Sinusoidal Pulse width modulation

The switches in the voltage source inverter (See Fig. 1)can be turned on and off as required. In the simplest approach, the top switch is turned on If turned on and off only once in each cycle, a square wave waveform results. However, if turned on several times in a cycle an improved harmonic profile may be achieved.

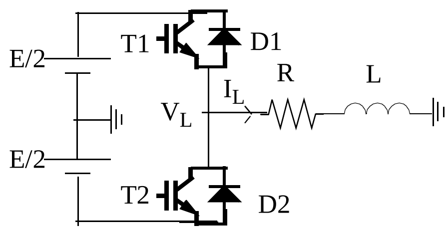

Fig 1: Simple Voltage Sourced Inverter

In the most straightforward implementation, generation of the desired output voltage is achieved by comparing the desired reference waveform (modulating signal) with a high-frequency triangular 'carrier' wave as depicted schematically in Fig.2. Depending on whether the signal voltage is larger or smaller than the carrier waveform, either the positive or negative dc bus voltage is applied at the output. Note that over the period of one triangle wave, the average voltage applied to the load is proportional to the amplitude of the signal (assumed constant) during this period. The resulting chopped square waveform contains a replica of the desired waveform in its low frequency components, with the higher frequency components being at frequencies of an close to the carrier frequency. Notice that the root mean square value of the ac voltage waveform is still equal to the dc bus voltage, and hence the total harmonic distortion is not affected by the PWM process. The harmonic components are merely shifted into the higher frequency range and are automatically filtered due to inductances in the ac system.

When the modulating signal is a sinusoid of amplitude Am, and the amplitude of the triangular carrier is Ac, the ratio m=Am/Ac is known as the modulation index. Note that controlling the modulation index therefor controls the amplitude of the applied output voltage. With a sufficiently high carrier frequency (see Fig. 3 drawn for fc/fm = 21 and t = L/R = T/3; T = period of fundamental), the high frequency components do not propagate significantly in the ac network (or load) due the presence of the inductive elements. However, a higher carrier frequency does result in a larger number of switchings per cycle and hence in an increased power loss. Typically switching frequencies in the 2-15 kHz range are considered adequate for power systems applications. Also in three-phase systems it is advisable to use $\frac{fc}{fm} = 3k, (k \in N)$ so that all three waveforms are symmetric.

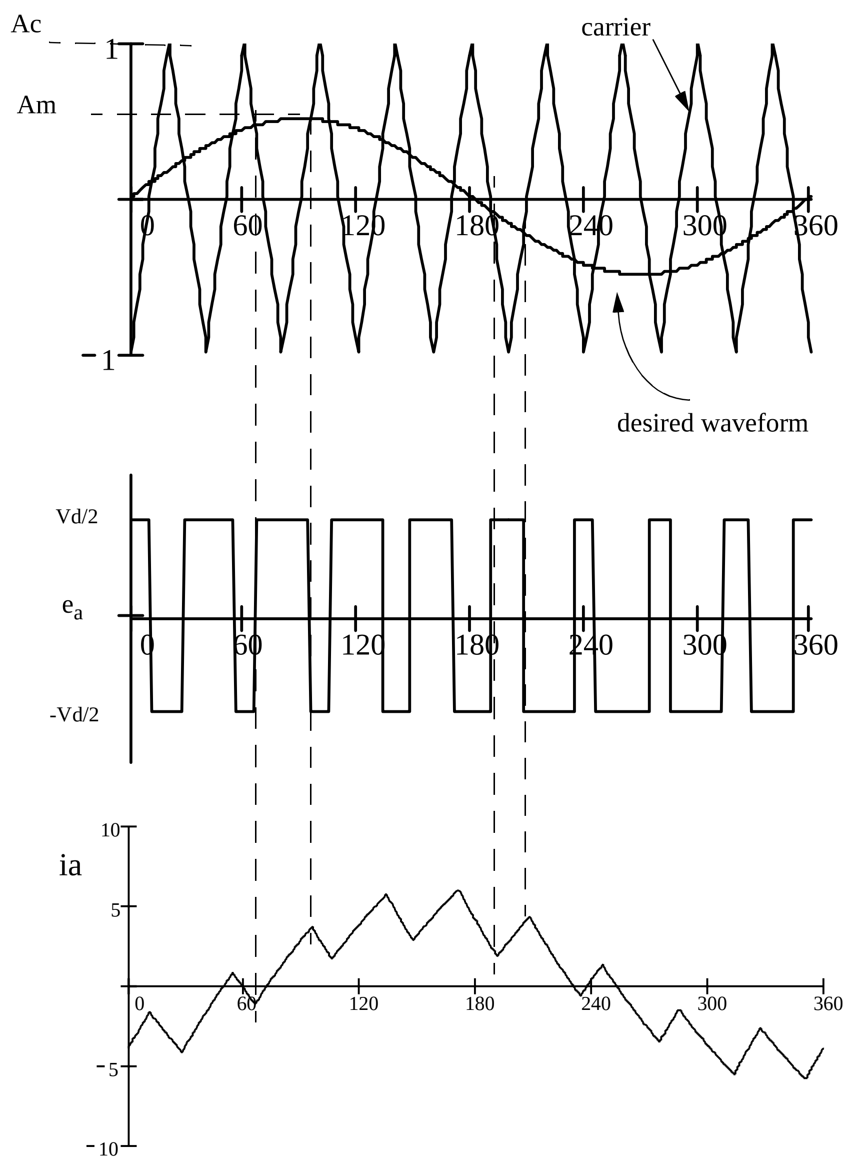

Fig 2: Principal of Pulse Width Modulation

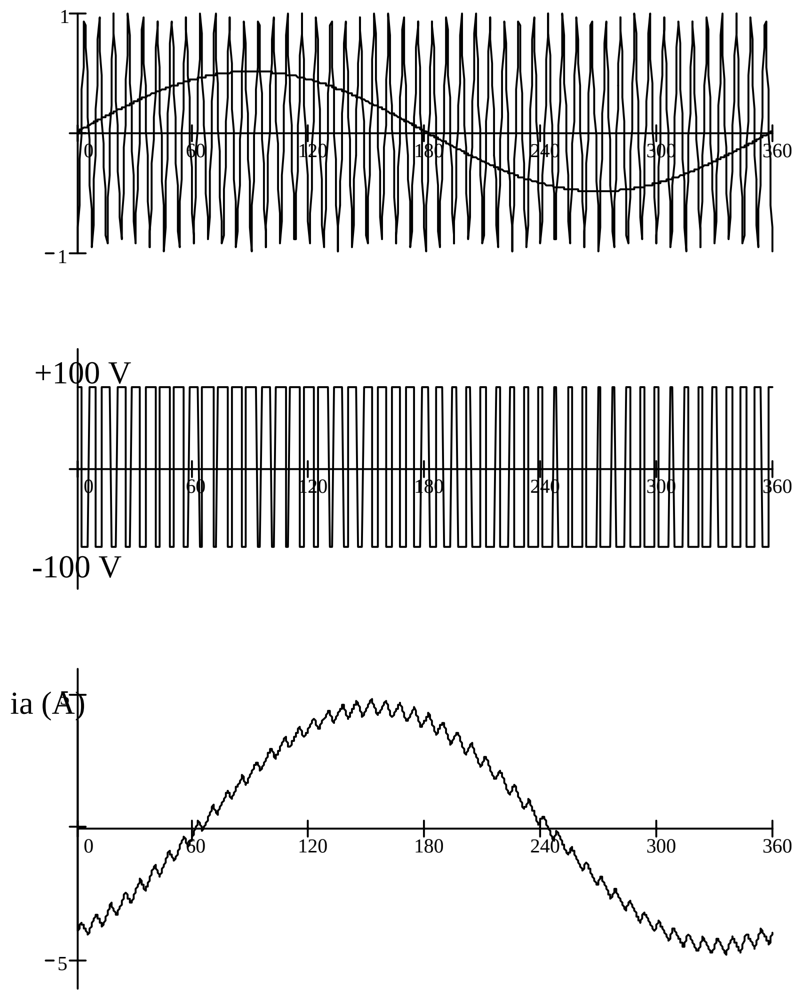

Fig. 3: SPWM with fc/fm = 48, L/R = T/3

Note that the process works well for $m \leq 1$. For $m > 1$, there are periods of the triangle wave in which there is no intersection of the carrier and the signal as in Fig. 4. However, a certain amount of this "overmodulation" is often allowed in the interest of obtaining a larger ac voltage magnitude even though the spectral content of the voltage is rendered somewhat poorer.

Note that with an odd ratio for fc/fm, the waveform is anti-symmetric over a 360 degree cycle. With an even number, there are harmonics of even order, but in particular also a small dc component. Hence an even number is not recommended for single phase inverters, particularly for small ratios of fc/fm.

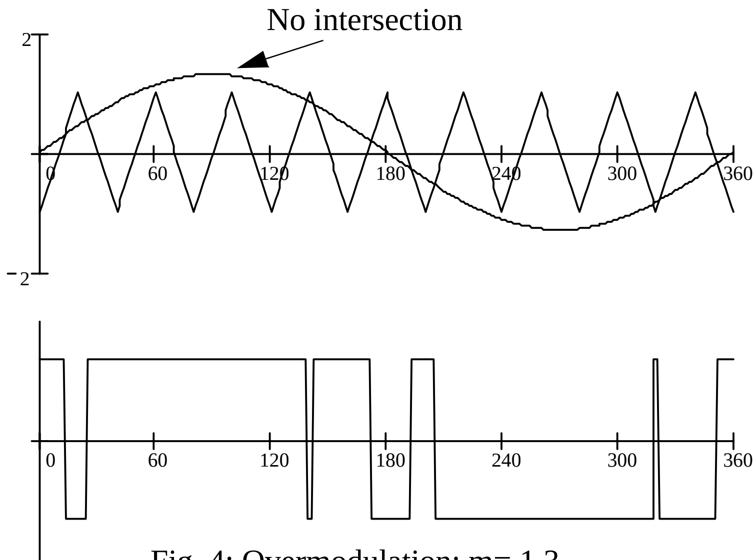

Fig. 4: Overmodulation: m= 1.3

SPWM Spectra:

Although the SPWM waveform has harmonics of several orders in the phase voltage waveform, the dominant ones other than the fundamental are of order n and n±2 where n = fc/fm. This is evident for the spectrum for n=15 and m = 0.8 shown in Fig.5. Note that if the other two phases are identically generated but 120o apart in phase, the line-line voltage will not have any triplen harmonics. Hence it is advisable to choose $\frac{fc}{fm} = 3k,(k \in N)$, as then the dominant harmonic will be eliminated. It is evident from Fig 5b, that the dominant 15th harmonic in Fig. 5a is effectively eliminated in the line voltage. Choosing a multiple of 3 is also convenient as then the same triangular waveform can be used as the carrier in all three phases, leading to some simplification in hardware.

It is readily seen that as the $(pwm(\theta))^2 = E^2$ where E is the dc bus voltage, that the rms value of the output voltage signal is unaffected by the PWM process. This is strictly true for the phase voltage as triplen harmonic orders are cancelled in the line voltage. However, the problematic harmonics are shifted to higher orders, thereby making filtering much easier. Often, the filtering is carried out via the natural high-impedance characteristic of the load.

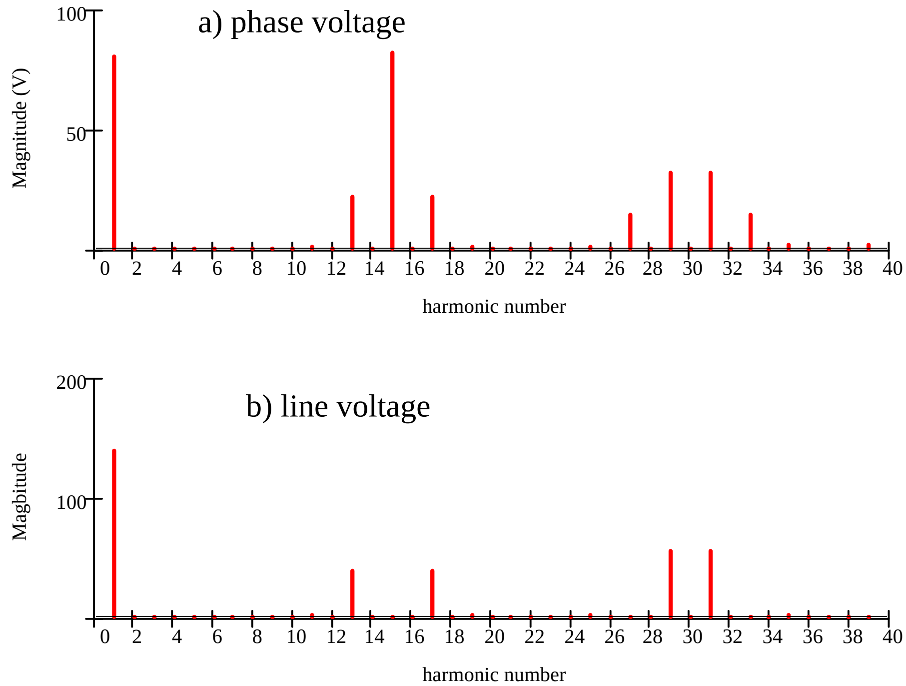

Fig. 5: SPWM Harmonic Spectra: n = 15, m =0.8

Selective Harmonic Elimination

(also called Optimal PWM)

Notice that in the SPWM strategy developed above, a large number of switchings are required, with the consequent associated switching losses. With the method of *Selective Harmonic Elimination*, only selected harmonics are eliminated with the smallest number of switchings. This method however can be difficult to implement on-line due to computation and memory requirements.

For a two level PWM waveform with odd and halfwave symmetries and n chops per quarter cycle as shown in Fig 4, the peak magnitude of the harmonic components including the fundamental, are given byEqn. 1:

$$h_1 = \left(4 \cdot \frac{E}{\pi}\right) \cdot [1 - 2\cos\alpha_1 + 2\cos\alpha_2 - 2\cos\alpha_3 \ldots 2\cos\alpha_n]$$
$$h_3 = \left(4 \cdot \frac{E}{3\pi}\right) \cdot [1 - 2\cos 3\alpha_1 + 2\cos 3\alpha_2 - 2\cos 3\alpha_3 \ldots 2\cos 3\alpha_n]$$
$$\vdots \qquad (1)$$
$$h_k = \left(4 \cdot \frac{E}{k\pi}\right) \cdot [1 - 2\cos k\alpha_1 + 2\cos k\alpha_2 - 2\cos k\alpha_3 \ldots 2\cos k\alpha_n]$$

Here h_i is the magnitude of the i^{th} harmonic and α_j is the j^{th} primary switching angle. Even harmonics do not show up because of the half-wave symmetry.

The n chops in the waveform afford n degrees of freedom. Several control options are thus possible. For example n selected harmonics can be eliminated. Another option which is used here is to eliminate n-1 selected harmonics and use the remaining degree of freedom to control the fundamental frequency ac voltage. To find the α's required to achieve this objective, it is sufficient to set the corresponding h's in the above equations to the desired values (0 for the n-1 harmonics to be eliminated and the desired per-unit ac magnitude for the fundamental) and solve for the α's.

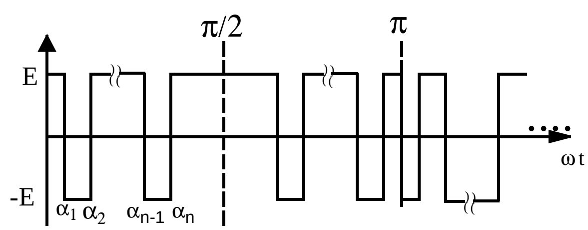

Fig 4: A two-level PWM waveform with odd and halfwave symm

Equation 1 can be readily proved by finding the fourier coefficients of the waveform shown in Fig. 4. In general, for a periodic waveform with period 2π, the Fourier Cosine and Sine Coefficients are given by:

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)d\theta$$

$$a_k = \frac{1}{\pi}\int_0^{2\pi} f(k\theta)\cos(k\theta)d\theta \qquad (2)$$

$$b_k = \frac{1}{\pi}\int_0^{2\pi} f(k\theta)\sin(k\theta)d\theta)$$

Because of the half-cycle symmetry of the waveform of Fig. 4, only odd order harmonics exist. Also, it is easy to see that the Fourier Cosine coefficients disappear with the choice of coordinate axes used. Utilizing the quarter cycle symmetry, the Fourier Sine coefficients become:

$$b_k = \frac{4}{\pi}\int_0^{2\pi} f(k\theta)\sin(k\theta)d\theta) \qquad (3)$$

Substituting the two-valued pwm waveform for $f(\theta)$, one obtains (see Fig. 4):

$$b_n = \frac{4E}{\pi}\left(\int_0^{\alpha_1}\sin(k\theta)d\theta - \int_{\alpha_1}^{\alpha_2}\sin(k\theta)d\theta + \int_{\alpha_2}^{\alpha_3}\sin(nk)d\theta\ldots\int_{\alpha_n}^{\frac{\pi}{2}}\sin(k\theta)d\theta\right)$$

$$= \frac{4E}{\pi k}\left(-\cos(k\theta)\Big|_0^{\alpha_1} + \cos(k\theta)\Big|_{\alpha_1}^{\alpha_2} - \cos(k\theta)\Big|_{\alpha_2}^{\alpha_3}\ldots\right) \qquad (4)$$

$$= \frac{4E}{\pi n}[1 - 2\cos n\alpha_1 + 2\cos k\alpha_2 \quad - 2\cos k\alpha_3\ldots 2\cos k\alpha_n]$$

The following example illustrates the use of three chops per quarter cycle which allow for three degrees of freedom. We may use these to eliminate two harmonics and control the magnitude of the fundamental to any desired value:

Example:

Selective Harmonic Elimination is applied with a view to controlling the fundamental component of voltage to 50V (rms) and eliminating the 3rd and 5th harmonics. The source voltage is 100 V. Calculate the required chopping angles.

As three objectives are to be achieved, we need 3 chops. The fundamental, 3rd and 5th harmonic magnitudes are given by:

(5)

$$v1 = \frac{4 \cdot E}{\pi} \cdot (1 - 2 \cdot \cos(\alpha 1) + 2 \cdot \cos(\alpha 2) - 2 \cdot \cos(\alpha 3))$$

$$v3 = \frac{4 \cdot E}{\pi \cdot 3} \cdot (1 - 2 \cdot \cos(3 \cdot \alpha 1) + 2 \cdot \cos(3 \cdot \alpha 2) - 2 \cdot \cos(3 \cdot \alpha 3))$$

$$v5 = \frac{4 \cdot E}{\pi \cdot 5} \cdot (1 - 2 \cdot \cos(5 \cdot \alpha 1) + 2 \cdot \cos(5 \cdot \alpha 2) - 2 \cdot \cos(5 \cdot \alpha 3))$$

We require:

$v1 := 50 \cdot \sqrt{2}$ (peak)

$v3 := 0$

$v5 := 0$

This gives us three equations in the three unknowns α1 α2 and α3. Solving numerically we get:

$\alpha 1 = 27.432 \circ deg$ $\alpha 2 = 42.131 \circ deg$ $\alpha 3 = 85.62 \circ deg$