Part 11: Power In AC Circuits

11.1 AC/DC

Power is the rate of doing work or expending energy (section 3.1). The electrical unit of energy is the watt (W) and one watt is the rate of expending energy at the rate of one joule per second. In DC circuit the power dissipated in a resistive circuit is given by:

$$P = UI = I^2R = \frac{U^2}{R}$$

where:
P = power (W)
U = potential difference (PD) (V)
I = current (A)
R = resistance (Ω)

In AC circuits the instantaneous values of voltage, current and therefore power are constantly changing. However, at any instant we can still say that:

$$p = vi$$

where:
p = instantaneous power (W)
v = instantaneous voltage (V)
i = instantaneous current (A)

The RMS values (U, I and P) can be easily used in AC circuits with only resistance however, matters are more complicated when capacitance and inductance are involved. Remember that the RMS values are defined so that a current of RMS 1A AC will produce the same heating effect in a resistor as 1A DC.

11.2 Power In The Resistance AC Circuit

For a resistive circuit current and voltage are in phase and the power at any instant can be found by multiplying the voltage by the current at that instant. Figure 11.1 shows the voltage, current and power waves and it is clear that when the voltage is positive, so is the current and the power is positive. Also, when the voltage in negative, so is the current and therefore the power is again positive. Because of this in-phase relationship, RMS values can be used in the DC power equation:

$$\mathbf{P = UI = I^2R = \frac{U_2}{R}}$$

Many electrical load, such as heaters, irons, kettles and filament bulbs can be considered to be wholly resistive.

<u>Example</u>
A 3kW immersion heater is connected to a 240V AC supply. Calculate the current.

$$I = \frac{P}{V} = \frac{3000}{240} = 12.5A$$

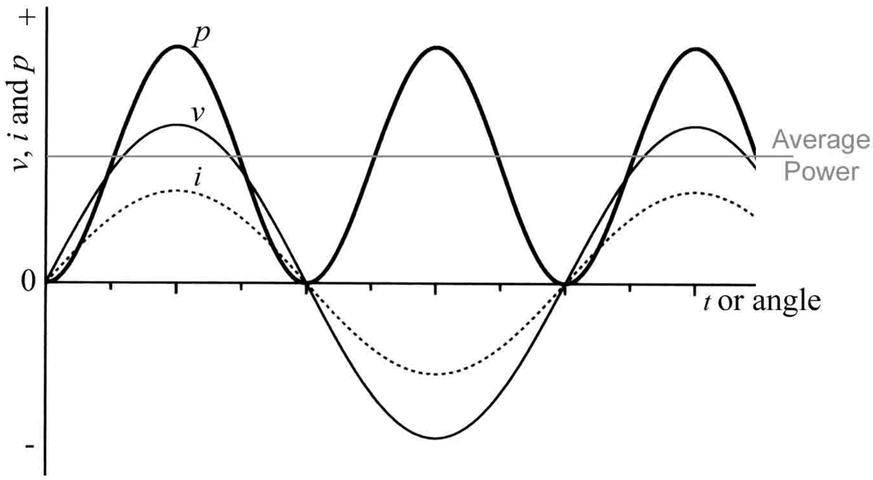

Figure 11.1: Power in an AC resistive circuit. Note that *v*, *i* and *p* are not plotted on the same scale.

11.3 Power In The Capacitive AC Circuit

Figure 11.2 contains the wave diagram for an AC capacitor circuit and shows the current leading the voltage by 90°. In the first-quarter cycle both v and i are positive, therefore the power is also positive (since $p = vi$, at any instant). In the second quarter-cycle *v* stays positive while *i* has gone negative, therefore *p* is negative. In the third-quadrant both *i* and *v* are negative and so *p* is positive. Finally, in the fourth-quadrant *i* is positive and *v* is still negative resulting in *p* being negative. The power wave is thus a series of identical positive and negative pulses whose average value over an half-cycle of voltage is zero, also note that its frequency is twice the frequency of the voltage.

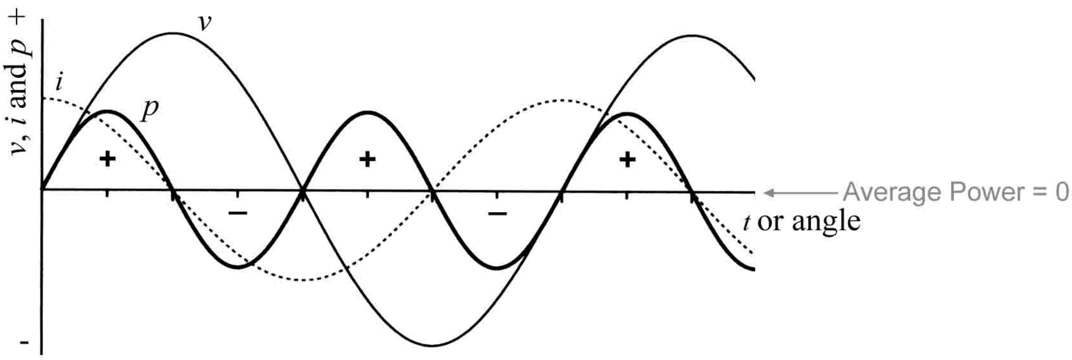

Figure 11.2: Power in an AC capacitive circuit. Note that *v*, *i* and *p* are not plotted on the same scale.

During the first and third quarter-cycles the power is positive meaning that power is supplied by the circuit to charge the capacitor. In the second and fourth quarter-cycles the capacitor is discharging and thus supplies the energy stored in it (section 7.6) back to the circuit, thus *p* has a negative value. Although negative power may seem like an odd concept the minus or plus signs simply indicate the direction in which the power is flowing. Since this interchange of energy dissipates no average power no heating will occur and no power is lost (for a perfect capacitor, that experiences no current leakage, at least).

Since we have a voltage and current but no power dissipated, the expression P = IV (using RMS value) is no longer valid. The product of current and voltage in this case is called *reactive power* or *reactive voltamperes* and is measured in voltamperes (VA_r). (The reactive power is not really power at all and the name is slightly deceptive). The current that flows through the capacitor (that does not have resistance), causes no heating and is called *reactive current*.

Example

10μF capacitor is connected to a 240V, 50Hz supply. Calculate the reactive current and the reactive voltamperes.

$$X_C = \frac{10^6}{2\pi f C'} = \frac{10^6}{2\pi \times 50 \times 10} = 318\Omega$$

$$I = \frac{U}{X_C} = \frac{240}{318} = 0.755A$$

reactive voltamperes = 240 × 0.755 = 181VA_r

11.4 Power In The Inductive AC Circuit

Figure 11.3 contains the wave diagram from an inductive AC circuit and shows the current lagging the voltage by 90°. In a similar state of affairs to that found for the power in capacitors, the power in an inductive circuit consists of positive and negative pulses. The average of these pulses over an half-cycle is zero and therefore no heating occurs (for a perfect inductor, although in practice the wire that forms the coil will have resistance). In this case the negative power is due to energy, that has been stored in the magnetic field, being fed back into the circuit.

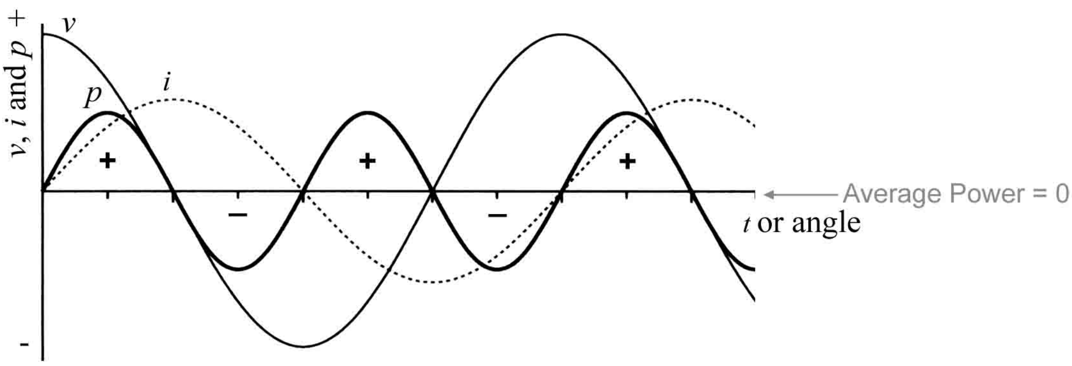

Figure 11.3: Power in an AC inductive circuit. Note that *v*, *i* and *p* are not plotted on the same scale.

11.5 Power In Resistive And Capacitive Circuits (RC Circuits)

In a circuit consisting of a resistance and a capacitive reactance in series, the voltage and current will have a relative phase angle between 0° and 90°, depending on the ratio of resistance to reactance (section 9.3). Figure 11.4 shows the wave diagram for such a circuit, with the current leading the voltage by ϕ°. Because energy is dissipated in the resistor, less energy is returned to the circuit than for a pure capacitive circuit and the negative pulses are considerably smaller. Thus there is a net power drawn from the supply which will be dissipated as heat. However, the energy dissipated will be less than that dissipated in a purely resistive circuit because the current is out of phase with the voltage.

Figure 11.4 shows the phasor diagram that corresponds to the wave diagram shown in figure 11.3.

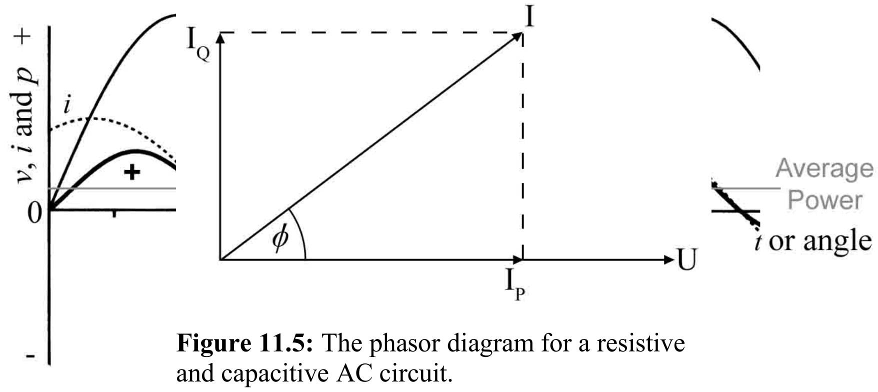

Figure 11.5: The phasor diagram for a resistive and capacitive AC circuit.

Figure 11.4: Power in an AC resistive and capacitive circuit. Note that *v*, *i* and *p* are not plotted on the same scale.

The current phasor can be split into two component currents; these do not actually exits but as demonstrated in section 8.1 vector quantities can be split into components to make sums easier. I_P is the *in-phase* or *active* component of current, being in-phase with the voltage it is the part of the current that is considered to causes heat to be dissipated. While the *quadrature* or *reactive* component I_Q leads the voltage by 90° and is considered to be the part of the current that causes energy to be stored and then returned to the circuit but causing no heating. In figure 11.5 the supply voltage (U) is taken as the reference and so in a RC circuit the supply current (I) leads this by ϕ°. We can see from figure 11.5 that since I_P is in-phase with U the power dissipated in the resistive part of the circuit is given by:

$$P = UI_P$$

But from figure 11.5: $\text{coz}\phi = \frac{I_P}{I}$

$$\mathbf{I_P = I\cos\phi} \quad \text{and} \quad \mathbf{P = UI\cos\phi}$$

Cos ϕ is known as the power factor of the circuit. Since this result has been derived from a phasor diagram it only applies to sin waves and in such cases average power = RMS voltage × RMS current × the cosine of the phase angle between voltage and current.

It is often useful to be able to calculate the circuit phase angle from the resistance, reactance and impedance of the circuit. From section 10.1:

$$\cos\phi = \frac{U_R}{U} = \frac{IR}{IZ} \quad \text{so} \quad \cos\phi = \frac{R}{Z}$$

$$\sin\phi = \frac{U_C}{U} = \frac{IX_C}{IZ} \quad \text{so} \quad \cos\phi = \frac{X_C}{Z}$$

$$\tan\phi = \frac{U_C}{U_R} = \frac{IX_C}{IR} \quad \text{so} \quad \tan\phi = \frac{X_C}{R}$$

We can also see that since the power is only dissipated in the resistive part of the circuit, it follows that:

$$\mathbf{P = I^2R = \frac{U_R{}^2}{R}}$$

Note that the above equations do not mix current an voltage (unlike P = UI) and so there is no problem with phase differences. For this reason the power losses in wires and cables carrying AC supplies are calculated using $P = I^2R$.

Example

A circuit connected to a 240V AC supply consists of a resistance of 28.8Ω in series with a capacitor of reactance 38.4Ω. Calculate (i) the circuit current, (ii) the circuit phase angle, and (iii) the power dissipated.

(i) $Z = \sqrt{(R^2 + X_C{}^2)} = \sqrt{(28.8^2 + 38.4^2)} = 48\Omega$

(ii) $\cos\phi = \frac{R}{Z} = \frac{28.8}{48} = 0.6, \quad \phi = 53.1°$ current leading voltage because this is an RC circuit

(iii) $P = UI\cos\phi = 240 \times 5 \times 0.6 = 720W$

or, $P = I^2R = 5^2 \times 28.8 = 720W$

$U_R = IR = 5 \times 28.8 = 144$

$$P = \frac{U_R{}^2}{R} = \frac{144^2}{28.8} = 780W$$

11.6 Power In Resistive And Inductive Circuits

When resistance and inductive reactance are in series, current lags supply voltage by an angle of ϕ° which will vary from almost 0° to nearly 90°. Figure 11.6 shows the wave diagram for such a circuit. Like the RC circuit the amount of energy taken from the supply is greater that the energy returned to the circuit by the inductor and a net energy is consumed. The average power (P) consumed is illustrated in figure 11.6.

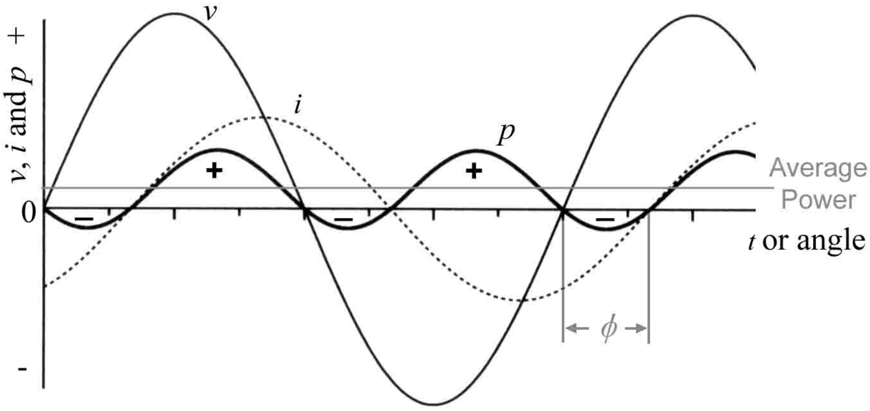

Figure 11.6: Power in an AC resistive and inductive circuit. Note that *v*, *i* and *p* are not plotted on the same scale.

Figure 11.7 shows the phasor diagram which corresponds to the wave diagram in figure 11.6. Again, since I and U are out of phase P = IU is not valid, however once again the power consumed can be found from:

$$\mathbf{P = UI_P = UI\cos\phi}$$

This equation only holds true for sine waves.

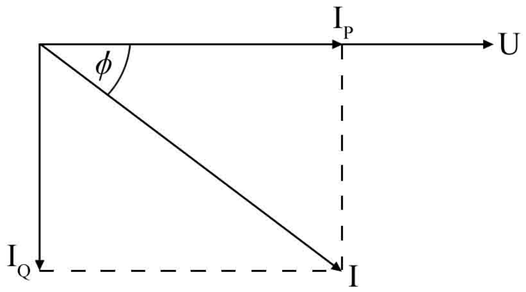

Figure 11.7: The phasor diagram for a resistive and inductive AC circuit.

From section 10.1 and figure 11.7 the following relationships can be deduced:

$$\cos\phi = \frac{U_R}{U} = \frac{IR}{IZ} \quad \text{so} \quad \cos\phi = \frac{R}{Z}$$

$$\sin\phi = \frac{U_L}{U} = \frac{IX_L}{IZ} \quad \text{so} \quad \cos\phi = \frac{X_L}{Z}$$

$$\tan\phi = \frac{U_L}{U_R} = \frac{IX_L}{IR} \quad \text{so} \quad \tan\phi = \frac{X_L}{R}$$

11.7 Power In General Series Circuits (RCL Circuits)

These circuits follow the same rules as RC and RL circuits because power is still only dissipated in the resistive part of the circuit. Therefore:

$$\mathbf{P = VI\cos\phi \quad P = IR^2 \quad P = \frac{V_R{}^2}{R}}$$

<u>Example</u>
A choke of inductance 0.318 H resistance 30Ω is connected in series with 53µF capacitance across a 24V, 50Hz supply. Calculate the power dissipated.

This is the same circuit as that used in the example in section 10.3, where it was calculated that the circuit current is 0.48A, $\phi = 54.1°$ ($\cos\phi = 0.6$) and $V_R = 14.4V$.

$P = I^2R = 0.48^2 \times 30 = 6.91W$

The two alternative methods are:

$P = UI\cos\phi = 24 \times 0.48 \times 0.6 = 6.91W$

$$P = \frac{U_R^{\ 2}}{R} = \frac{14.4^2}{30} = 6.91W$$