistor in the common (to ground) terminal of primary and secondary, or for a "trans-resistance" to be inserted in each of the primary and secondary. It is critically important to note that the contact does not force an autotransformer model. The shared segment is not an inductor, only the complete current loops of primary and secondary are true inductors. An autotransformer model would be appropriate only if a loop of primary were drawn inside the secondary.

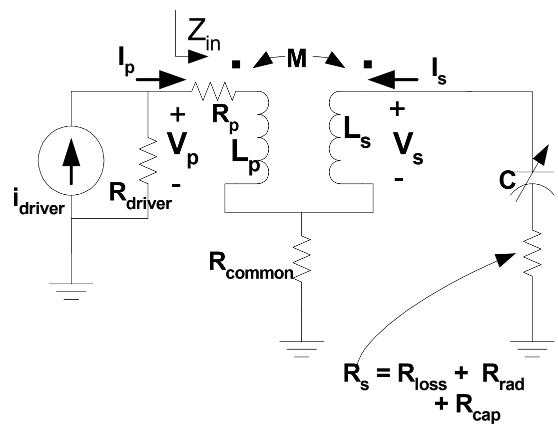

Figure 15: Circuit model of transformer matched loop antenna acts as a separated transformer with the minor exception of the shared resistance over the common section of trace.

Fig. 15 is the desired circuit model for the transformer matching case using the concepts developed above. The small loop is designated as the primary and the large loop as the secondary of the transformer. Despite the fact that the loops of Fig. 13 are touching and share a side, the structure truly functions as a separated transformer with the exception that the shared side has a loss and radiation resistance that is represented as R_{common}. Normally this common resistance is so small that it may be set to zero in calculations.

The transformer of Figure 15 is not an "ideal" transformer with infinite inductance and winding ratio N that gives impedance transform N^2. It is a linear transformer of winding ratio one for which full circuit equations must be written. Neglecting R_{common}, we may write primary and secondary KVL equations as:

$$I_p(j\omega L_p + R_p) + I_s j\omega M = V_p \quad (81)$$

$$I_p j\omega M + I_s\left[j(\omega L_s - \frac{1}{\omega C}) + R_s \right] = 0 \quad (82)$$

Solving this equation set for primary voltage and current and then taking their ratio as input impedance yields (see Hayt, ref. 15):

$$Z_{IN} = \left(R_P + \omega^2 M^2 \frac{R_S}{R_S^2 + X_S^2} \right) \quad + \quad j \left(X_P - \omega^2 M^2 \frac{X_S}{R_S^2 + X_S^2} \right) \quad (83)$$

Z_{IN} is the total complex input impedance. We have used X_P as the magnitude of the reactance of the primary and X_S as the magnitude of the reactance of the secondary to simplify. M is the transfer inductance between the two loops, with units in Henrys. From eq. 83 we note that when X_S is zero (secondary resonance), Z_{IN} still contains some reactance from the primary inductor impedance X_P. In practice, an extremely small amount of secondary reactance change is required (by varying say the loop capacitance slightly) to obtain a purely real Z_{IN}. From (72) we find that the input impedance at resonance is approximately

$$Z_{IN} \approx R_P + \omega^2 M^2 \frac{1}{R_S} \quad (84)$$

Equation 84 provides a simple method to match the low resistance R_S of a resonant loop antenna to the KOhms required by CMOS integrated circuits. Initially eq. 84 is used to calculate the needed transfer inductance M to achieve a specific input impedance $Z_{IN} = R_D$ for matching. Secondly, using eq. 80, L_a, L_b and offset are adjusted until the required M is achieved. Eq. 80 will generally be found to be accurate within about 10%, but if the greatest possible accuracy is desired, an electromagnetic simulator can be used to refine the geometry more closely. As we will see later, to minimize radiation from the primary loop and to lower primary loop reactance, L_a should be made as large as possible, and L_b and offset should be made as small as possible.

Harmonic Behavior. At higher frequencies input impedance eq. 83 simplifies to:

$$Z_{inH} \approx R_P + \frac{M^2}{L_S^2} \cdot R_S \quad + \quad j\omega L_p \qquad (85)$$

Table 7: Calculated performance of the unmatched and matched 12 X 34 mm loop antenna at 434 MHz. The loop has inductance of 52.9 nH, series loss resistance of 0.125 ohm, and capacitor series loss of 0.138 ohm. Parasitic harmonics such as off power supply lines are not included.

	Mismatch Loss (dB)	Total Eff.	2nd Harm Rej (dB)	3rd Harm Rej (dB)
Unmatched	11.3	0.59%	22.1	23.6
Tapped Capacitor	0	8%	50.6	52.0
Transformer	0	8%	41.5	36.0

In the transformer case we have current flowing through the primary and secondary loops. Like any current loop, the primary loop is an unavoidable contributor to radiation. Also, except for right around the fundamental, the primary exhibits a broadband response with little filtering of the first few harmonics (up to the point where $j\omega L_p$ exerts a pole), unless additional filtering such as a parallel tank is used in the driver output. The primary can thus dominate over the secondary as a harmonic radiator, though if the primary area is kept as small as possible (large L_a to get necessary mutual inductance, small L_b to keep primary area small) it will normally fall a few dB under the secondary. To calculate R_{rad} for the primary and secondary loop use eq. 46 twice. Then use eq. 48 twice to calculate loss resistance for both loops. We may then rewrite the input impedance at the harmonics in terms of the primary and secondary resistances of eq. 85 as

$$Z_{inH} = (R_{lossPH} + R_{radPH}) + \frac{M^2}{L_S^2} \cdot (R_{lossSH} + R_{radSH} + R_{cSH}) \quad + \quad j\omega L_p \quad (86)$$

In eq. 86 the terms are: R_{lossPH} = primary loop series ohmic loss at harmonic H, R_{radPH} = primary loop series radiation resistance at H, R_{lossSH} = secondary loop series loss at H, R_{radSH} = secondary loop radiation resistance at H, and R_{cSH} = secondary tune capacitor series loss at H.

Assuming that the source resistance $Z_D >> j\omega L_p$, we may use the real part of eq. 86 to write the ratio of radiated harmonic power P_H to carrier power P_1 as

$$\frac{P_H}{P_1} = \frac{\eta_H \, i_{rmsH}^2 \, \mathrm{Re}(Z_{INH})}{\eta_1 \frac{i_{rms1}^2}{2} R_D} \qquad (87)$$

The harmonic radiation efficiency is given by:

$$\eta_H = \frac{R_{radPH} + \frac{M^2}{L_S^2}\left(R_{radSH}\right)}{(R_{lossPH} + R_{radPH}) + \frac{M^2}{L_S^2} \cdot (R_{lossSH} + R_{radSH} + R_{cSH})} \qquad (88)$$

Further assuming harmonic power to be 10dB below the carrier and then adding back 5dB for harmonic directivity we reduce (79) to

$$\frac{P_H}{P_1} = 0.632 \cdot \frac{\eta_H}{\eta_1} \cdot \frac{\mathrm{Re}(Z_{INH})}{R_D} \quad (89)$$

If Z_D is not >> $j\omega L_p$, then a current divider function may be written similarly to the tapped capacitor case and added to eqns. 87 and 89.

The basic performance of the unmatched, tapped capacitor, and inductor antennas with the same 12 X 34 mm radiating loop is summarized in Table 7. For the transformer loop, harmonic rejection of 41.5 dB for the second harmonic and 36 dB for the third harmonic was calculated. A 7dB increase in power between the second and third harmonic was expected due to radiation resistance being a fourth-order function of frequency as in eq. 45. It is important to note that the harmonic rejection of the transformer loop antenna is not based on parallel LC filtering, but on extreme mismatching at the harmonic frequency. The loop capacitor brings about the resonance condition which in cooperation with the mutual inductance of the transformer leads to a good match at the fundamental. Away from the fundamental, this match is not supported and the input impedance of the primary is extremely small, so that i^2R radiated power is also small.

Working with Differential Drivers. Most discrete short-range transmitter designs use single ended RF output based on a discrete transistor, and are thus easy to visualize in terms of a driver model referenced to the same ground as RF test instruments. We have used single ended drive in the analyses of this article as it is more illustrative in introducing the basic matching forms. But most integrated transmitters use a differential output that is not as intuitively clear. The desire to carry signals in differential mode is a consequence of the need to maintain amplifier stability in the presence of a relatively poor RF ground inside the chip (separated from board ground by bond wire and pin inductances), the need to maintain power supply and ground common mode noise rejection (the RF circuitry is very close to the digital control circuitry in an integrated transmitter), and the convenience of matched devices on the die that can meet these needs. A secondary benefit is the extra transmit power that can be provided if voltage swing limits with a single device are the limiting power factor.

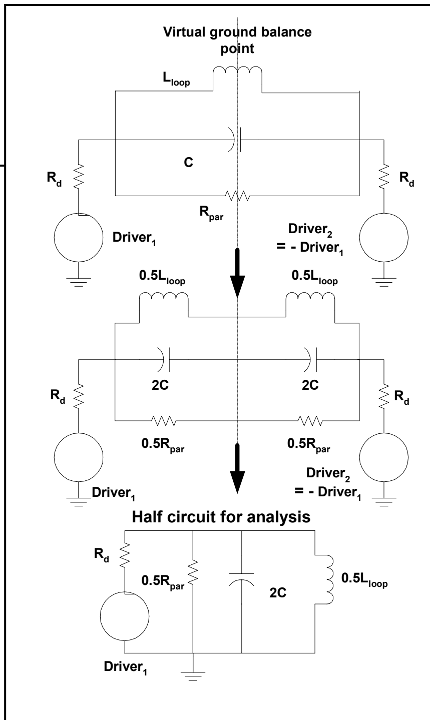

Figure 16: The Half Circuit Concept provides for understanding the application of single ended drive analysis to differentially driven loop antennas.

The easiest way to visualize differential drivers with a loop antenna is to use the "half circuit concept" depicted in Fig. 16. This concept is based on acknowledging the fact that the drivers are matched but have voltage outputs that are 180 degrees out of phase. This results in points on the circuit where the voltage does not swing relative to ground, and these points can be viewed as artificial grounds. This allows us to consider the full antenna as consisting of two half circuits that are each driven single ended, and that each remain resonant at the desired frequency with half the inductance and resistance, and twice the capacitance of the full circuit. Each half circuit also maintains the same Q.

It is not necessary to maintain a perfect geometric balance in a loop antenna to use differential drive. To reduce components parts may be combined. This may result in loop antennas whose functionality is not apparent at a glance, but by breaking the parts back up into the symmetric circuit needed for visualization the operation and matching will become clear. For example, the circuit in Fig. 17 that at first appears to have no matching is seen to actually be the excellent tapped capacitor form, highly efficiently implemented with just two capacitors.

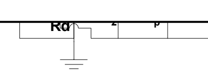

Figure 17: The Half Circuit Concept applied to understand an efficient differentially driven tapped capacitor loop antenna.

Laboratory Measurements. To certify a short-range transmitter as regulatory compliant requires testing and a report generated by a government agency approved laboratory. It normally cost approximately $2,000 per day to conduct such testing, though a clearly passing product might require only a half day for complete testing. In any event, it is expensive in time and money to have to loop through the test lab several times to get a product to pass with empirical experimentation. A far better procedure is to deliberately design the product to pass and perform confirmation testing in an internal lab. The first order analytic methods given in this article allow a good approximation of fundamental power and a fair approximation of what the loop is capable of in terms of low order harmonic rejection. Electromagnetic simulation is the most feasible way to attack harmonic analysis at the higher harmonics or anywhere the small loop approximation does not hold. A future article will present detailed layout guidance to ensure that actual harmonics achieved are not too much worse than what the loop is capable of. With the fundamental set several dB below maximum and with some degree of safety margin on harmonic emissions, one may go to the test lab confident of first pass success.

FCC Measurements. The basic mathematics of field strength measurement are as follows. As mentioned in Part 2 of this series, the Part 15.231 the rms field strength emissions from intentional radiators shall not exceed

$$E_{ss}(f) = .041667(f-260)+3.75 \quad (8) \quad \text{(control operations)}$$

at 3 meters, with spurious emissions required to be down a factor of 10 (more if spurious emissions fall into restricted frequencies specified in Part 15.209). The designer needs a convenient means to measure the field strength emitted by their product. A simple test set up can exist of a quarter wave whip antenna for the fundamental and each harmonic of interest, and a spectrum analyzer. Spectrum analyzers measure received power in watts. Therefore, we need to convert from field strength (V/m) to power (W).

Part 1 presents the "effective aperture" of an antenna given as a function of frequency (wavelength) and directivity, as eq. 1.

$$A_{em} = \frac{\lambda^2 D_0}{4\pi} \quad (1)$$

The directivity D0 of a quarterwave whip is about 1.5. The received power from a test antenna as a function of rms electric field, effective aperture, and free space impedance "η" was given in eq. 2, which may be solved for field strength as:

$$E_{rms} = \sqrt{\frac{\eta P_{rec}}{A_e}} \quad (2A)$$

The FCC emission limits specify received field strength at 3 meters, which may be a bit inconvenient to set up in a cramped lab. The field strength limits may be extrapolated to a convenient range using:

$$E_2 = \frac{R_1 E_1}{R_2} \quad (89)$$

About the minimum range usable is 1 meter, where near field effects are starting to be seen. This set of equations is all that is needed for basic FCC type measurements, though harmonics above the 4th or 5th will be difficult to measure because of declining antenna aperture and their low level compared to the spectrum analyzer noise floor. Higher order harmonics generally require an LNA ahead of the spectrum analyzer and a directional test antenna to improve aperture. Vertical movement and polarization shift of the directional receive antenna will also be needed to emulate the normal measurement practice of certified labs in seeking spacial maximums.

Fig. 18 illustrates the field strength limits for the fundamental frequency 434 MHz and harmonics up to the 10th converted to power in dBm at 1 meter that would be measured with a quarterwave whip.

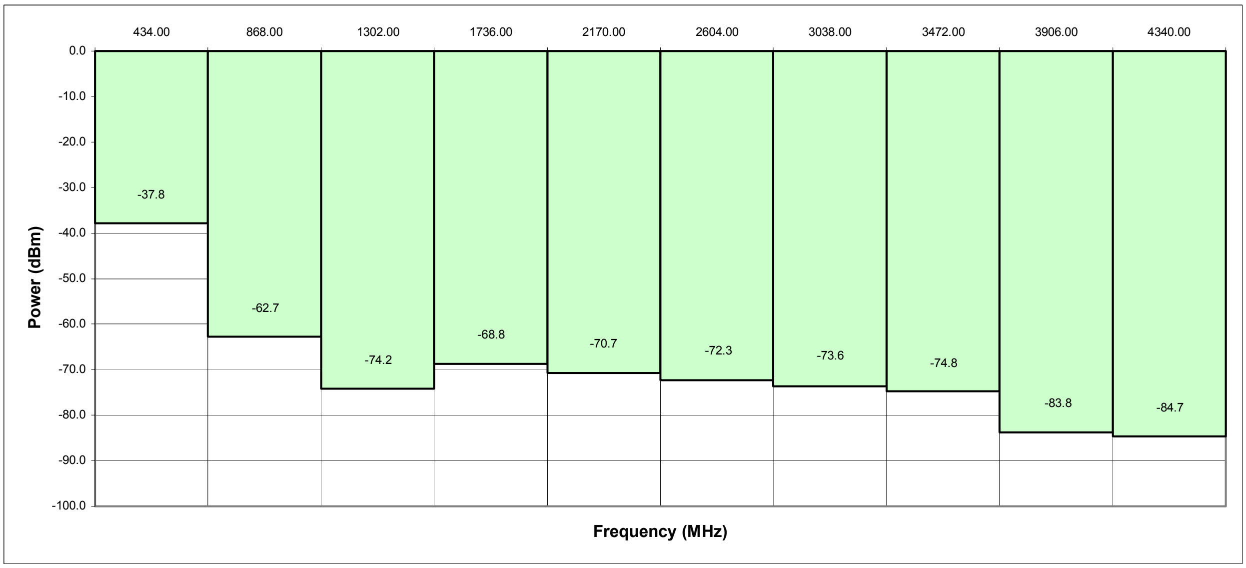

Figure 18: Power limits for 434 MHz fundamental and harmonics as measured with a quarterwave whip at 1 meter that meet the requirements of FCC 15.231. The Excel file to provide this graph for any desired carrier will be made available on the Microchip web site. Note the 3rd harmonic falls into a restricted band and has a more stringent requirement.

European Measurements. European nations generally require measurement of effective radiated power in watts as opposed to field strength. For polarization matched antennas that are aligned on directionality maximums the Friss Transmission Equation given as eq. 3 may be rewritten as:

$$P_{tranERP} = \frac{P_{rec} R^n}{G_{or}} \left(\frac{4\pi}{\lambda} \right)^2 \quad (3A)$$

Here P_{rec} is receive power, $P_{tranERP}$ is effective transmit power (transmitted power times transmit antenna gain), R is range in meters, n is the path loss exponent (2.0 in free space), and G_{0r} is the gain of the receive antenna (directivity multiplied by efficiency loss). The below equations are also useful for transforming between U.S. and European regulatory limits.

$$P_{tranERP} = 0.03333 R^2 E_{rms}{}^2 \quad (91)$$

$$E_{rms} = \frac{5.477}{R}\sqrt{P_{tranERP}} \quad (92)$$

Conclusion. The material contained in this article is sufficient for good accuracy in matching and in predicting fundamental efficiency. We believe the transformer model of the loop antenna to be previously unpublished with the exception of our own recent application note, and that this method for the first time provides a correct basic model of the tapped loop antenna. Here the circuit designer's intuition can lead to erroneous conclusions, and reversion to the underlying electromagnetics is required. Based on the terminal behavior of the loop antenna and its behavior over the first few harmonics where the loop is still electrically small, the relations given should allow approximate prediction of radiated harmonics. We find the tapped capacitor antenna to be capable of excellent harmonic suppression, and the transformer loop antenna to be capable of good suppression. However, one often finds that a given board layout does not meet the predicted harmonic suppression. This is probably most often due to non-ideal effects in the layout, such as harmonic leakage onto power lines that then radiate above the level of the loop. Future articles and application notes will expand the antenna design information presented here with detailed advice on frequency selection, parts tolerance, layout methods, and cost trade-offs. We also plan to extend the general knowledge presented in this series to also cover modulated crystal oscillators, receiver design, protocols, error detection and correction, and the design of firmware specifically for short-range radio applications.

References

8. K. Fujimoto and J.R. James, *Mobile Antenna Systems Handbook*, Second Edition, Artech House, 2001.

9. K. Fujimoto, A. Henderson, K. Hirasawa, and J.R. James, *Small Antennas*, Research Studies Press LTD, John Wiley & Sons, 1987.

10. Frederick Grover, *Inductance Calculations Working Formulas and Tables*, Dover Publications, 1946.

11. Thomas Lee, *The Design of CMOS Radio Frequency Integrated Circuits*, Cambridge University Press, Cambridge, England, 1998.

12. Krauss, Bostian, and Raab, *Solid State Radio Engineering*, John Wiley & Sons, 1980.

13. Constantine Balanis, *Antenna Theory, Analysis and Design*, Second Edition, John Wiley & Sons, 1997

14. Jan van Niekerk, *AN831: Matching Small Loop Antennas to rfPIC™ Devices*, Microchip Technology INC., 2002.

15. W. Hayt and J. Kemmerly, *Engineering Circuit Analysis, third edition*, McGraw-Hill, 1978.

16. W. Hayt, *Engineering Electromagnetics,* Fourth Edition, McGraw-Hill, 1981.