

THE SPECTRAL NATURE OF LIGHT

The Bohr Hydrogen Atom

Atoms have a diameter of about 10^{-10} m and are made up of a small (about 10^{-14} m), heavy, and positively charged nucleus, surrounded by orbiting electrons. The nucleus consists of protons (positive electric charge) and neutrons (no electric charge) bound together by nuclear forces. In a neutral atom there are as many negatively charged electrons as there are protons. Each electron occupies one of the

many possible well defined atomic orbit states. Each particular orbit state has an associated total energy (kinetic + potential.) Electrons “jumping” from one orbit with less energy to one with more (binding) energy will release discrete quantities of energy in the form of light. The spectrum of quantized frequencies observed in the light emitted by excited atoms is a direct measurement of the allowed transitions of electrons among different orbits in the atom. The emission spectrum of a given atom is a unique signature of this atom.

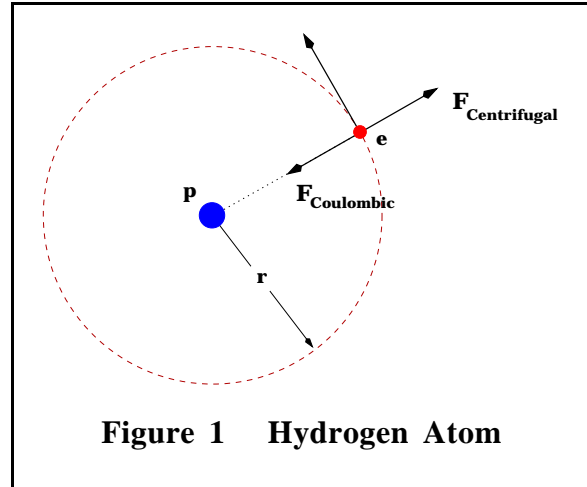


Figure 1 Hydrogen Atom

Although the existence of discrete line spectra emitted by atoms has been known for a long time, it was Niels Bohr who first calculated the spectrum of the hydrogen atom. This is the simplest atomic system and consists of one proton as nucleus and one orbiting electron. The electron describes a circular orbit in the electric field of the proton; the forces acting on the electron are:

$$F_{\text{electrostatic}} = F_{\text{centrifugal}}$$
$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \quad (1)$$

From (1) the kinetic energy of the electron is:

$$KE = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\epsilon_0 r} \quad (2)$$

The electrostatic potential of the proton at a distance r , is:

$$V = \frac{e}{4\pi\epsilon_0 r}$$

and the potential energy of the electron is:

$$U = (-e)V = -\frac{e^2}{4\pi\epsilon_0 r}$$

The total energy E of the system is:

$$E = KE + U = -\frac{e^2}{8\pi\epsilon_0 r} \quad (3)$$

Bohr quantized the value of r by postulating that the electronic orbit must be a standing wave with wavelength λ , consequently the length of the circle describing the electron's trajectory must be the product of an integer and λ ,

$$n\lambda = 2\pi r_n,$$

$$n = 1, 2, 3 \dots$$

The French physicist Louis de Broglie had postulated that one can assign a wavelength λ to any object having a momentum $p = mv$;

$$\lambda = \frac{h}{p} = \frac{h}{mv},$$

where h is Planck's constant (6.63×10^{-34} Joule sec). Combining the last two equations we obtain,

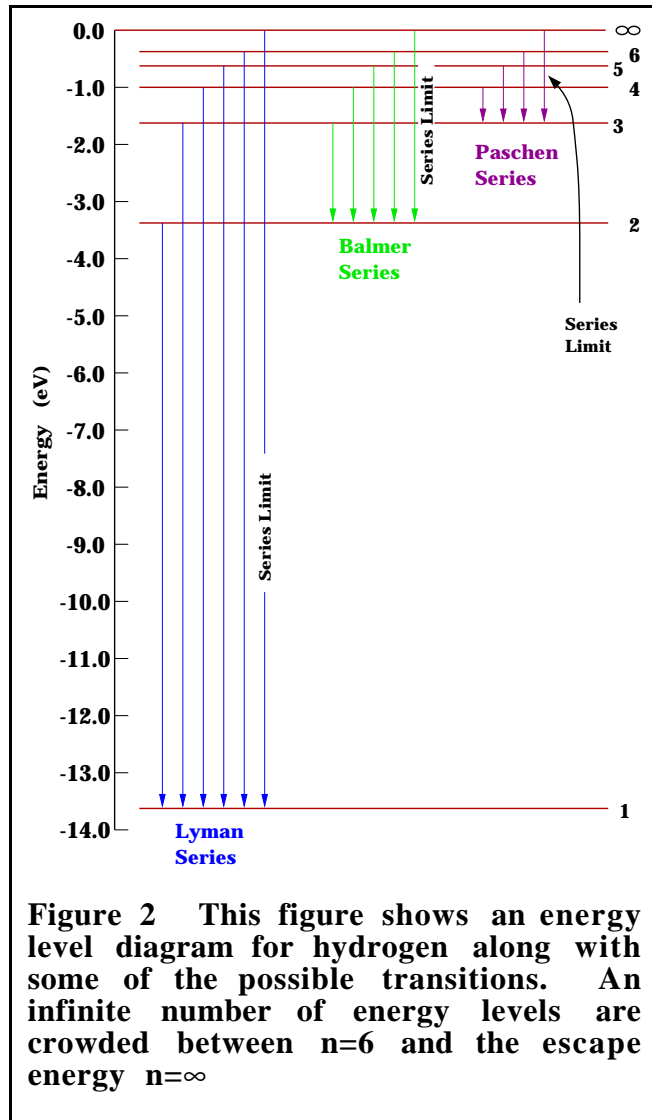


Figure 2 This figure shows an energy level diagram for hydrogen along with some of the possible transitions. An infinite number of energy levels are crowded between $n=6$ and the escape energy $n=\infty$

Name of Series	Quantum Number		Wavelength,
	j (lower state)	k (upper state)	
Lyman	1	2	1216
	1	3	1026
	1	4	970
	1	5	949
	1	6	940
	1	∞	912
Balmer	2	3	6563
	2	4	4861
	2	5	4341
	2	6	4102
	2	7	3970
	2	∞	3650
Paschen	3	4	18751
	3	5	12818
	3	6	10938
	3	7	10050
	3	8	9546
	3	∞	8220

Table 1 Selected Lines of the Hydrogen Spectrum

$$n \frac{h}{mv} = 2\pi r_n \quad n = 1, 2, 3, \dots \quad (5)$$

which gives, with equation (2), for the orbit of the electron

$$r_n = \frac{\epsilon_0}{\pi} \frac{h^2}{me^2} n^2 \quad n = 1, 2, 3, \dots \quad (6)$$

and, with equation (3), the energy of the electron

$$E_n = -\frac{1}{8\epsilon_0^2} \frac{me^4}{h^2} \frac{1}{n^2} = -R_y \frac{h}{n^2} \quad (7)$$

R_y is called the Rydberg constant:

$$R_y = \frac{1}{8\epsilon_0^2} \frac{me^4}{h^3} \frac{1}{n^2} = 3.29 \times 10^{15} \text{ sec}^{-1} .$$

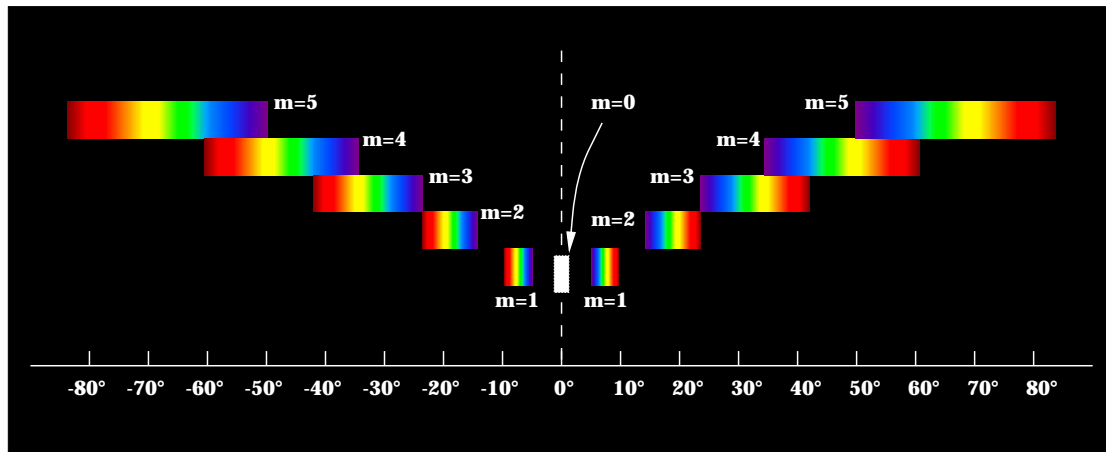


Figure 3 The spectrum of white light as viewed using a grating instrument like the one in this experiment. The different orders identified by the order number m , are shown separated vertically for clarity. In actuality they would overlap.

The radius r_n of the orbiting electron and the binding energy E_n are quantized: they can assume only discrete values! The quantity n is called the principal quantum number. The radius r_n increases as n^2 . The energy values are negative because we are calculating a binding energy (one has to supply energy in order to excite the atom.) Figure 2 shows the energy of stationary state and their associated quantum numbers. The upper level marked $n = \infty$, corresponds to a state in which the electron is completely removed from the atom; the atom is ionized.

An atom radiates when, after being excited, it makes a transition from one state with the principle quantum number k , and consequently with energy E_k , to a state j with lower energy E_j . The quantum energy carried by the photon is

$$E = h \nu = E_k - E_j = R_y \left(\frac{h}{j^2} - \frac{h}{k^2} \right) = \frac{me^4}{8\epsilon_0^2 h^2} \left(\frac{1}{j^2} - \frac{1}{k^2} \right)$$

The frequency of the emitted radiation is

$$\nu = R_y \left(\frac{1}{j^2} - \frac{1}{k^2} \right) = \frac{me^4}{8\epsilon_0^2 h^3} \left(\frac{1}{j^2} - \frac{1}{k^2} \right). \quad (8)$$

Table 1 gives some wavelengths calculated from the last equation and $\lambda = c/\nu$.

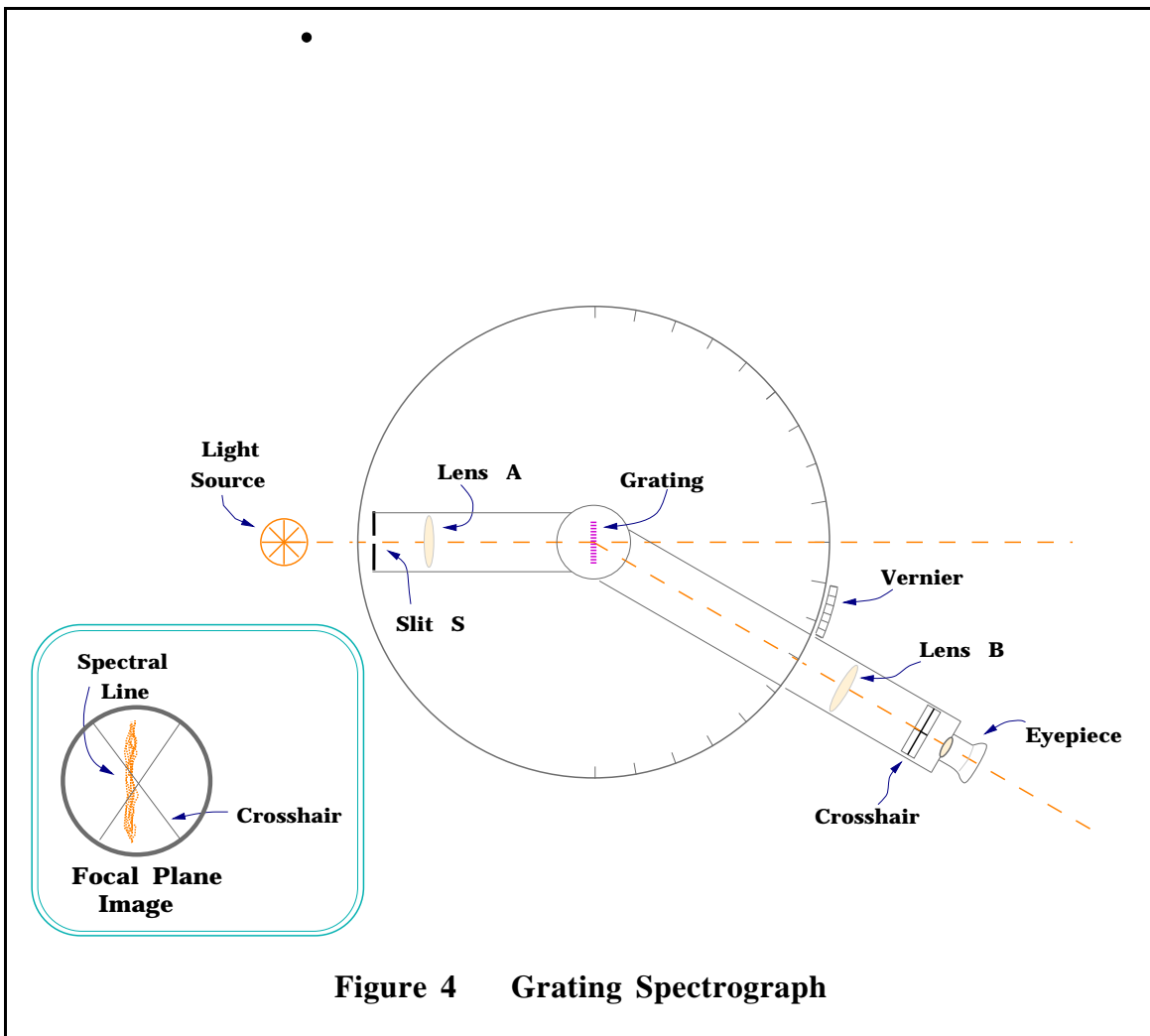
The Grating Spectrometer

As already discussed in the second laboratory, gratings are used to analyze the frequency spectrum of light. Light of a given wavelength, λ , will be deflected by a grating with a line spacing constant d , by an angle θ , according to the relation:

$$d \sin \theta = m \lambda , \quad m = 0, 1, 2, \dots \quad (9)$$

where m is the order number.

Figure 3 shows how a grating spectrometer diffracts visible light in groups of intensity



distributions as a function of θ , and each of these groups corresponds to a different order m . Adjacent distributions do overlap.

Figure 4 shows the spectrometer used for this experiment. It consists of a circular table with a diffraction grating at its center and a scale along its circumference. Two pipes are mounted on the table and these pipes intersect in the region of the grating. The mounting of the first pipe is permanent. It is designed to carry parallel light from an external source to shine upon the grating.

The second pipe is mounted in such a way as to be able to pivot about the center of the circular table. This rotation makes it possible to separate the light,

collecting only what has been diffracted through some chosen angle, θ . If this pipe is set to an angle such as $\theta = 10^\circ$, then whatever light the grating diffracts through an angle of 10° will travel down the length of the pipe. It will be focused by the adjustable lens (B) and the eyepiece. A pair of crosshairs identify the center of the pipe and, together with the vernier scale located on the equipment, allow the accurate measurement of the angle, θ .

The grating constant d is engraved on the frame holding the grating. Slit S, in Figure 4 defines the light source; lens A is separated from the slit by a distance equal to its focal

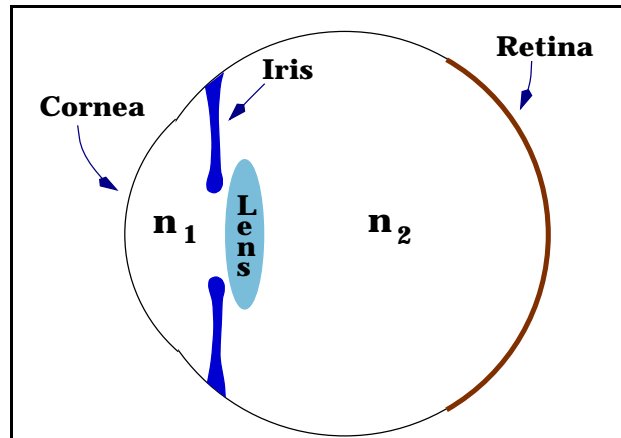


Figure 5 Focusing system of the eye. Most of the refraction occurs at the first surface, where the cornea C separates a liquid with index of refraction n_1 from the air. The lens L, which has a refractive index varying from 1.42 in the center to 1.37 at the edge, is focused by tension at the edges. The main volume contains a jelly like 'vitreous humour' ($n=1.34$). The image is formed on the retina. Illumination levels are controlled by the aperture of the iris.

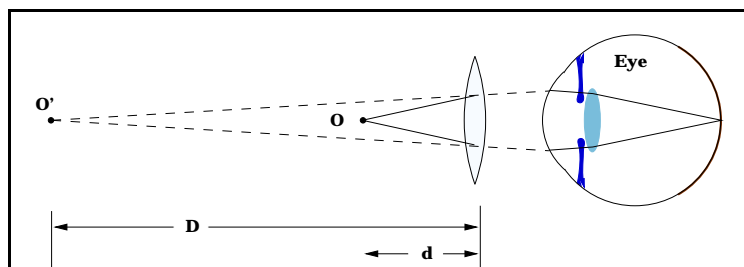


Figure 6 Simple lens used as a magnifier. An object at O close to the eye can be focused by the eye to appear to be much larger and at a more distant point O'

length, f , and consequently it projects a beam of parallel light on to the grating. Lens B focuses the light diffracted by the grating into a focal plane which contains the crosshair. Both positions of lens B and the crosshair can be adjusted in order to focus the image better.

The optics of the human eye is shown schematically in Figure 5. The optimum focal distance of our eyes is about 25 cm. In order to observe an image at a closer distance, a lens or an eyepiece may be used as indicated in Figure 6. The spectrometer has an eyepiece with focal length of 2.5 cm. Each student will have to adjust the position of the of lens B to see a sharp image the spectral fringe. The placement of the crosshairs should then be adjusted so that the too are in focus. A small light bulb controlled by a push-button switch allows the crosshairs to be illuminated

To determine the axis ($\theta = 0$) of the spectrometer, place the table light in front of the spectrometer slit, S, remove the diffraction grating and look through the pivoting arm of the spectrometer directly at the light source. Adjust the position of the slit, of lens B, and of the crosshair in order to see a sharp, white image (order $m = 0$). When a sharp vertical image of the slit is centered on the crosshair, you can read, with the help of the vernier, the angle corresponding to the axis of the spectrometer. All measurements of θ that you will make must be with respect of the spectrometer axis.

The Light Sources

In this laboratory you will observe the spectrum of four gases: H, He, Ne, and Hg. The light source

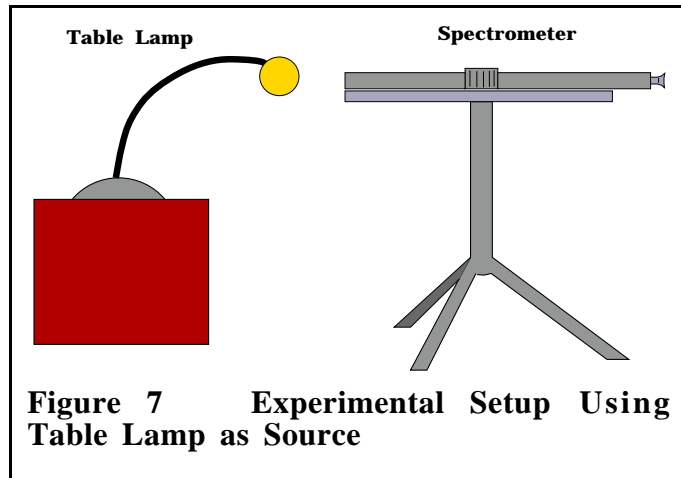


Figure 7 Experimental Setup Using Table Lamp as Source

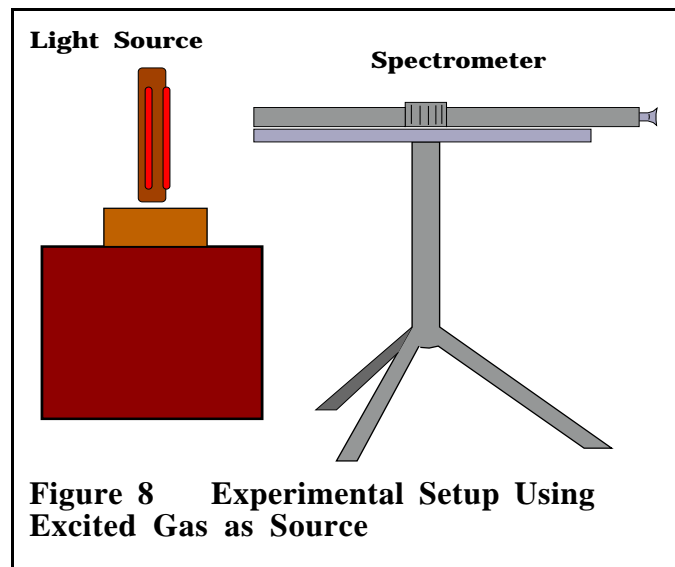


Figure 8 Experimental Setup Using Excited Gas as Source

consists of a glass tube which contains one of these gases at a pressure of about 20-30 cm Hg. A 60 Hz voltage of a few kilovolts is applied at electrodes entering the gas tube at each end. The variable electric field between the two electrodes will excite the gas in the tube. An electron will remain in an excited state for a time on the order of 10^{-8} sec before jumping down to a state with higher binding energy. The excited state, with a lifetime $\tau \cong 10^{-8}$ sec, has then decayed into a lower energy state. In order to prolong the usefulness of these tubes do not keep the light source on longer than necessary.

Experiment 1 The Spectrum of White Light

The electromagnetic spectrum emitted by the hot filament of the table light is a continuous spectrum whose relative intensity depends upon its temperature. The shape of the spectrum of a black body is shown in Figure 10. It was Planck who first calculated the shape of this spectrum. Planck introduced a constant h , now known as Planck's constant, to reproduce mathematically the observed distribution of this spectrum.

Observe the visible part of the electromagnetic spectrum emitted by your incandescent lamp. Look at the spectrum associated with the order numbers $m = 0, 1, \text{ and } 2$. Observe the different colors associated with different frequencies or wavelengths.

Measure the spectral sensitivity of your eye by observing the highest frequency your eye can discern. Use the order $m = 1$ to make these measurements. Measure the angle θ with respect to the spectrometer axis and with equation (9) calculate the corresponding λ .

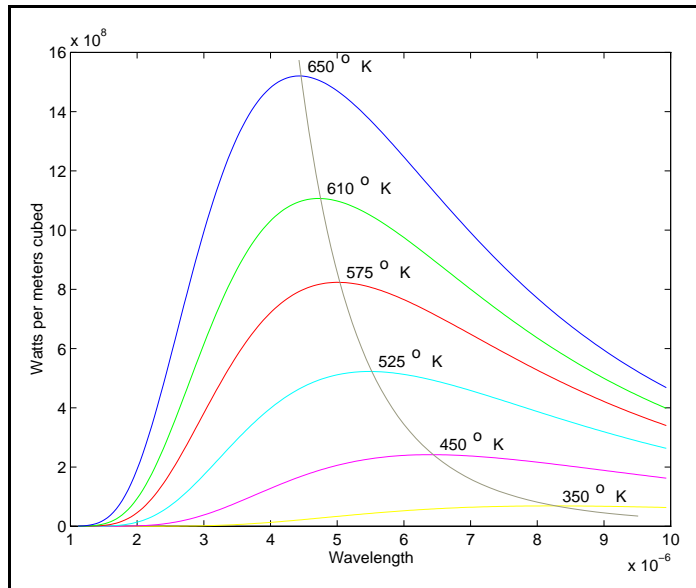


Figure 9 Blackbody Radiation Spectrum. Planck's energy density of blackbody radiation at various temperatures as a function of wavelength. Note that the wavelength at which the curve is a maximum decreases with temperature.

HYDROGEN	HELIUM	NEON	MERCURY
λ ()	λ ()	λ ()	λ ()
6562.8	7065	7245	7345*
6562.7	6678	7170	6905
4861	5875	7032	6156
4340	5015	6929	5790
4104	4471	6402	5769 [!]
	3888	6143	5460
		5881	4358
		5852	4046*
		5400	
		5350	
		4790	
		4725	
		4576	

Table 2 Wavelengths and Intensities of Prominent Lines in Simple Spectra
(Lines with asterisk are very dim. (!) denotes a double line.

Experiment 2 The Hydrogen Spectrum

Observe and record as many spectral lines as you can see in the spectrum of hydrogen, using a hydrogen gas tube. Make sure that the light source is properly aligned in front of the spectrometer slit. Compare the wavelength of the lines you observe with the value shown in Table 1. Which series of lines do you observe? Table 1 also gives the principal quantum numbers involved in the observed transitions. Using equation (8) and the frequency of measured transition calculate the value of the Rydberg constant.

Hints for measuring lines:

- 1) Align and center the light source to make the lines as bright as possible.
- 2) Optimize the focus for each measurement. Make sure the diffraction grating is aligned (a peg on the slide's bottom should fit into a groove on the mount.)
- 3) Use some intuition in matching the measured wavelengths to the actual chart. Even with careful measurement procedures the actual values may easily vary as much as a couple tenths of a micron. Also the chart only lists the brighter lines, other dim lines may also be present.

Experiment 3 The Spectrum of He, Ne, and Hg

The He gas tube is labeled while the tubes with Ne and Hg are only marked with colored tape. Record the wavelength of 5 to 10 spectral lines of each one of these elements. Notice that the spectrum of these gases is much more complicated, with many more lines than the one of hydrogen. This is due to the larger number of electrons orbiting the nuclei of these elements and consequently the larger number of possible transitions corresponding to spectral lines. Table 2 gives the wavelengths and the relative intensity of the line spectra of these elements. By comparing your observed values with those on the table you will be able to tell which gas is enclosed in a tube marked with a given color.

QUESTIONS

Following is a list of questions intended to help you prepare for this laboratory session. If you have read and understood this write-up, you should be able to answer most of these questions. The TA may decide to check your degree of preparedness by asking you some of these various questions:

- Why does the binding energy of an atomic electron have a negative value?
- What did Bohr postulate in calculating the energy spectrum of the hydrogen atom?
- What does quantization mean? Name five quantities that are quantized in nature. Show, using equation (8), that the dimensions of the Rydberg constant are sec^{-1} .
- What is the difference between the electromagnetic spectrum produced by an incandescent object and the spectrum generated by excited atoms?
- How do you excite an atom and how long does it stay excited?
- Is the light emitted from atoms polarized?
- The light of which one of the hydrogen series is in the visible spectrum?
- What is the physical significance of the order m of the spectrum produced by a grating?
- Draw the path of two parallel light rays diverging from the spectrometer slit and entering the observer's eye.

Sections of this write up were taken from:

Physics Laboratory: Third Quarter, Bruno Gobi, Northwestern University

APPENDIX: The Vernier Principle

A vernier consists of a movable scale next to a stationary scale. The movable scale is marked off in divisions that are slightly smaller than divisions on the main scale. If you line up the zero mark of the vernier with the zero mark of the fixed scale, you will notice that not all of the other divisions are aligned. However, if you examine the two scales closely, you should observe that the n th division of the movable scale exactly coincides with the $n-1$ division of the fixed scale. This implies that $n-1$ divisions of the main scale has the same length as n divisions on the vernier scale; so each vernier division is $\frac{n-1}{n}$ or $1 - \frac{1}{n}$ times the length of a main scale division. Determining the quantity $\frac{1}{n}$ is an important

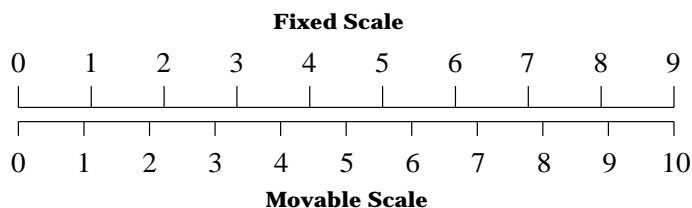


Figure 10 Vernier Scale

first step in making any measurement with a vernier: $\frac{1}{n}$ is called the “least count”.

The least count gives an indication of the accuracy of the particular measuring device. For example, if we can read the main scale of our device to the nearest degree, and the device has a least count of $1/4$ (i.e. 4 vernier divisions equal 3 main scale divisions) then we can make measurement with an instrumental limit of error of $\pm 1/4$ of a degree. Examine Figure 10. What is n ? What is the least count of this vernier. If each division of the main scale corresponds to the one degree, what is the accuracy of the measurements we can make with this device?

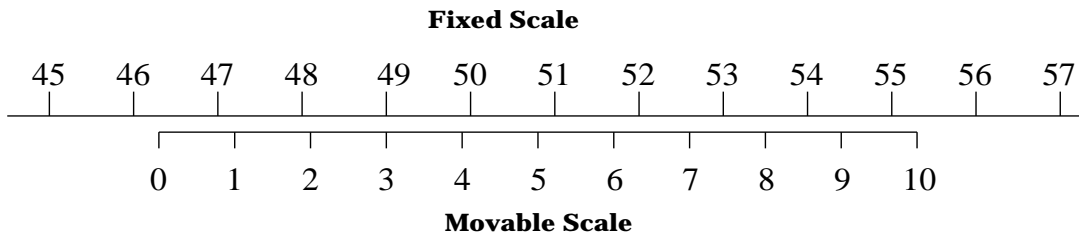


Figure 11 Vernier Scale - Reading 46.3

To read a measurement from a vernier such as the one in Figure 11, follow these steps:

- 1) Determine the least count of your device.
- 2) Look opposite the zero mark of the movable scale and read off what division of the fixed scale is at the zero mark. If the zero mark does not exactly align and is situated between two divisions, then read off the value of the lower of these two divisions. In Figure 11, we read 46 degrees.
- 3) Examine each mark along the movable scale and determine which division (from zero to n) is aligned *exactly* with a division of the main scale. Read off this number from the movable scale. In Figure 11, we can see that 3 on the movable scale is aligned with a division on the main scale. So we must add 3 least counts of $\frac{3}{10}$ of a degree to the number read from the main scale earlier, 46. Thus our final answer is 46.3 degrees. If the zero mark of the movable scale had exactly coincided with a division of the main scale, as in Figure 10, then we would add no least counts and arrive at an answer of 0.0 degrees.

Incidentally, all of the above is brought to you courtesy of Pierre Vernier (1585-1638), a French engineer who spent most of his life building fortifications, but who designed the vernier scale in about 1630.