Part 9: Basic AC Theory

9.1 Advantages Of AC Systems

Dealing with *alternating current* (AC) supplies is on the whole more complicated than dealing with DC current, However there are certain advantages of AC that have lead to it being the standard for electrical supplies:

(a) An alternating-current generator (often called an alternator) is more robust, less expensive, requires less maintenance, and can deliver higher voltages the its DC counterpart.
(b) The power loss in a transmission lines depends on the square of the current carried ($P = I^2R$). If the voltage used is increased, the current is decreased, and losses can be made very small. The simplest way of stepping up the voltage at the sending end of a line, and stepping it down again at the receiving end, is to use transformers, which will only operate efficiently from AC supplies.
(c) Three-phase AC induction motors are cheap, robust and easily maintained.
(d) Energy metres, to record the amount of electrical energy used, are much simpler for AC supplies than for DC supplies.
(e) Discharge lamps (florescent, sodium, mercury vapour etc.) operate more efficiently from AC supplies, although filament lamps are equally effective on either type of supply.
(f) Direct-current systems are subject to severe corrosion, which is hardly present with AC supplies.

9.2 Waves

As was noted in section 6.2 a coil rotating in a magnetic field will produce an alternating current (AC) which is made to flow by an alternating EMF. Such generators produce sine wave currents and voltages (section 8.5) and the equations for a sine wave AC voltage source and the current it delivers are:

$$e = E_m \sin \theta = E_m \sin f\,2\pi t = E_m \sin \omega t$$

and: $$i = I_m \sin \theta = I_m \sin f\,2\pi t = I_m \sin \omega t$$

where:
e = instantaneous EMF (V)
E_m = maximum or peak voltage (V)
θ = coil angle relative to magnetic flux (section 6.2)
f = frequency of the supply (Hz)
t = time, (s)
ω = angular velocity of the coil rotating in the magnetic field (rads/s)
i = instantaneous current (A)
I_m = maximum or peak current (A)

In section 8.5 we considered a stick rotating which produced sine waves but for AC generators we are considering the coil rotating in a magnetic field. Although nearly all AC supplies are sine waves other wave shapes are encountered occasionally. Figure 9.1 shows a simple EMF sine wave (a current sine wave would be similar) and we can see that several values are important:

- *Instantaneous values* - are values at particular instants in time, and will be different form instant to instant. Symbols for instantaneous values are lower case symbols, *v* for voltage and *i* for current, *e* for EMF and so on. These values can be calculated from the above equations if the maximum values are known.
- *Maximum or Peak Values* - are the greatest values reached during alternation, usually occurring once in each half-cycle. Maximum values are indicated by U_m for voltage, I_m for current and so

on.

- *Average or Mean Values* - are the average value of current or voltage. If an average value is found over a full cycle, the positive and negative half-cycles will cancel out to give a zero result if they are identical. In such cases, it is customary to take the average value over a half-cycle. Mean values have symbols, U_{av} and I_{av} etc.
- *Root Mean Square Values (RMS) or equivalent Values* - RMS values are a method of averaging sine waves to give a DC equivalent. The heat dissipated in a DC circuit is proportional to the current squared ($P = I^2R$), the equivalent in AC is the RMS current and for a given resistance a DC current of 1A will dissipate the same heat as an AC RMS current of 1A. The symbols used fro RMS value are the same as DC symbols, that is U, I etc. Note that from this point on, unless otherwise stated, all values followed by the symbol V or A (e.g. 240V and 13A) are RMS values.

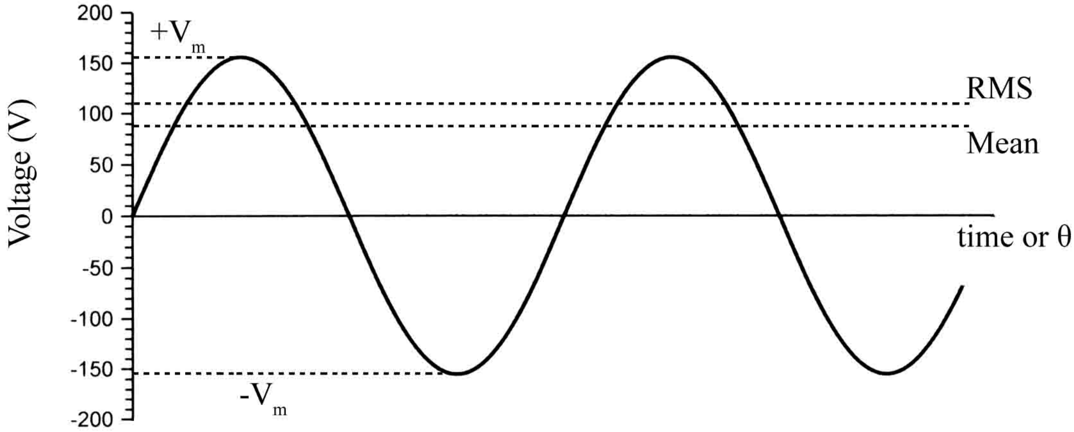

Figure 9.1: A sine wave for a 110V AC supply, showing the maximum, RMS and mean values.

In figure 9.1 we can see that the voltage becomes alternately positive and negative, meaning that the current also alternates. In a DC circuit the current flows around the circuit in one direction only and is always positive. In an AC circuit the current flows first in one direction and then in the other. In a sine wave supply the current and voltage are constantly changing, this change only ceases for the very instant that the peak values are reached; at these points the *rates of change* in current or voltage are said to be zero. The maximum rate of change occurs as the wave crosses the *x*-axis (i.e. *v* or *i* are zero).

The concept of AC may seem less intuitive than that of DC. In DC circuits it seems obvious that electrons can carry energy around a circuit and that this energy can be used to power motors and light bulbs etc. It may help to consider an analogy with a circular saw and a hand saw. The circular saw is like DC supplies and its teeth fly around in one direction using the supplied energy to cut through a plank. A hand saw is like AC supplies and moves the teeth of the saw backwards and forwards using the supplied energy to cut through a plank. Either way the same amount of energy is needed to cut through the plank.

<u>Example</u>
Figure 9.1 shows a sine wave for an AC supply with a peak voltage of 155.6V. Find the mean and RMS values.

By either taking measurements from the graph or using the sin wave equations ($v = 155.6 \sin \theta$) we can generate values of for v and v^2 for a half-cycle:

θ	**0°**	**15°**	**30°**	**45°**	**60°**	**75°**	**90°**	**105°**	**120°**	**135°**	**150°**	**165°**	**Total**
***v* (volts)**	0	40	78	110	135	150	156	150	141	78	110	78	**1226**
***v*²**	0	1600	6084	12 100	18 225	22 500	24 336	22 500	18 225	12 100	6084	1600	**145 354**

The mean value can be found from:

$$U_{av} = \frac{\text{the total of the } v \text{ values}}{\text{the number of values}} = \frac{1226}{12} = 99.0\text{V}$$

Root mean square is calculated from the square root of the mean of the *v* values squared. The mean of the squared values is given by:

$$\text{mean of the squared values} = \frac{\text{the total of } v^2 \text{ values}}{\text{the number of values}} = \frac{145\,354}{12} = 12\,112.8$$

The Root Mean Square value is given by:

$$\text{RMS} = \sqrt{12\,112.8} = 110\text{V}$$

This example illustrates that a RMS 110V supply actually has a peak voltage of 155.6V. Normally we need not take this into account since AC equipment rated at 110V means that it is rated at RMS 110V.

Another value which is sometimes calculated to indicate the shape of a waveform is the *form factor*: the higher its value the more 'peaky' the wave shape. The form factor is the ratio:

$$\text{form factor} = \frac{\text{RMS}}{\text{mean value}}$$

For the above example:

$$\text{form factor} = \frac{110}{99.0} = 1.11$$

The above method can be used to find U_{av} and the RMS values for any wave shape, not just sine waves. The following short cuts can be used for sine waves only:

$$\textbf{average value} = \frac{\mathbf{2 \times maximum\ value}}{\boldsymbol{\pi}} = \mathbf{0.637 \times maximum\ vlaue}$$

$$\textbf{RMS values} = \frac{\textbf{maximum values}}{\sqrt{\mathbf{2}}} = \mathbf{0.707 \times maximum\ values}$$

Therefore, the maximum value of a 240V supply will be 240 ÷ 0.707 = 339.5V. Note that the form factor of a sine wave is always 1.11 and that sin 45° = 0.707.

9.3 Adding AC Sine Waves

In the first example in section 8.7, we found that waves that are in phase can be simply added together. We can also perform simple addition on RMS value, so that two in-phase 110V supplies will have a total RMS voltage of 220V and a total maximum voltage of (155.6 × 2) 311.2V. The same applies to currents that are in-phase.
If voltages are out of phase they can not be simply added. We can add together a series of instantaneous value however, a quicker method is to use the parallelogram rule, as illustrated in the

second example in section 8.7.

Example
Two 110V AC supplies, U_1 and U_2, are 60° out of phase. Add these two supplies together.

The maximum voltage for a 110V supply, $U_m = 110 \div 0.707 = 155.6V$

The parallelogram diagram (also known as a phasor diagram), drawn to scale, is shown in figure 9.2. From it we can measure that the resulting supply, U_R, has a maximum voltage of 269.7V, lags U_1 by 30° and leads U_2 by 30°. The RMS of U_R is:

RMS = 269.7 × 0.707 = 190.7V

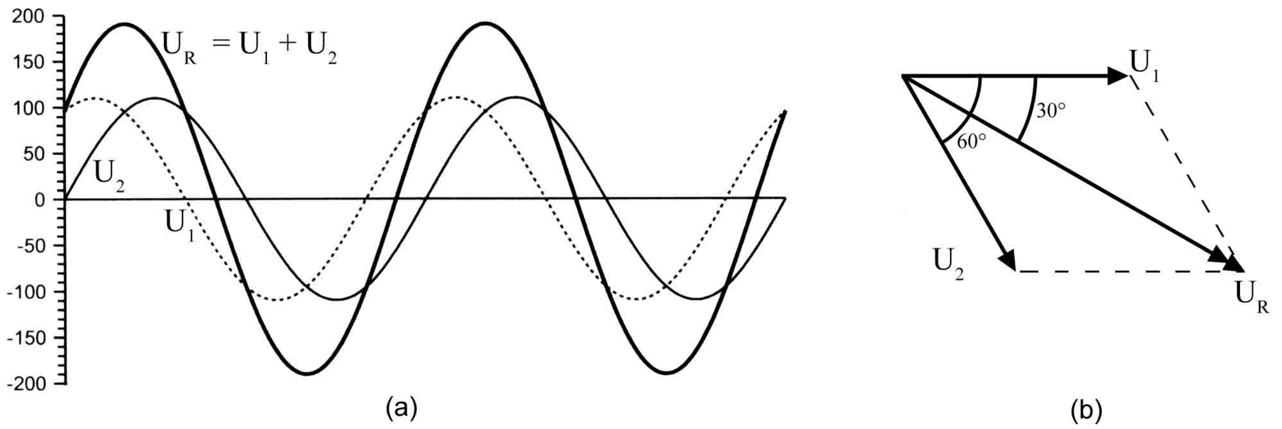

Figure 9.2: The addition of two AC supplies, each with a voltage of 110V but 60° out of phase.

In the above example we used maximum voltage values, although we could equally have use RMS values all the way through. To do this we can use the same parallelogram (figure 9.2b), but we must rescale it using $U_1 = U_2 = 110V$. Now $U_R = 190.7V$.

9.4 Resistive AC Circuits

Current and voltage stay in-phase in a purely resistive circuit (i.e. a circuit without inductance or capacitance). The circuit, wave and phase diagrams are shown in figure 9.3. If the alternating voltage of:

$$v = U_m \sin \omega t$$

is applied to a resistor, the instantaneous current:

$$i = \frac{v}{R} = \frac{U_m \sin \omega t}{R} = I_m \sin \omega t$$

Thus: $$I_m = \frac{U_m}{R} \quad \text{or, using RMS Values,} \quad \mathbf{I} = \frac{\mathbf{U}}{\mathbf{R}}$$

Example
A 240V AC supply is connected to an 80Ω resistor. Calculate the resulting current flow.

$$I = \frac{U}{R} = \frac{240}{801} = 3A$$

Note that these Current and voltage values are RMS values.

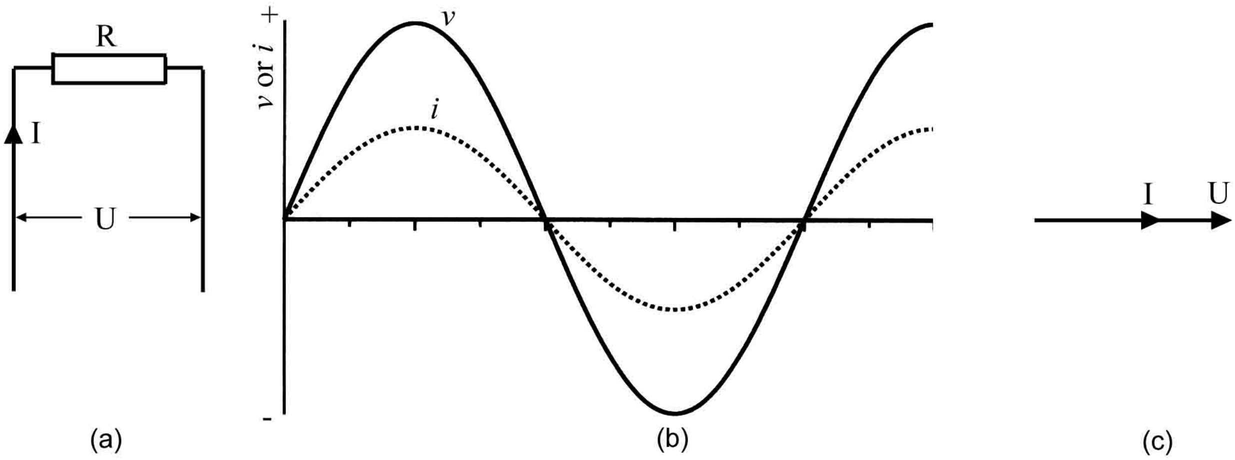

Figure 9.3: AC resistive circuit; (a) the circuit; (b) the wave diagram, *v* and *i* are in phase; (c) the phasor diagram. *v* is the instantaneous voltage and *i* is the instantaneous current. I and U are RMS values. Note that *v* and *i* are not plotted on the same scale.

9.5 Inductive AC Circuit

As we note in section 6.5, any coil of wire carrying a current will set up a magnetic field and consequently exhibit self inductance. Due to Lenz's law (section 6.3), if the current in the coil is increasing, the induced EMF will oppose will oppose the supply voltage limiting the rate of increase. Similarly, if the current in the coil is decreasing the induced EMF will try to keep it flowing. In a DC circuit the current reaches a steady value at which point the magnetic field becomes steady and self induction ceases, thus the induced EMF can not prevent a change in current only slow it down. In an AC circuit the current is constantly changing and matters are more complex.

Figure 9.3 shows the circuit, wave and phasor diagrams for a purely inductive circuit. The induced EMF in the coil will always oppose the applied voltage because it is always trying to oppose the change in current that the supply is causing. Figure 9.3 therefore shows the induced EMF (*e*) and the supply voltage (*v*) out of phase by 180°. The instantaneous value of the induced EMF depends on the rate of change of the current (section 6.5):

$$e = \frac{L(I_2 - I_1)}{t}$$

As mentioned in section 8.5, the rate of change of a value will be zero at the maximum and minimum points on the graph, and have its greatest value when the wave crosses the *x*-axis (i.e. the value is zero). Therefore, when *i* is at a maximum or minimum, the rate of change of *i* is zero and *e* is zero. Also when $i = 0$, the rate of change of *i* peaks so that *e* reaches a maximum. Due to Lenz's law, when the current is going positive, the EMF must oppose this change and will therefore be negative. The current wave diagram can therefore be drawn, and figure 9.4b shows that current *lags* the applied voltage by 90° and voltage *leads* current; it is normal the currents relationship to voltage to be stated, therefore the form description is used. The phasor diagram (figure 9.4c) thus shows the current under the voltage with an angle of 90° between them.

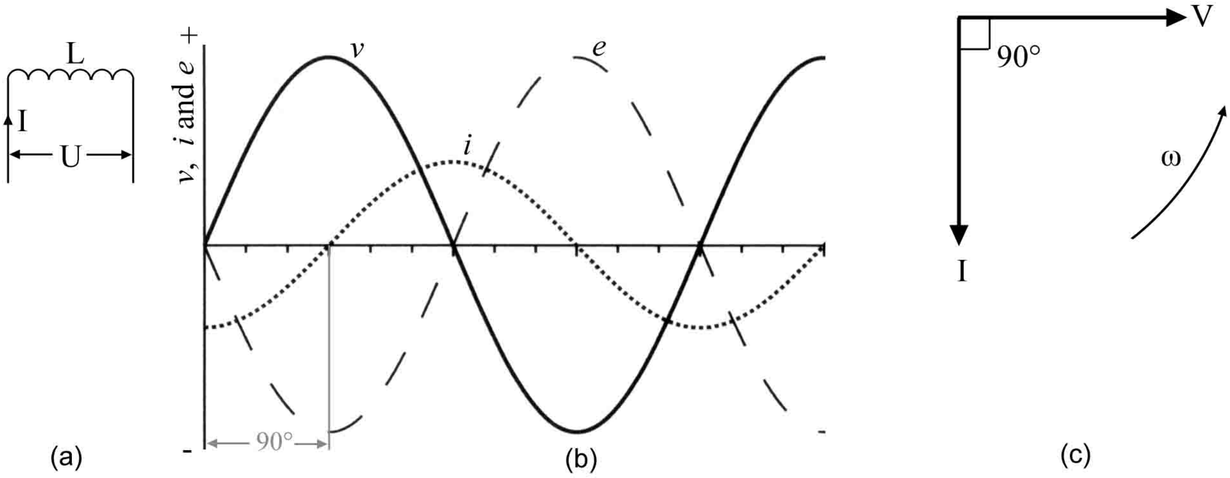

Figure 9.4: AC inductive circuit; (a) the circuit; (b) the wave diagram, *e* and *v* are 180° out of phase, *i* and *v* are 90° out of phase; (c) the phasor diagram. *e* is the instantaneous induced EMF, *v* is the instantaneous voltage and *i* is the instantaneous current. I and U are RMS values. Note that *v* and *i* are not plotted on the same scale.

Note that we are assuming that the circuit is a perfect inductor and therefore has no resistance. This is not possible, since the wire that forms the coil will have some resistance, however to examine the effects of inductance we will ignore this. In a circuit that only has resistance the current is limited by that resistance and I = U/R. If you connected a wire with a small resistance to the terminals of a battery, the wire would get very hot and the battery would go flat quit quickly because a very high current would flow. Also, if the wire had no resistance at all, I = U/0 and an infinite current would flow.

In a circuit with inductance but no resistance an infinite current does not flow and something else is limiting the current. Obviously, from the above discussion the current is limited because it lags the supply voltage and this does not increase infinitely, however it is useful to deal with a simple property which is similar to resistance. This property is called the *inductive reactance* of the coil (X_L) and it can be shown that:

$$\mathbf{X_L = \frac{U}{I} = 2\pi \, \mathit{f} \, L = \omega L}$$

where:
X_L = inductive reactance of the coil (Ω)
U = voltage applied to a coil (V)
I = resulting current flow (A)
f = supply frequency (Hz)
L = coil inductance (H)
$\omega = 2\pi f$

Note that, when $f = 0$, the inductive reactance will be zero. Thus if a coil is connected to a DC supply a steady current will flow through it, which is limited only by the coil's resistance.

Also (for circuits with resistance only): $$\mathbf{I = \frac{U}{X_L}}$$

9.6 Capacitive AC Circuit

If a direct voltage is applied to a capacitor the current gradually falls off until the capacitor is fully charged at which point no more current flows (section 7.7). If the capacitor is connected to an AC supply however, the current constantly changes direction and the capacitor will charge and discharge accordingly. In AC circuits, although no current flows right through the capacitor, an alternating current does exist in the circuit.

If an alternating voltage is applied to an uncharged capacitor, as the voltage passes through zero going positive, the current will immediately reach its maximum value as the capacitor starts to charge. As the charge increases, charging current will fall, reaching zero when the voltage becomes steady, which it does for an instant at its maximum value. As the voltage falls, the capacitor will discharge, and a negative current results. This pattern is shown in figure 9.5, which shows the circuit, wave and phasor diagrams for a capacitive circuit. It is clear from these diagrams that in a capacitive circuit current *leads* voltage by 90° (and so voltage lags current by 90°).

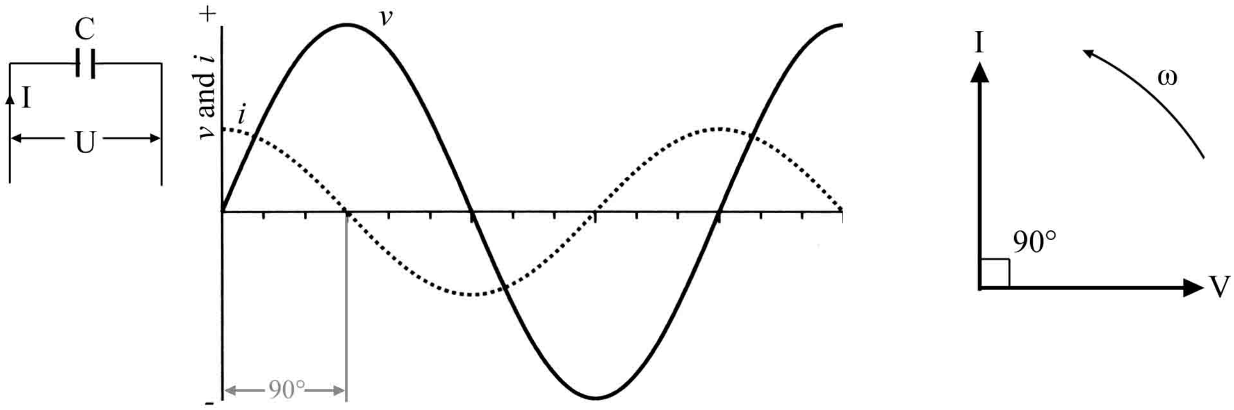

Figure 9.5: AC capacitive circuit; (a) the circuit; (b) the wave diagram, *v* and *i* are 90° out of phase; (c) the phasor diagram. *v* is the instantaneous voltage and *i* is the instantaneous current. I and U are RMS values. Note that *v* and *i* are not plotted on the same scale.

Like inductive circuits it is clear that the current is limited by a property other than resistance. This property is called *capacitive reactance* (X_C). It can be shown that:

$$\mathbf{X_C = \frac{U}{I} = \frac{1}{2\pi f C} = \frac{1}{\omega C}}$$

where:
Xc = capacitive reactance (Ω)
U = supply voltage (V)
f = supply frequency (Hz)
I = circuit current (A)
C = capacitance (F)
$\omega = 2\pi f$

With capacitance in microfarads (C'): $$\mathbf{X_L = \frac{10^6}{2\pi f C'}}$$

And (for circuits with capacitance only): $$\mathbf{I = \frac{U}{X_C}}$$