

## Part 12: AC Power Factor And Apparent Power

### 12.1 Power Factor

In sections 11.3 and 11.4 we saw that, for perfect capacitors and inductors, it is possible for a current to flow and no power to be dissipated. In section 11.5, 11.6 and 11.7 we found that even when the circuit has some resistance, if the phase angle ( $\phi$ ) between voltage and current is large then very little power is dissipated. In such cases  $P = UI$  is not valid as a method for finding the dissipated power, neither is it a valid method of finding the current that flows in the circuit. From figures 11.7 and 11.5 it can be seen that with a large phase angle the *in-phase* or *active* component of current will be smaller than the *quadrature* or *reactive* component. Thus, the current that is in-phase with the voltage, and therefore responsible for the dissipated power, will be considerably smaller than the total current flowing in the circuit.

Despite all this the product of current and voltage is still used in AC circuits and is called *apparent power* (VA), giving a value of *voltamperes* (VA). The term apparent power is misleading because it suggests that the apparent power is dissipated, however as we found previously the power dissipated in an AC circuit, called *active, true* or *real power* (in the units of watts), is given by:

$$P = IU \cos \phi = \text{watts (W)}$$

The apparent power is given by:

$$VA = UI = \text{voltamperes (VA)}$$

Real and apparent power are linked by the *power factor* (PF) which is defined as:

$$\frac{\text{active power}}{\text{apparent power}} \quad \text{or} \quad \frac{\text{watts}}{\text{voltamperes}}$$

These definitions are true under all circumstances and if the supply is sinusoidal:

$$\text{Power Factor} = \frac{\text{active power}}{\text{apparent power}} = \frac{UI \cos \phi}{\text{apparent power}} = \cos \phi$$

From section 10.1 we can add that:

$$PF = \frac{P}{UI} = \cos \phi = \frac{U_R}{Z}$$

In a predominantly inductive series circuit, where current lags voltage, the power factor is called a *lagging power factor*. Similarly, in a predominately capacitive series circuit, where current leads voltage, the power factor is called a *leading power factor*. The power factor can vary between definite limits, being 1 (unity) for purely resistive circuits, where the phase angle is  $0^\circ$  and  $P = UI$ ; or 0 for purely reactive (inductive or capacitance) circuits, where the phase angle is  $90^\circ$  and  $P = 0$ .  
Note:

- if  $PF = 1$  (i.e. purely resistive circuit) active power = apparent power =  $UI$
- if  $PF = 0$  (i.e. purely inductive or capacitive circuit) active power = reactive power =  $UI$  (section 11.3)

### Example

An AC single-phase motor takes 5A at 0.7 lagging power factor when connected to a 240V, 50Hz supply. Calculate the power input to the motor. If the motor efficiency is 70%, calculate the output.

$$P = UI \cos \phi = 240 \times 5 \times 0.7 = 840W$$

$$\text{Output power} = \text{input power} \times \text{efficiency} = 840 \times 70/100 = 588W$$

### Example

A 200V AC circuit comprises a 40Ω resistor in series with a capacitor of reactance 30Ω. Calculate the current and the power factor.

$$Z = \sqrt{R^2 + X_c^2} = 50\Omega$$

$$I = \frac{U}{Z} = \frac{200}{50} = 4A$$

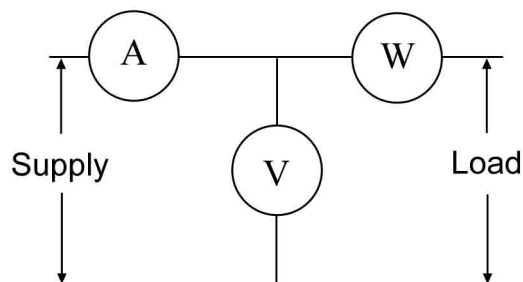
$$PF = \frac{R}{Z} = \frac{40}{50} = 0.8 \quad \text{leading}$$

Alternative, power factor could have been calculated from values of true and apparent power.

$$VI = 200 \times 4 = 800VA$$

$$P = UI \cos \phi = UI \frac{R}{Z} = 800 \times \frac{40}{50} = 640W$$

$$PF = \frac{P}{VI} = \frac{640}{800} = 0.8 \quad \text{leading}$$



**Figure 12.1:** The placement of a voltmeter (V), an ammeter (A) and a wattmeter (W) for measuring the power factor.

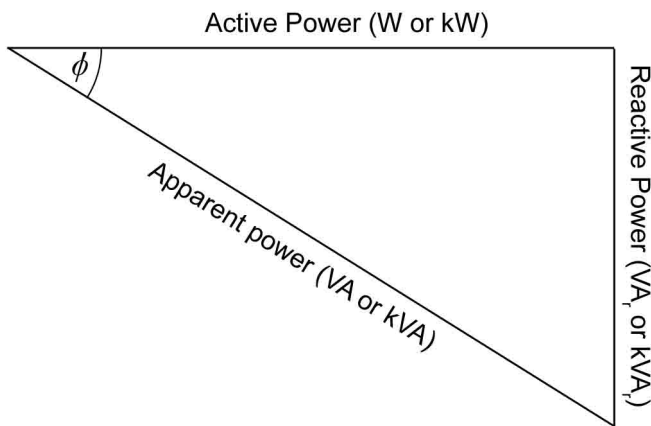
The power factor can be found if a voltmeter, ammeter and wattmeter are connected to a circuit (figure 12.1). Then, the power factor equals the voltmeter reading multiplied by the ammeter reading, divided by the wattmeter reading. An instrument called a power-factor meter is also available but they are not common.

## 12.2 Components Of Power

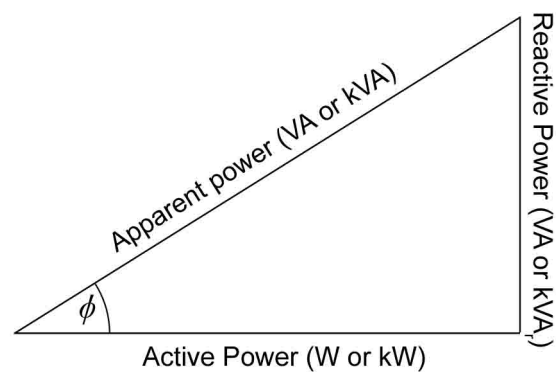
We have already seen in figures 11.5 and 11.7 that the circuit current may be considered to have *in-*

*phase and quadrature components.* The voltamperes or apparent power can similarly be broken down into components. Figure 12.2 shows a power triangle for a resistive-inductive circuit, here the reactive power and apparent power are drawn below the active power because the circuit current lags the supply voltage. In this instance the apparent power is said to be lagging. From simple trigonometry, since  $\cos \phi = W/VA$ , the true power (real or active power) makes an angle of  $\phi$  with the apparent power, this angle is also the phase angle for the circuit concerned.

Figure 12.3 shows a power triangle from a resistive and capacitive circuit, here the reactive power and apparent power are drawn above the active power because the circuit current leads the supply voltage. The reactive power is said to be leading. If the circuit contains capacitive and inductive elements, whether the reactive power leads or lags will depend on the balance between the capacitive and inductive reactances.



**Figure 12.2:** The power diagram of a resistive and inductive circuit.



**Figure 12.3:** The power diagram of a resistive and inductive circuit.

From figure 12.2:

- *Active Power* (or true power, or real) – the power dissipated or consumed: this will be dissipated in the resistive part of the circuit. Calculate by multiplying the in-phase current by the supply voltage or  $P = UI \cos \phi$ . The symbol is P and units are watts (W) or kilowatts (kW).
- *Apparent Power* (or voltamperes) – the product of the supply volts and circuit current ( $P = UI$ ). The symbol is VA and the units are voltamperes (VA) or kilovoltamperes (kVA).
- *Reactive Power* - the power that is constantly recycled through the non-resistive parts of the circuit (i.e. the inductances and capacitances). Calculated by multiplying the quadrature current by the supply voltage or  $VA_r = UI \sin \phi$ . The symbol is  $VA_r$  and the units are voltamperes (VA) or kilovoltamperes (kVA).

From Pythagoras:

$$(VA)^2 = (W)^2 + (VA_r)^2$$

### Example

A  $10\Omega$  resistor and a capacitive reactance of  $20\Omega$  are connected in series to a 240V supply. Calculate the apparent power, the true power, the reactive power and the power factor.

$$Z = \sqrt{(R^2 + X_c^2)} = \sqrt{(10^2 + 20^2)} = 22.4\Omega$$

$$I = \frac{U}{Z} = \frac{240}{22.4} = 10.7A$$

$$\text{apparent power} = UI = 240 \times 10.7 = 2570VA \text{ or } 2.57kVA \text{ leading}$$

$$\text{Power Factor} = \frac{R}{Z} = \frac{10}{22.4} = 0.446 \quad \text{leading}$$

$$\text{true power} = \text{apparent power} \times \text{PF} = 2570 \times 0.446 = 1150\text{W or } 1.150\text{kW}$$

$$\text{or, true power} = I^2R = 10.7^2 \times 10 = 1.150\text{kW}$$

$$\text{reactive power} = \sqrt{[(\text{VA})^2 \times W^2]} = \sqrt{[2570^2 \times 1150^2]} = 2.30\text{kVA}_r$$

### 12.3 Adding Power Factors

Loads at differing power factors on the same supply can be added using a power diagram to show the resultant voltamperes and power factor. The sum is performed using the apparent power of each load.

#### Example

A single-phase load consists of:

(i) 12kW of lighting and heating at unity power factor,

(ii) 8kW of motor at 0.8 power factor lagging, and

(iii) 10kVA of motors at 0.7 power factor lagging.

Calculate (a) the total Kw, (b) the total kVA<sub>r</sub>, (c) the total kVA, (d) the overall power factor, and (e) the total supply current at 240V.

The sum is shown in figure 12.4a, b and c, where all of the values are drawn to scale. Figures 12.4a, b and c are power diagrams for loads that are resistive or inductive, therefore: the true power is drawn horizontally, the reactive power is drawn vertical below this and the apparent power is also below the horizontal but at an angle.

Load (i): At unity power factor kW = kVA, so the load of 12kW = 12kVA and is drawn as a horizontal line to a suitable scale 12 units long.

$$\text{Load(ii): } \text{PF} = \frac{\text{kW}}{\text{kVA}} \quad \text{so} \quad \text{kVA} = \frac{8}{0.8} = 10\text{kVA}$$

The angle of lag has a cosine of 0.8, so equals 36.9°. A line equal to 10 units is drawn to represent the 10kVA, making an angle of 37° with the horizontal.

These first two loads are added together by completing the parallelogram, to give the resultant 'A' shown as a broken line in figure 12.4a.

Load (iii): is given in kVA, the angle being that which has a cosine of 0.7, therefore 45.6°, so a line of length 10 units is drawn at this angle to the horizontal. This load is then added to the resultant A, and gives a total kVA of B, measured off as 28.1 kVA (figure 12.4b). The in-phase (horizontal) component of this load is 25.4kW, and represents the true power consumed. The quadrature (vertical) component is 11.9kVA<sub>r</sub>, and represents the reactive kilovoltamperes. The angle made by the load is 25°, and the cosine of this angle is the power factor, which is 0.91 lagging (figure 12.4c).

$$I = \frac{\text{kVA} \times 10^3}{V} = \frac{28100}{240} = 117\text{A}$$

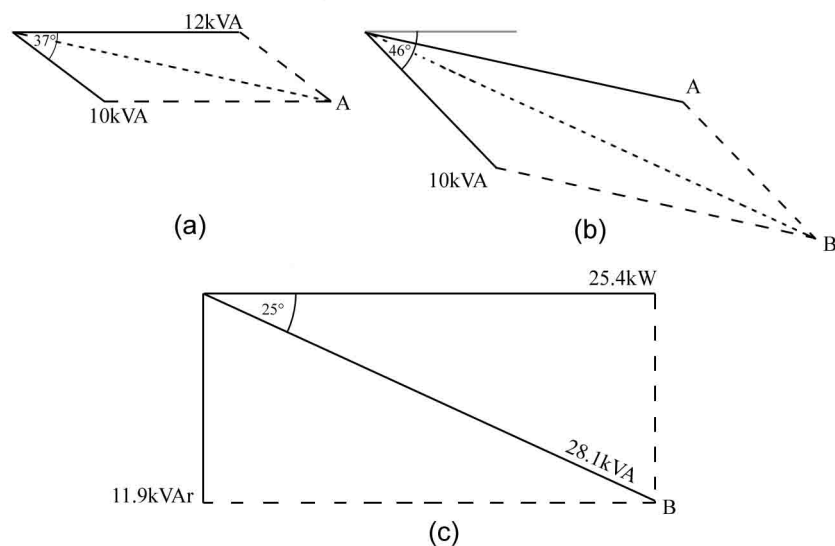


Figure 12.4:

### 12.4 kVA And Current Ratings

There may not seem much point in calculating the apparent power but it is a very useful quantity. In DC systems it is easy to calculate the current that will flow through a piece of equipment since we would know the voltage of the supply and power rating of the equipment would probably be quoted. The equation  $P = UI$  can be used to either find the current that will be drawn, for say a 60W bulb connected to a 24V DC supply.

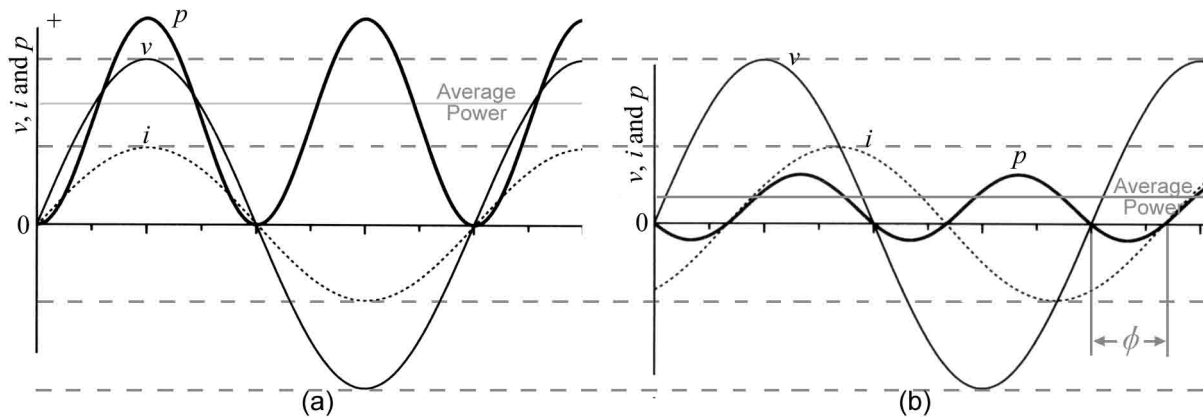


Figure 12.5: The wave diagrams for the power in: (a) a resistive circuit and (b) a resistive and inductive circuit. Note that  $v$ ,  $i$  and  $p$  are not plotted on the same scale.

The instantaneous values can be used in any circuit, so that  $p = vi$  is always valid. In an AC circuit that is purely resistive current and voltage are in-phase, the power factor is unity and the RMS value can be used, so that  $P = UI$  holds true giving the real power in watts. Figure 12.5 shows a wave diagram for such a circuit and demonstrates that the peak voltage coincides with the peak current.

Figure 12.5b shows the wave diagram for an inductive circuit with the same current and voltage curves as figure 12.5a but a phase angle ( $\phi$ ) between them. Figure 12.5 demonstrates that when the current and voltage waves are out of phase, and the peak current does not correspond to the peak voltage, less power is dissipated from the same amount of current than if the waves are in phase. Thus, while at any instant  $p = vi$ ,  $P = UI$  is not valid. Therefore, for an AC circuit with inductive and/or capacitive components the product  $UI$  gives the apparent power ( $VA = UI$ ) and as long as the power factor is not unity  $VA$  is greater than  $P$ .

AC supplies are often rated in kVA to avoid confusion over the power factor. For example if the 240V output of a transformer is rated at 30kW it can deliver  $30\text{kW}/240\text{V} = 125\text{A}$ , however it can only deliver 30kW if it delivers this power to a resistive load. If the load is inductive and resistive (a motor for example) the current will lag voltage and a smaller amount of active power is available (although the same current can still be delivered, figure 12.5), therefore the transformer does not actually deliver 30kW of active power but 30kVA of apparent power. Therefore, a rating 30kVA will hold true for loads with any power factor whereas, a rating of 30kW really means that a maximum of 30kW can be delivered to a purely resistive load but to other loads, where the power factor is less than unity, the power delivered will be less. Using kVA and knowing the supply voltage we can still work out how much current can be delivered, even if we don't know the power factor of the load. To work out the active power delivered we need to know the power factor of the load.

AC machines are also often rated using kVA because they may work with different supply voltages. For example, a 4kVA single-phase motor will draw  $4\text{kVA}/240\text{V} = 16.7\text{A}$  from a 240V supply and 36.36A from a 110V supply. We have calculated this without knowing the power factor, and therefore without knowing the active power consumption of the motor. If the motor were rated in kW we would need to know the power factor to find out how much current it would draw.

Example

A single-phase 3.73kW motor is 85% efficient at full and is fed from a 240V supply. Calculate its full-load current if it operates at a power factor of (i) unity (ii) 0.85 lag (iii) 0.6 lag.

$$\text{input power} = 0.73 \times 1000 \times \frac{100}{85} = 4390\text{W}$$

$$\text{(i) apparent power} = \frac{4390}{1} = 4390\text{VA}$$

$$\text{Therefore: } I = \frac{\text{VA}}{\text{V}} = \frac{4390}{240} = 18.3\text{A}$$

$$\text{(ii) apparent power} = \frac{4390}{0.85} = 5170\text{VA}$$

$$\text{Therefore: } I = \frac{\text{VA}}{\text{V}} = \frac{5170}{240} = 21.5\text{A}$$

$$\text{(iii) apparent power} = \frac{4390}{0.6} = 7320\text{VA}$$

$$\text{Therefore: } I = \frac{\text{VA}}{\text{V}} = \frac{7320}{240} = 30.5\text{A}$$

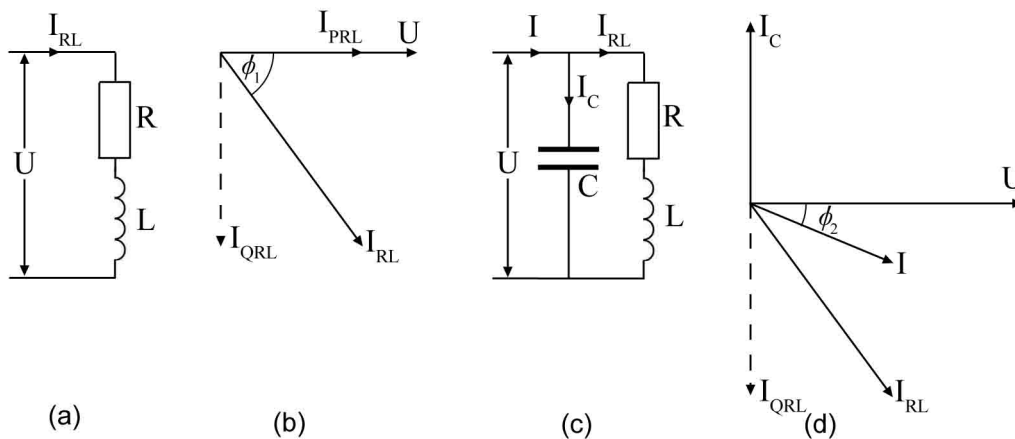
Note that the output power rating of a machine will be in kW, therefore it is the active power consumed - losses. In this example the output power is corrected for the efficiency and then the apparent power found so that the current may be calculated. Notice that the current needed to supply the same output power increases as the power factor decreases from unity, so that you are effectively getting less kW per ampere.

Although the power factor of a motor varies with load, it is usually higher at full load than at lower loadings. Full-load power factor can be safely used to calculate the current rating of cables, since the rise in current owing to reducing power factor is offset by the fall in load current.

## 12.5 Disadvantages Of Low Power Factor

The disadvantages of a low power factor stem from the fact that a load with a low power factor will draw a large current:

- (i) Large cables, switchgear and transformers may be necessary both within the installation and in the supply mains feeding it.
- (ii) Low-power-factor working causes operating difficulties on high-voltage transmission lines.
- (iii) Because of the effects of items (i) and (ii), electricity companies usually penalise the consumer whose load is at a poor power factor by charging more for the electrical energy used.
- (iv) Large cables may be needed within an installation to carry the extra current at low power factor. Alternatively, extra load can be connected to a cable if the power factor of the existing load it carries is improved.
- (v) Higher currents give rise to higher copper losses in cables and transformers.
- (vi) Higher currents give larger voltage drop in cables, and a change in load gives a larger change in voltage drop if the power factor is low. This is called 'poor voltage regulation'



**Figure 12.6:** (a) the resistive and inductive circuit; (b) the phasor diagram for the resistive and inductive circuit; (c) a capacitor added to increase the power factor; (d) the phasor diagram for the same circuit.

## 12.6 Power Factor Correction

The majority of low power factors are lagging, since they are caused by motors and transformers that have inductance and resistance but no capacitance. In such machines the power factor can be increased by connecting a capacitor in parallel across the coil's terminals as illustrated in figure 12.6c. Figure 12.6b shows the phasor diagram for an inductive and resistive load, such as a motor, and the current  $I_L$  can be seen to lag the voltage by  $\phi_1$ . Figure 12.6d shows the phasor diagram when the capacitor is added, the resultant current ( $I$ ) is the phasor sum of  $I_{RL}$  and  $I_C$ , and the phase angle is reduced to  $\phi_2$ ; thus there is an improvement in the power factor.

The power factor will be to unity when the capacitor current is equal and opposite to the quadrature component ( $I_{QRL}$ ) of the uncorrected current ( $I_{RL}$ ). If  $I_C$  exceeds  $I_{RQ}$ , over correction occurs and this situation should be avoided.

As figure 12.6c illustrates, the corrected current (I) only flows in the circuit up to the point where the capacitor branches off, therefore it is preferable to have the capacitor as close to the motor as possible. However, economics may dictate that a large capacitor is used to correct the power supply of a whole installation at one point. Such capacitors need to be variable so that over correction does not occur if some of the equipment is turned off.

Example

A 240V, 5kW, 50Hz single-phase motor working at full load with an efficiency of 85% has a power factor of 0.5 lagging.

- (a) Calculate the current taken by the motor on full load.
- (b) If a capacitor is connected across the motor terminals to raise the overall power factor to unity, calculate;
  - (i) the current to be carried by the capacitor, and
  - (ii) the capacitance of the capacitor in microfarads.

(a) the output of the motor is 5kW

$$\text{input power} = \frac{\text{output power}}{\text{efficiency}} = 5 \times \frac{100}{85} = 5.88\text{kW}$$

$$P = UI \cos \phi \quad \text{so} \quad I = \frac{P}{U \cos \phi} = \frac{5.88 \times 10^3}{240 \times 0.7} = 35\text{A}$$

(b)(i) A phasor diagram is drawn to scale (figure 12.7). Firstly, a voltage phasor drawn along the horizontal, to an arbitrary length, to act as a reference. The 35A current phasor (the uncorrected current draw by the the motor, I) is then added at an angle of  $\cos 0.7 = 45.5^\circ$  lagging. The capacitor current (I<sub>c</sub>) needed to correct the circuit current (I<sub>t</sub>) to unity is added as a vertical line, the length of which is found by completing the parallelogram. By measurement, the current carried by the capacitor is found to be I<sub>c</sub> = 25A.

$$(ii) X_L = \frac{U}{I_c} = \frac{10^6}{2\pi f C'}$$

$$\text{Therefore: } C' = \frac{I_c \times 10^6}{2\pi f U} = \frac{25 \times 10^6}{2\pi \times 50 \times 240} = 331\mu\text{F}$$