Part 15: Transformers

15.1 Single-Phase Transformers

Transformers are made up from *primary* and *secondary* coils (called windings) that are made from turns of insulated wire. The coils are arranged on a core of magnetic material that increases the amount of magnetic flux set up by one coil and will make sure that most of it links with the other coil; in this way mutual inductance is increased. A simple transformer is illustrated in figure 15.1a and the corresponding circuit diagram is shown in figure 15.1b. The primary winding has a voltage of U_1 across it and is made of N_1 turns of wire. The secondary coil has a voltage of U_2 across it and is made from N_2 turns.

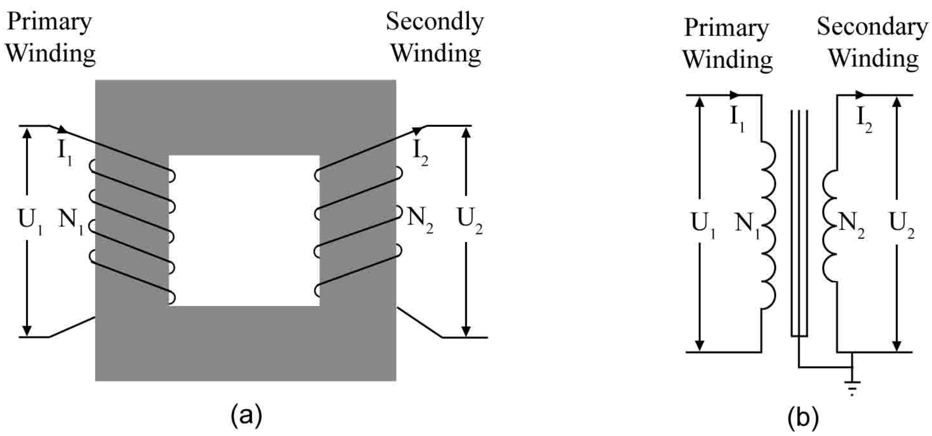


Figure 15.1: (a) a simple transformer; (b) the circuit diagram for a simple transformer.

An alternating current in the primary coil will set up an alternating magnetic flux in the core and the primary's self inductance induces an EMF which opposes the supply voltage that is driving the current (sections 6.5 and 9.5). The induced EMF will almost be the same magnitude as the supply voltage and although there will be a small difference, this is normally ignored. The changing EMF from the primary coil links with the second coil and mutual inductance causes an EMF to be induce (section 6.4). It is normally assumed that all of the flux from the primary links with the secondary, and although this is not strictly true it is a good approximation and the induced EMF per turn in the primary is taken to be the same as the induced EMF per turn in the secondary. Thus

$$\text{primary volts per turn} = \frac{\text{primary volts}}{\text{primary turns}} = \frac{U_1}{N_1}$$

$$\text{secondary volts per turn} = \frac{\text{secondary volts}}{\text{secondary turns}} = \frac{U_2}{N_2}$$

$$\frac{\mathbf{U_1}}{\mathbf{N_1}} = \frac{\mathbf{U_2}}{\mathbf{N_2}} \quad \text{and} \quad \frac{\mathbf{U_1}}{\mathbf{U_2}} = \frac{\mathbf{N_1}}{\mathbf{N_2}}$$

If 100% efficiency is assumed:

power in = power out

$U_1 I_1 \cos\phi = U_2 I_2 \cos\phi$

Where I_1 and I_2 are the primary and secondary currents, and cos ϕ_1 and cos ϕ_2 are the primary and secondary power factors. If the two power factors are assumed to be the same, they cancel and:

$$U_1 I_1 = U_2 I_2$$

Therefore: $$\frac{\mathbf{U_1}}{\mathbf{U_2}} = \frac{\mathbf{N_1}}{\mathbf{N_2}} = \frac{\mathbf{I_1}}{\mathbf{I_2}}$$

For a transformer with more turns in the primary than the secondary $U_1 > U_2$, such a transformer is called a *step-down* transformer. For a transformer with more turns in the secondary than the primary $U_1 < U_2$, such a transformer is called a *step-up* transformer.

Example

A 75kVA transformer has a step-down ratio of 12:1, with 2400 primary turns and a primary voltage of 3.3kV. Calculate:

(a) the number of secondary turns,
(b) the secondary voltage,
(c) the volts per turn,
(d) the full load primary and secondary currents.

(a) $$\frac{U_1}{U_2} = \frac{N_1}{N_2} \quad \text{so} \quad N_2 = N_1 \times \frac{U_1}{U_2}$$

$$N_2 = 2400 \times \frac{1}{12} = 200 \text{ turns}$$

(b) $$\frac{U_1}{U_2} = \frac{12}{1} \quad \text{so} \quad U_2 = \frac{U_1}{12}$$

$$U_2 = \frac{3300}{12} = 275\text{V}$$

(c) $$\text{primary volts/turns} = \frac{U_1}{N_1} = \frac{3300}{2400} = 1.38$$

$$\text{or, primary turns} = \frac{U_2}{N_2} = \frac{275}{200} = 1.38$$

(d) $$I_1 = \frac{\text{kVA} \times 1000}{U_1} = \frac{75000}{3300} = 22.7\text{A}$$

$$I_2 = \frac{\text{kVA} \times 1000}{U_2} = \frac{75000}{275} = 273\text{A}$$

15.2 Tapped Windings

The voltage ratio depends on the ratio of the number turns in the primary to the number of turns in the secondary. If more than one secondary voltage is needed from one transformer a number of secondary windings can be provided. All of the windings are placed on the same magnetic core and this arrangement is shown in figure 15.2a.

In some cases a single adjustable voltage is needed, and this can be accomplished by tapping the primary or secondary winding. Figure 15.2b and c show tapped windings, where the number of coils in winding that has been tapped can be changed by switching between terminals. It is unusual to tap both the primary and secondary windings, transformers with small outputs are usually tapped on the secondary winding.

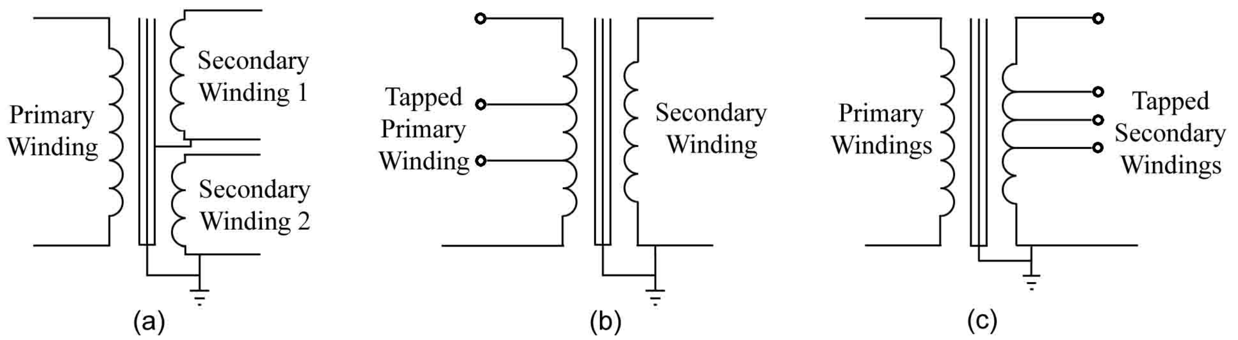


Figure 15.2: Methods for obtaining multiple voltages from a single transformer: (a) multiple secondary windings; (d) tapped primary winding; (c) tapped secondary winding.

15.3 Three-phase Transformers

A three phase transformer is effectively the same as three single-phase transformers connected in a three-phase arrangement and it is possible to use three separate single-phase transformers, although it is far more usual to have all the windings on the same core. Three-phase transformers have six windings, three primary and three secondary, that can be connected in star (Y) or delta (D) configurations. The primary winding is commonly denote by a capital Y or D and the secondary windings are denoted by a lower case y or d.

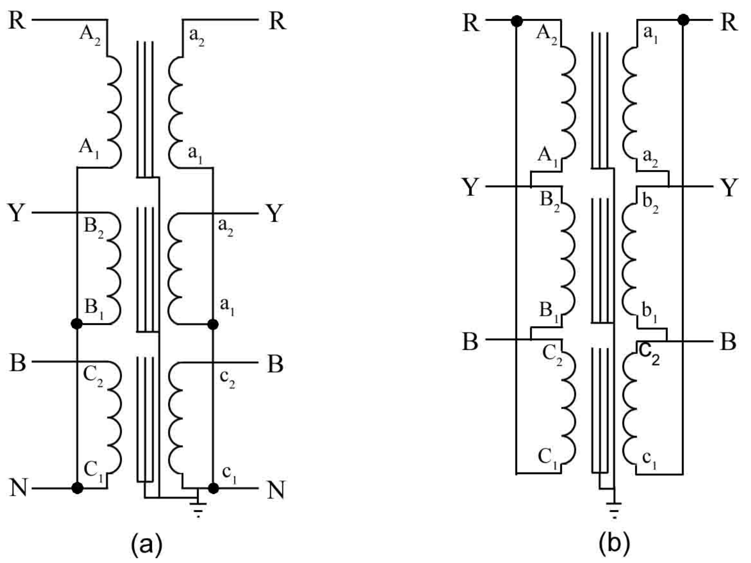


Figure 15.3: Three-phase transformers similarly connected: (a) star-star - for the primary and secondary one end from each winding is connected together forming a neutral; (b) delta-delta – one end of each winding connected to the next winding.

Figure 15.3a shows a transformer where both primary and secondary windings are star connected, such a transformer is called a *star-star*, *wye-wye* or *Yy* transformer. Figure 15.3b shows a *delta-delta*, *mesh-mesh* or *Dd* transformer, where both primary and secondary cores are delta connected. The secondary and primary coils need not be connected in the same configuration so that star-delta (Yd) and delta-star (Dy) are also possible and are shown in figure 15.4. Transformers with delta connected secondaries are seldom used to supply consumer's loads because there is no position for a neutral wire, such transformers are used for high voltage transmission between substations.

Figure 15.3 shows the standard method for marking three-phase transformer windings. The three primary windings are labelled with a capital A, B and C. The three secondary windings are labled with a lower case a, b and c. Each winding has two ends and labelled 1 and 2 so that the ends of the primary on the second winding are labelled B_1 and B_2.

If the primary and secondary windings are connected similarly (i.e. star-star or delta-delta), calculations are the same as those for single phase transformers, as long as the system is balanced (section 13.4). When the primary and secondary have different types of connection, the overall turns ratio of the transformer is more complicated. For example, consider a single-phase transformer with a 1:1 turns ratio, the input and output voltages from the windings are the same. This will also be true for a three-phase transformer with the primary and secondary windings connected similarly. However, if the three-phase transformer is connected in star-delta (figure 15.4a), and has a primary line voltage of U, each of the star connected primaries will have the phase voltage across it, which is U/√3 (the voltage between any line and the neutral point). Each of the secondary windings will then have this same voltage induced in it, and since these windings are delta-connected, the voltage U/√3 will be the secondary. Thus, a star-delta transformer with a turns ratio of 1:1 provides a √3:1 step-down. For figure 15.a:

$$\frac{N_1}{N_2} = \frac{U_1}{U_2\sqrt{3}}$$

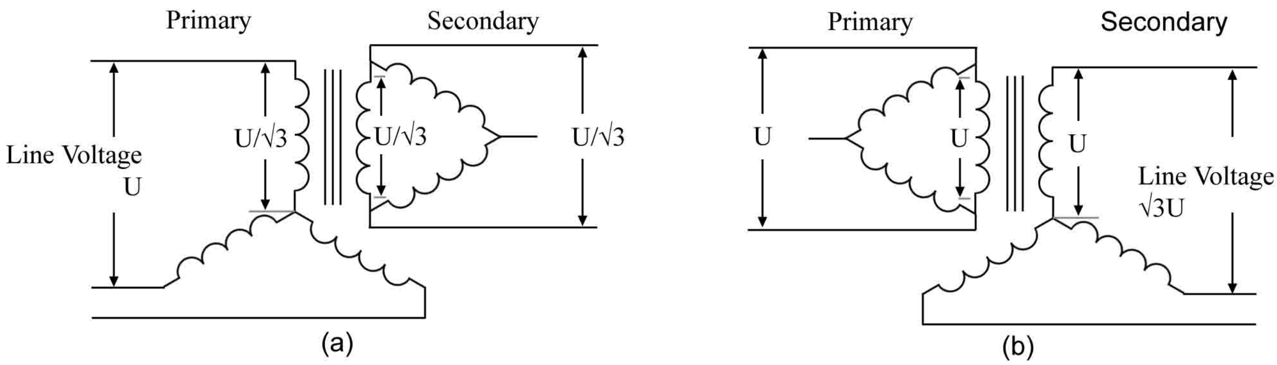


Figure 15.4: Three-phase transformers with primary and secondary windings connected differently: (a) star-delta; (b) delta-star.

For a delta-star transformer a similar effect happens but there is a 1:√3 step-up for line voltage in addition to the effect of the turns. Thus, from figure 15.4b:

$$\frac{N_1}{N_2} = \frac{U_1\sqrt{3}}{U_2}$$

Only identical transformers should ever be connected in parallel. Transformers are identical when their turn ratios are the same and when the primary and secondary windings are connected in the same way.

Example
A 660 kVA three-phase transformer is star-star connected, and each of the three primary windings has 6500 turns. The primary windings are fed with a line voltage of 11kV, and the secondary windings are to provide a line voltage of 415V. Calculate:
(a) the primary and secondary phase voltages,
(b) the number of turns on each secondary winding,
(c) the primary and secondary full-load currents.

(a) $U_P = \frac{U_L}{\sqrt{3}} = \frac{11000}{1.73} = 63500\text{V (primary)}$

$$U_P = \frac{U_L}{\sqrt{3}} = \frac{415}{1.73} = 240\text{V (secondary)}$$

Since the transformer is star-star connected, the phase voltage ratio strictly should be used. although line voltages have the same ratio.

(b) $\frac{U_1}{U_2} = \frac{N_1}{N_2}$ so $N_2 = N_1 \times \frac{U_2}{U_1} = 6500 \times \frac{240}{6350} = 246\text{ turns}$

(c) The power in a three-phase balanced system is given by (section 14.5):

$P = (\sqrt{3})U_L I_L \cos\phi$

The voltamperes can be calculated by dividing the power by the power factor, so:

$$VA = \frac{P}{\cos\phi} = \frac{(\sqrt{3})U_L I_L \cos\phi}{\cos\phi} = (\sqrt{3})U_L I_L$$

$$\text{primary } I_L = \frac{VA}{(\sqrt{3})U_L} = \frac{660 \times 10^3}{1.73 \times 11 \times 10^3} = 34.7\text{A (primary)}$$

$$\text{secondary } I_L = \frac{VA}{(\sqrt{3})U_L} = \frac{660 \times 10^3}{1.73 \times 415} = 34.7\text{A (secondary)}$$

Example
A 3000/415 delta-star connected three phase transformer delivers 100kVA on full-load. Calculate:
(a) the turns ratio, primary to secondary,
(b) the full-load current in each secondary winding,
(c) the full-load primary line current,
(d) the full-load current in each primary winding.

(a) $\frac{N_1}{N_2} = \frac{(\sqrt{3})U_1}{U_2} = \frac{1.73 \times 3300}{415} = 1142.83$

(b) For the star-connected secondary winding, line and phase currents are the same, so winding current:

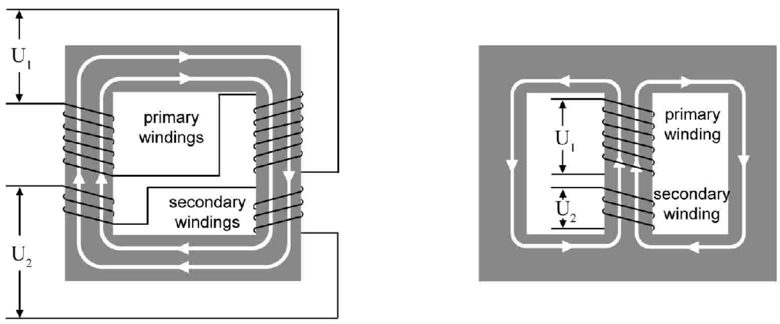
$$\text{secondary } I_L = \frac{VA}{(\sqrt{3})U_L} = \frac{100 \times 10^3}{1.73 \times 415} = 139\,A \text{ (primary)}$$

$$\text{(c) primary } I_L = \frac{VA}{(\sqrt{3})U_L} = \frac{100 \times 10^3}{1.73 \times 33000} = 17.5A \text{ (primary)}$$

$$\text{(d) primary } I_P = \frac{I_L}{\sqrt{3}} = \frac{17.5}{1.73} = 10.1A$$

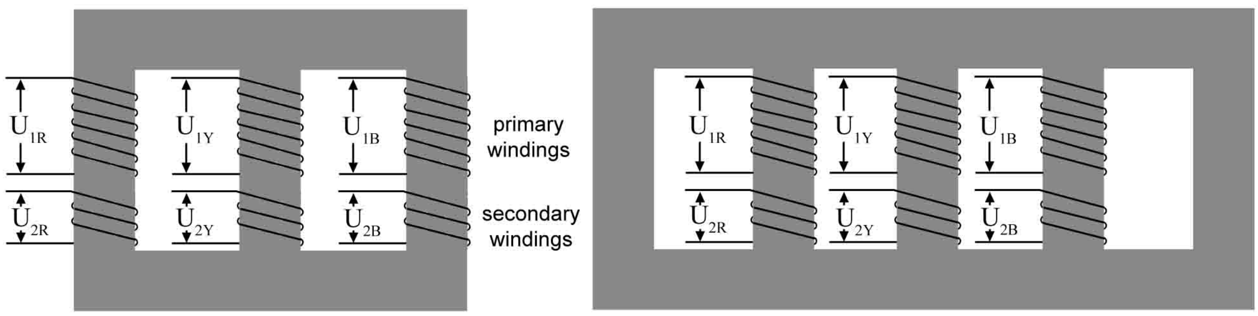
15.4 Transformer Construction

If a transformer was constructed like the one shown in figure 15.1a. with the primary and secondary windings on separate limbs, a proportion of the magnetic flux produced by the primary core would not link to the secondary core. A better arrangement, called a *core-type* transformer is shown in figure 15.5a. If weight and cost are not issues, an even better arrangement called a *shell-type* transformer van be made and is illustrated in figure 15.5b.



(a)

Figure 15.5: single-phase transformer construction: (a) a core-type transformer and; (b) a shell-type transformer. The magnetic flux path through the core is shown in white.



(a) (b)

Figure 15.6: three-phase transformer construction: (a) core-type, three limb transformer and; (b) a shell-type five limb

Core-type and shell-type transformers can also be constructed for three-phase transformers (figure 15.6). The advantages of each type are:

- core-type (or three limb) is the most commonly used method of construction, the smaller core means less weight and expense.
- shell-type (or five limb) is used for larger transformers because they can be made with a reduced height.

15.5 Autotransformers

An autotransformer has only one tapped winding, which is both the primary and the secondary of

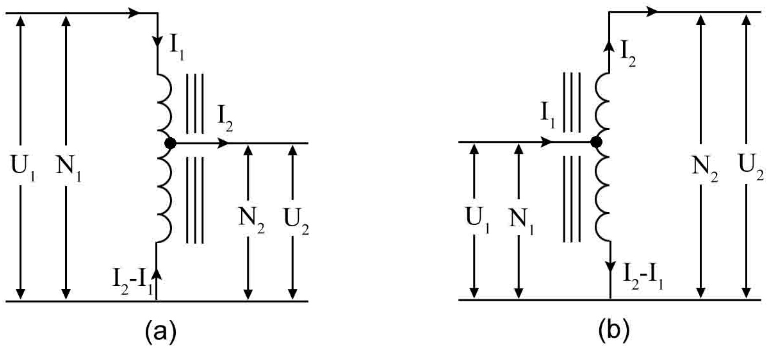


Figure 15.7: Autotransformers: (a) step-down; (b) step-up.

the machine. Since only one winding is needed the autotransformer is cheaper than a normal transformer. Step-down and step-up transformers are illustrated in figure 15.7.

If losses are neglected, turns, voltage and current ratios are the same for double-wound transformers, that is :

$$\frac{U_1}{U_2} = \frac{N_1}{N_2} = \frac{I_1}{I_2}$$

where:
N_1 = number of turns connected to input voltage
N_2 = number of turns connected to output voltage
U_1 = input (primary) voltage
U_2 = output (secondary) voltage
I_1 = input (primary) voltage
I_2 = output (secondary) current

The disadvantages of the autotransformer are that:

- There is a direct metallic connection between the input and the output, whereas the coupling in a double-wound transformer is magnetic only, giving electrical isolation of the two windings.
- In the event of an open-circuit fault in the common part of the winding, the input voltage of a step-down autotransformer would appear on the output terminals.

<u>Example</u>
A step-down autotransformer has a winding of 200 turns, a supply of 240V being connected across it. An output is taken from the neutral of the supply, and from a tapping 80 turns from the neutral end of the winding. Calculate:
(a) the output voltage
(b) the current in each part of the winding if the current from the supply is 20A

(a) $\frac{U_1}{U_2} = \frac{N_1}{N_2}$ so $U_2 = U_1 \times \frac{N_1}{N_2} = 240 \times \frac{80}{200} = 96V$

(b) $\frac{I_1}{I_2} = \frac{U_1}{U_2}$ so $I_2 = I_1 \times \frac{U_1}{U_2} = 20 \times \frac{240}{96} = 50V$

Current in the common part of the winding = I_2 - I_1 = 50 - 20 = 30A

Note that the coil only carries 30A in the tapped (common) section and 20A in the section that is not tapped. In a conventional transformer the primary coil would have to carry 20A and the secondary coil would have to carry 50A (probably requiring thicker wire).

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15.6 Transformer Losses, Efficiency And Regulation

Iron Losses

Iron losses were covered in section 5.3 and 5.4. Since iron losses are dependent on the frequency of the supply they are assumed to remain constant and not depend on the transformer's loading.

Copper Losses

These are due to the resistance of the wire from which the windings are made. If the secondary winding of a transformer is connected to an open circuit, so that no current flows, the primary winding will still carry a small current which provides the ampere-turns necessary to set up the magnetic flux in the core. Since this magnetising current is very small it is normally assumed to be zero. When the secondary winding is connected to a closed circuit a current will flow in it, the power loss in a conductor is $P = I^2R$, therefore the copper losses will be proportional to the square of the current. Thus, a transformer operating on half load will have only one quarter of the copper loss it has when providing full load. We can say:

$$\text{copper loss } P_C = (\text{actual load/full load})^2 \times \text{full-load copper loss}$$

Efficiency

As well as providing for the output power, input to a transformer must supply the transformer losses. Thus:

$$\text{input power} = \text{output power} + \text{power losses}$$

$$\text{efficiency} = \frac{\text{output power}}{\text{input power}} \times 100\%$$

$$\text{efficiency} = \frac{\text{input power - power losses}}{\text{input power}} \times 100\%$$

$$\text{efficiency} = \frac{\text{output power}}{\text{output power} + \text{power losses}} \times 100\%$$

Regulation

The resistance and inductance reactance of a transformer winding provide an impedance through which output current must pass. A voltage drop thus occurs in the windings of the transformer, its value depending on the effective impedance and on the current (U = IZ). As the current drawn increase, the voltage drop increase and since the EMF induced in the secondary windings is constant (not depending on load current) the output voltage must drop. The difference between the no-load output voltage and the full-load output voltage, expressed as a percentage of no-load voltage, is called the *voltage regulation*:

$$\text{regulation} = \frac{\text{no - load volatge - full - load voltage}}{\text{no} - \text{load voltage}} \times 100\%$$

The value of voltage regulation is normally a few percent.