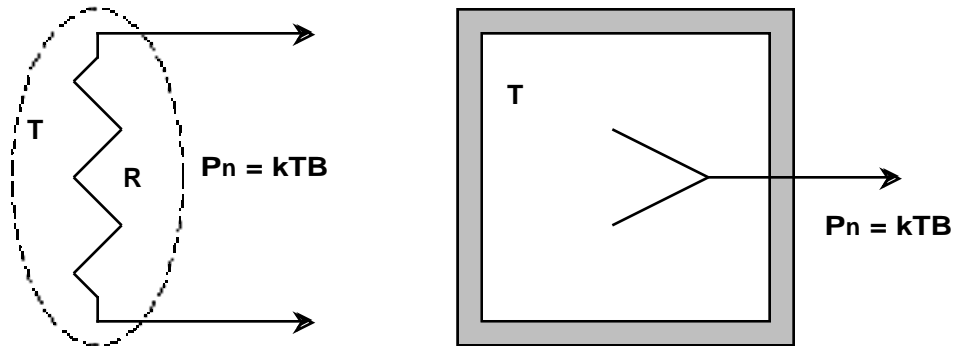


Antenna and Receiver Noise

Antenna Noise¹

Consider an ideally directive antenna contained within a shielded box at temperature T . The output of the antenna terminals is noise power that is identically the same as noise from a resistor at the same temperature T .

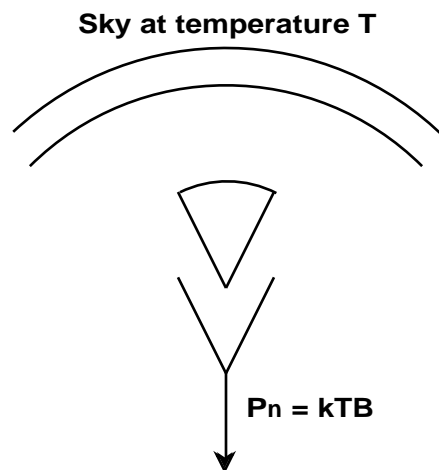


This noise power is

$$P_n = kTB \text{ Watts,}$$

where k is Boltzmann's constant $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$, T is the equivalent (or, in the case of non-luminous bodies, actual) temperature and B is the receiver bandwidth in Hertz.

Consider now a third situation, in which the antenna beam intercepts a uniform sky background at temperature T . In this case the antenna noise output will also be $P_n = kTB$ Watts.



¹ Kraus, *Antennas*, McGraw-Hill, 1988, pg. 774-790; see also *Reference Data for Radio Engineers*, 5th Ed., Ch. 34

A receiving antenna acts as a transducer for all incident radiation, including the desired signal but also many forms of noise and interference. For many systems, the principal issue is the noise other than the desired signal received from sources in the main lobe of the antenna.

All objects in view of the antenna radiate broadband "blackbody" noise corresponding to their surface temperature. In addition, there is a general background galactic noise level. Noise sources can be characterized by their equivalent temperature.

The net background noise temperature can be found in a practical case by integrating (averaging) the noise temperature per unit solid angle. If the beam of an antenna is narrower than the noise, it "sees" the background with noise temperature T_b . An example would be a directive satellite-borne antenna whose entire main lobe is aimed at the earth's surface, and whose gain is such that the earth subtends a larger angle than the beamwidth. In this case the background noise temperature would be $T_b = 290^\circ \text{K}$.

If however the beamwidth is such that the earth does not subtend the full beamwidth, then the effective noise energy would be the weighted average of the earth's thermal radiation and the galactic background, taken here to be the minimum of 3°K .

As an example, consider the case for which the earth subtends half the solid angle of the antenna main lobe. In this idealized case the average noise temperature will be simply

$T = 0.5T_p + 0.5T_g = 0.5(290) + 0.5(3) = 146.5^\circ\text{K}$ and the noise power per Hertz bandwidth is

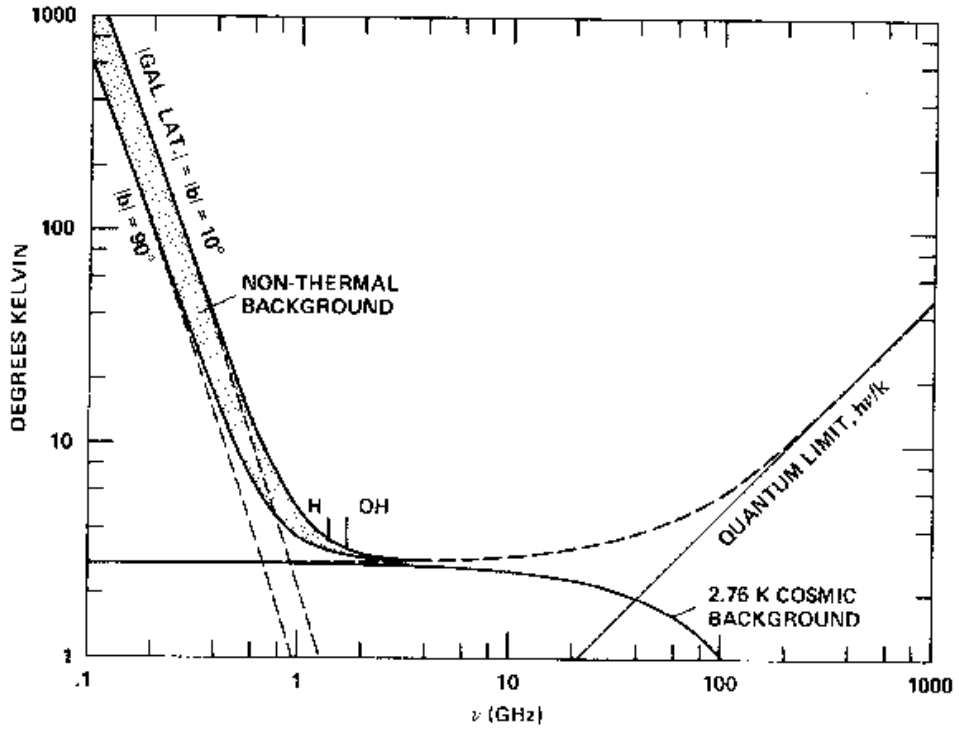
$$p_n = kT = 1.38 \times 10^{-23} (146.5) = 2.02 \times 10^{-21} \text{ or } -206.94 \text{ dBW per Hertz.}$$

This represents an example of a lower limit on the ability of the system to receive weak signals through this hypothetical antenna.

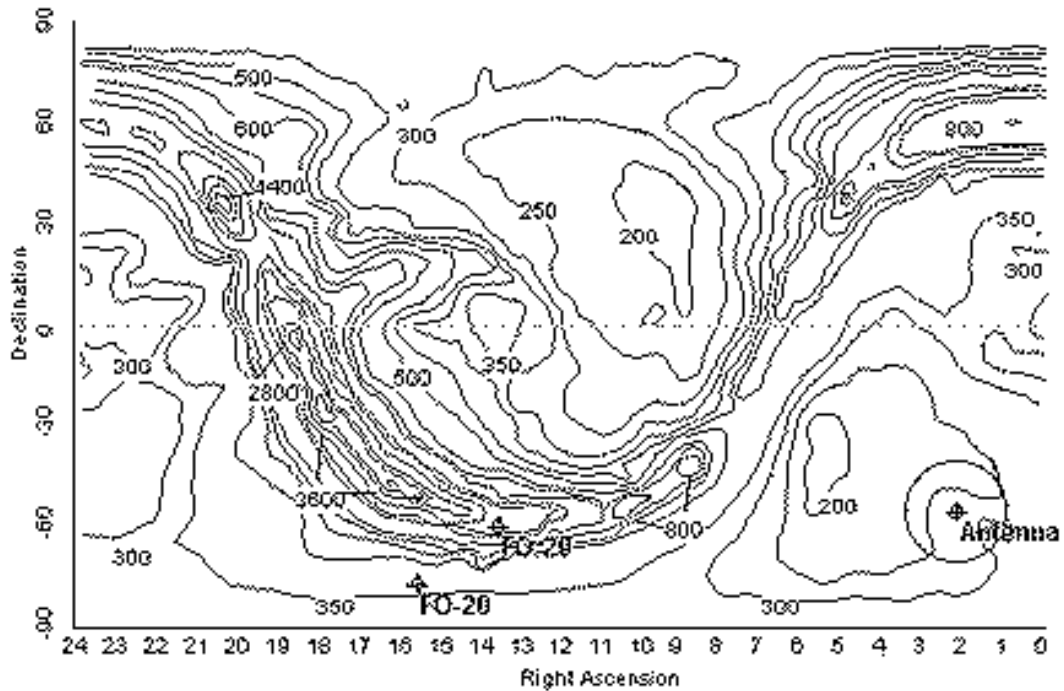
Sky Noise

A general picture of the noise received from deep space is shown in the figure here². In it you can see the free space window in the microwave range from 1-10 GHz in which the noise is a minimum.

² From *The Search for Extraterrestrial Intelligence*, NASA SP-419, 1977



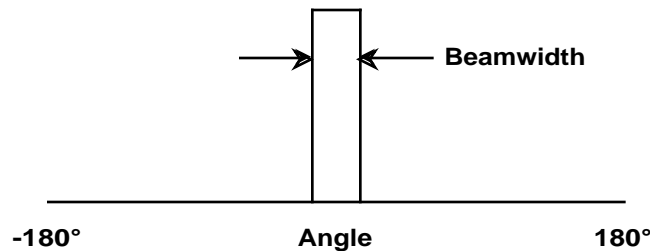
The noise background viewed from earth³, shown here for $f \approx 435$ MHz, varies with direction in the celestial sphere, with a maximum in the plane of our galaxy, the Milky Way



³ From <http://www.asahi-net.or.jp/~VQ3H-NKMR/satellite/noise.html>. FO-20 downlink $f = 436$ MHz.

Antenna Noise Temperature

Consider an antenna of arbitrary, but idealized, directional pattern as a function of angle shown here.



If the main lobe is directed toward a region of uniform background noise, the antenna noise temperature is

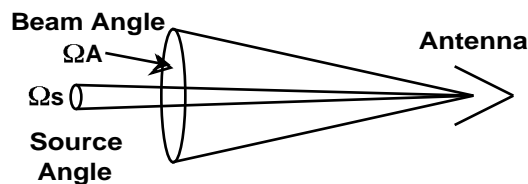
$T_a = T_b$, regardless of beamwidth.

If the background noise is non-uniform, the antenna noise temperature is the integral over the beamwidth of the background noise. For example, if the antenna is aimed at a radio noise source subtending a smaller solid angle Ω_S than that of the antenna beamwidth Ω_A , the antenna noise temperature will be the sum

$$T_a = \Omega_S T_S + (\Omega_A - \Omega_S) T_b \approx \Omega_S T_S + \Omega_A T_b$$

A method of measuring the source temperature T_S is to note the change in antenna temperature ΔT_a as the object transits the beam.

As an example (from Kraus⁴, which includes an excellent treatment of the entire subject of antenna noise), a measurement was made of Mars at $\lambda = 3.15$ cm with radio telescope. At the time of the measurement, Mars subtended an angle of 0.005° , and the antenna beamwidth (HPBW, defined as the width for half power) was 0.116° .



The solid area of the disk of Mars is $\Omega_S = \pi(0.005)^2/4 = 1.96 \times 10^{-5}$

Assuming that Ω_A is given by the solid angle within the HPBW, the solid area of the antenna beam is $\Omega_A = (0.116)^2 = 1.35 \times 10^{-2}$

The temperature of Mars at 3.15 cm was determined to be $T_M = 164^\circ\text{K}$.

The ratio of solid areas is $\Omega_A/\Omega_S = 1.35 \times 10^{-2}/1.96 \times 10^{-5} = 6.85 \times 10^2$, so when Mars transited the antenna beam the additional temperature above the background was

⁴ Kraus, *Antennas*, pg. 777

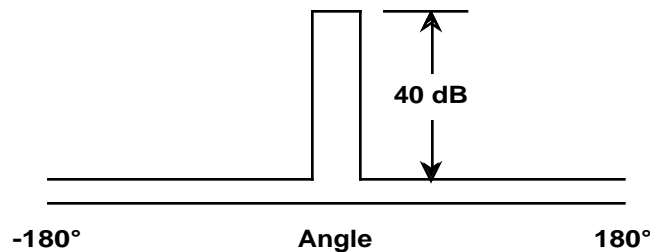
$$\Delta T = T(\Omega_S/\Omega_A) = 0.24^\circ\text{K}.$$

The example is summarized in the table here:

θ_S°	5.00E-03
θ_A°	1.16E-01
Ω_S square degrees	1.96E-05
Ω_A square degrees	1.35E-02
Ω_A/Ω_S	6.85E+02
$T^\circ\text{K}$	1.64E+02
$\Delta T^\circ\text{K}$	2.40E-01

This is an example of remote sensing radiometry.

Now consider an antenna with ideal main lobe, but also with idealized sidelobes.



In this case there will be a contribution to the antenna noise temperature from the (attenuated) background noise temperature in the backlobe direction, and also a minor contribution from the sidelobes directed at the sky noise. As an example, consider the contribution of black body radiation from the surface of the earth at $T_p = 290^\circ\text{K}$ occupying 180° of the sidelobe angles (1/2 the entire solid angle of the antenna radiation sphere).

Assume an antenna with 3° HPBW. This is a similar problem to the noise source problem above, only in this case the issue is that the power from the sidelobes is integrated over the solid angle corresponding to $\pm 90^\circ$. If the average sidelobe level is -40 dB as shown, the contribution from the backlobes will be

$$\Delta T = 290 (180/3)^2/10000 = 0.18 (290) = 52^\circ\text{K}$$

There will be an additional contribution from the 180° aimed at the sky (ignoring the small main lobe), calculated for $T_b = 20^\circ\text{K}$ to be

$$\Delta T = 20 (180/3)^2/10000 = 0.18 (20) = 3.6^\circ\text{K}$$

Since the main lobe contribution will be $T = 20^\circ\text{K}$, the total antenna noise temperature is

$$T_a = 20 + 52.2 + 3.6 = 75.8^\circ\text{K}$$

In the non-ideal case, the sidelobe levels will not be uniform, and the two noise contributions are evaluated by an integral over the solid angle of the entire antenna radiation sphere.

Noise Due to Attenuation

Attenuation in the atmosphere will reduce the received galactic noise level, but will replace it with noise from the hotter atmosphere. Other system losses such as antenna and feedline loss will act in the same manner.

Consider the case of a noise source of temperature T_b viewed through a lossy medium (sometimes called a "Grey Body"⁵) of loss τ_l and temperature T_p . In this case the effective temperature is

$$T_e = \tau_l T_b + (1 - \tau_l) T_p.$$

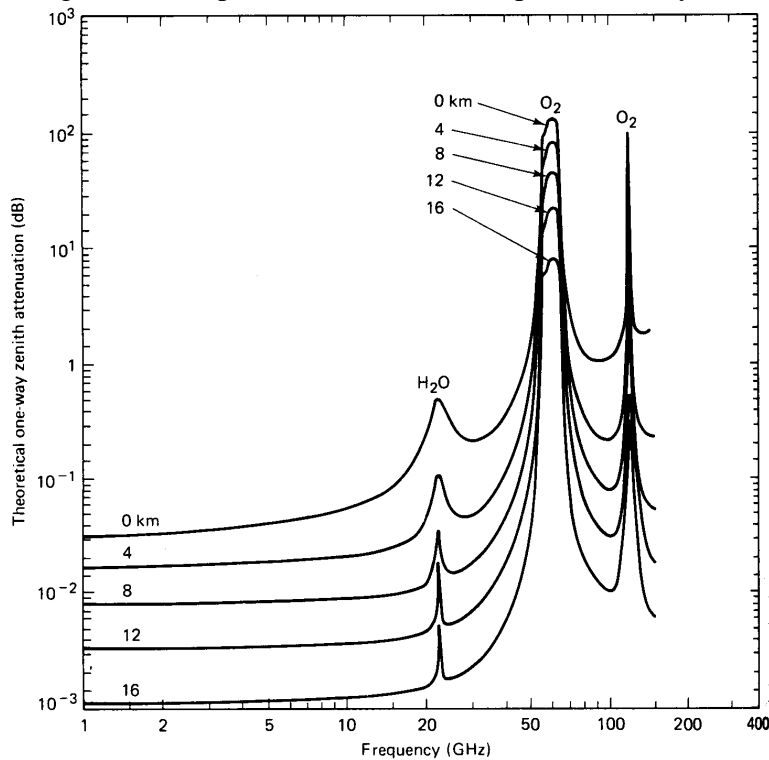
For the case of a vertical pass through the atmosphere, the loss at 12.7 GHz on a clear day with no rain attenuation is approximately 8×10^{-2} dB, or $\tau_l = 0.98$ and $(1 - \tau_l) = 0.02$. If the average temperature of the atmosphere is taken to be 150° K, then the noise contribution of the atmospheric loss is

$$(1 - \tau_l) T_p = 0.02(150) = 3^\circ \text{K}$$

If the background noise is $T_b = 15^\circ$ K, then the total effective noise at the earth's surface is

$$T_e = \tau_l T_b + (1 - \tau_l) T_p = 14.7 + 3 = 17.7^\circ \text{K}$$

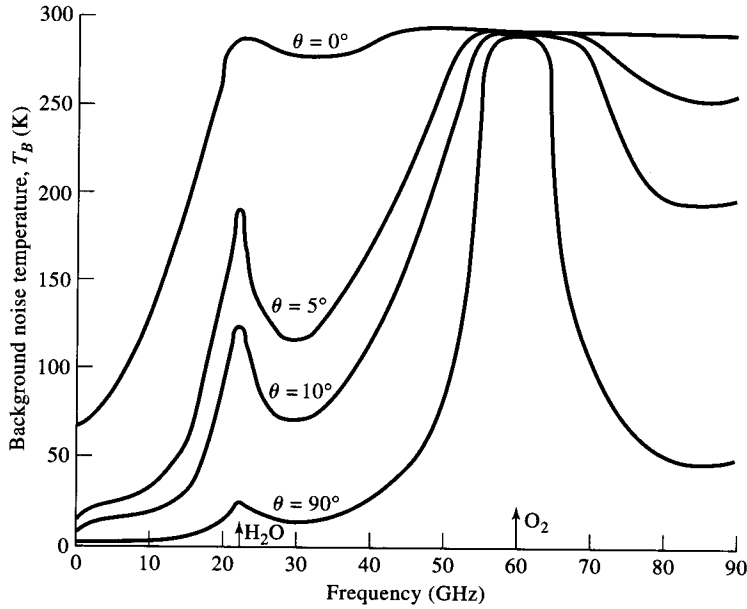
The effect of additional loss for oblique passage through the atmosphere results in higher noise contributions for such paths. The effect of atmospheric loss can be determined from the attenuation through the atmosphere, shown here (one pass vertically)⁶



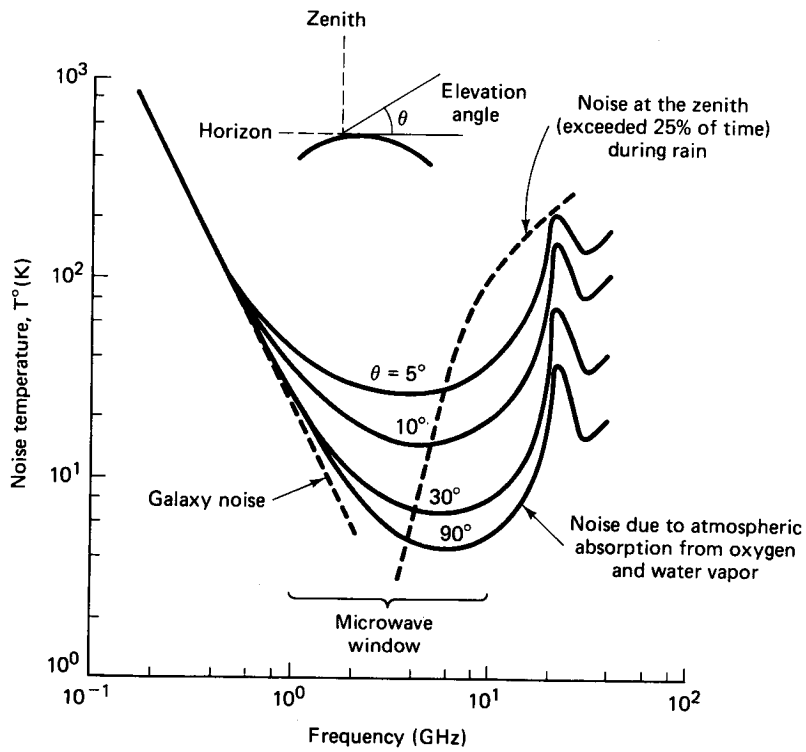
⁵ http://www.st-and.ac.uk/~www_pa/Scots_Guide/RadCom/part8/page3.html

⁶ Sklar, B., *Digital Communications*, Prentice Hall, 1988, pg. 194

Sky noise temperature vs. frequency⁷ shows that low angles and high frequencies are disadvantaged relative to frequencies below 10 GHz.



The microwave window⁸ is defined as frequencies lying above galactic noise and below absorption noise, as shown here.



⁷ Pozar, D., *Microwave and RF Design of Wireless Systems*, J. Wiley, 2001., pg. 127

⁸ Sklar, pg. 225

This same analysis can apply to ohmic losses in an antenna of physical temperature $T_p = 290^\circ\text{K}$, and also applies to the case of a lossy transmission line or other attenuator between the antenna and the receiver.

To summarize, consider the case of a radio astronomy or satellite communications antenna located on the earth's surface but with the main beam directed upward. No matter what the beamwidth (within reason) we would expect this antenna to have a background noise level set by the distributed galactic background noise level. At $f = 10\text{ GHz}$, this can range from 3°K to 100°K , depending on the region of the sky that is in the antenna beam.

If the antenna is pointed at a discrete noise source, such as a planetary surface, the noise temperature will be elevated above the background by an amount corresponding to the weighted average of the discrete source and the background.

An antenna may be directed toward the "cold" sky, but may have sidelobes that receive noise from the "hot" surface of the earth. This will result in an additional contribution to the noise from the antenna equal to the product of the sidelobe gain (a number substantially less than unity) and the earth temperature, 290°K . If the main lobe of the antenna intersects the ground, the noise temperature is dramatically raised.

Additionally, attenuation in the atmosphere will reduce the received galactic noise level, but will replace it with noise from the hotter atmosphere. Other system losses such as antenna and feedline loss will act in the same manner, as will noise generated in the first stages of the receiver itself.

Consider the worked example of the following parameters:

$T_s = 10^\circ\text{K}$, the noise temperature of the cold sky background in the main lobe direction
 $\eta_{\text{atm}} = 0.98$ (-0.1 dB), the transmissibility of the atmosphere at the antenna frequency
 $T_{\text{atm}} = 150^\circ\text{K}$, the average temperature of the atmosphere
 $\eta_{\text{ant}} = 0.98$, the ohmic efficiency of the antenna
 $T_e = 290^\circ\text{K}$, the temperature of the antenna, earth, feedlines and system components
 $G_s = 0.04$ (-14 dB), the integrated side lobe level relative to the main lobe
 $G_f = 0.94$ (-0.3 dB) The feedline and filter transmission gain (loss)

We can generate a table that accounts for all the sources of noise in the antenna system:

Space noise	$T_s \eta_{\text{atm}}$	$10^\circ\text{K} \times 0.98$	9.8°K
Atmospheric noise	$(1 - \eta_{\text{atm}}) T_{\text{atm}}$	$150^\circ\text{K} \times 0.02$	3.4°K
Antenna loss noise	$(1 - \eta_{\text{ant}}) T_e$	$290^\circ\text{K} \times 0.02$	6.8°K
Sidelobe noise	$G_s T_e$	$290^\circ\text{K} \times 0.04$	<u>11.6°K</u>
Total antenna noise	T_a		31.5°K
Feedline & filter loss	$G_f \times T_a + (1 - G_f) T_e$	$0.94 \times 31.5 + 0.06 \times 290$	48.8°K
Preamplifier and system noise	T_s		<u>35.0°K</u>
Total noise	T_{tot}		83.8°K
Total noise power density	kT_{tot}	$2.38 \times 10^{-23} \times 83.8$	$1.16 \times 10^{-21}\text{W/Hz}$
Total noise power density	$10 \log_{10} kT_{\text{tot}}$		-209.4 dBW/Hz or -179.4 dBm/Hz

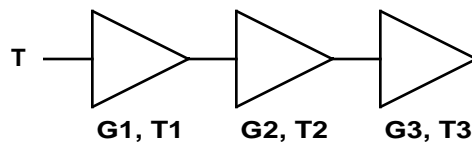
For this example, the system noise level is -179.4 dBm/Hz , so the ultimate sensitivity for non-noise-like signals depends on the choice of system filter bandwidth B , which will ideally be matched to the bandwidth of the desired signal.

Receiver Noise Temperature

An ideal receiver would add no noise to the signal and noise at the antenna. However, in the practical case a receiver does add additional noise power, which can be characterized in terms of a noise temperature T_r . T_r is defined as the temperature of a source resistor that would provide, from a noiseless receiver, output power equal to that generated in the stages of the receiver. Again, the noise power referred to the receiver input is

$$P_n = kT_r B$$

If we know the noise temperature of each of several cascaded amplifiers, we can determine the equivalent noise temperature of the combination.



Consider a series of amplifiers of gain G_i and noise temperature T_i . The noise temperature of the combination is given by

$$T = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2}, \text{ where the gain } G_i \text{ is the numerical gain, not dB gain.}$$

If one of the gains is fractional (a loss condition), the relationship holds. We can make use of the concept that loss L

$$L = \frac{1}{G}, \text{ so if for example the second stage is a mixer with a } G_2 < 1, \text{ we can use the form}$$

$$T = T_1 + \frac{T_2}{G_1} + \frac{L_2 T_3}{G_1}$$

As an example, consider the case of a three-stage amplifier, each stage with 8 dB gain and 60°K noise temperature.

The numerical gain of each stage is $G = 10^{8/10} = 6$

The noise temperature of the combined stages is

$$T = 60 + \frac{60}{6} + \frac{60}{6(6)} = 60 + 10 + 1.5 = 71.5^\circ\text{K}$$

If there are a large number of cascaded identical stages, the noise temperature approaches the quantity called *noise measure*.

The quantity *noise figure* F is the ratio of the input signal-to-noise to the output signal-to-noise (which is less). The noise figure F of an receiver or individual stage is also defined as the ratio of the total available noise power at the output of the amplifier to the available noise

power at the output that would result only from the thermal noise in the source resistance. Thus F is a measure of the excess noise added by the amplifier, and this quantity is equal to

$$F = 1 + \frac{T}{T_0}, \text{ where } T_0 \text{ is specifically defined as room temperature, } 290^\circ\text{K}$$

Noise figure is often expressed in dB form.

The noise figure of cascaded amplifiers is given by the numerical (not dB) relationship

$$F = F_1 + \frac{F_2 - 1}{G_{A1}},$$

where F_1 and G_{A1} are the noise figure and available gain of the first stage, and F_2 is the noise figure of the second stage. This applies to lossy stages and networks as well.

A loss L dB ahead of the first amplifying stage of a receiver results in exactly the same increase in receiver noise figure. As an example, if a cable at room temperature having 1 dB attenuation is placed before a receiver with a noise figure of 2 dB, the resulting overall noise figure will be 3 dB.

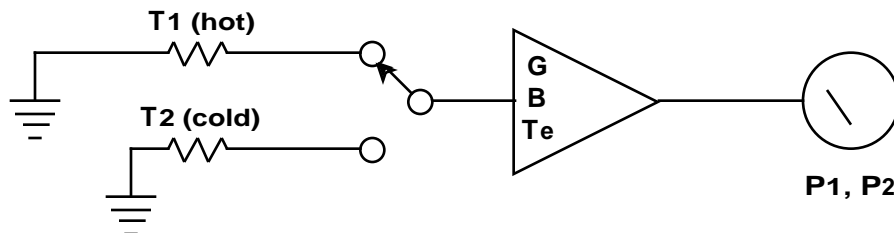
Equivalent noise temperature in terms of noise figure F and T_0 is given by

$$T = (F - 1)T_0$$

We have seen in earlier notes that noise temperature and noise figure are related quantities, and that noise temperature sources can include loss ahead of the first low-noise amplifier as well as noise from subsequent stages. This is particularly an issue with mixers, which typically have loss of the order of 6 dB, rather than gain. This places a requirement for sufficient preamplifier gain to preclude an excessive mixer contribution to system noise temperature.

Measuring Noise Temperature

The measurement of system noise temperature can, in principle, be accomplished by knowing the system gain and observing the output noise level. However, a more satisfactory measurement is accomplished by terminating the receiver input with two noise sources of differing noise temperatures⁹. These can be physical resistances (or even a single resistance, a "hot-cold source", with provision to change its temperature by means of a heater/cooler arrangement that can use liquid nitrogen, $T=77^\circ\text{K}$, or liquid helium, $T=4^\circ\text{K}$) or active noise sources such as ionized gas tubes or diodes that can be switched on and off.



⁹ Pozar, pg. 554

With the input switched to the hot source, the output power is

$P_1 = GkT_1B + GkT_eB = Gk(T_1 + T_e)B$, while in the other condition the output power is

$P_2 = GkT_2B + GkT_eB = Gk(T_2 + T_e)B$

Eliminating GB (assuming it is the same for each condition) and defining the measured power ratio, the Y-factor as

$$Y = \frac{P_1}{P_2} = \frac{T_1 + T_e}{T_2 + T_e} > 1 \text{ we have}$$

$$T_e = \frac{T_1 - YT_2}{Y - 1}$$

This same measurement can be made using a directive antenna. If pointed at cold sky, then at hot earth, the same hot and cold temperatures can be used to calculate receiver noise temperature T_r .

Suppose we let $T_p = 290^\circ\text{K}$ (hot earth) and $T_b = 20^\circ\text{K}$ (cold space). In this case we can calculate the Y factor from

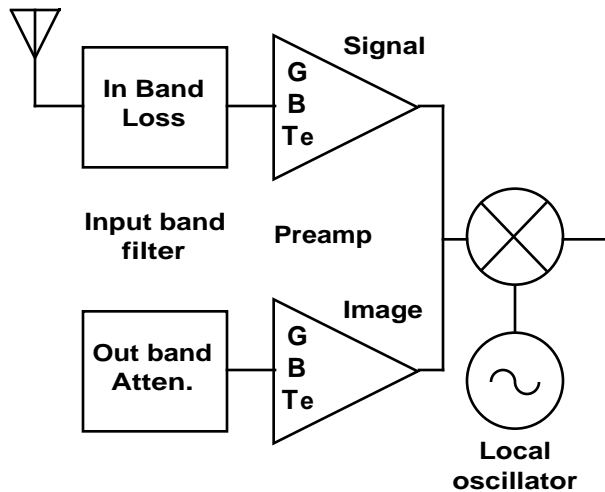
$$Y = \frac{P_1}{P_2} = \frac{T_p + T_r}{T_b + T_r}, \text{ so we have receiver noise temperature}$$

$$T_r = \frac{T_p - YT_b}{Y - 1}$$

It should be noted that the sky background temperature is often specified such that the temperature defined as received by a polarized antenna is half the total power radiated from a black body of brightness $2T$. The receiver noise is not polarized. This factor of 2 can cause endless problems.

Image Noise

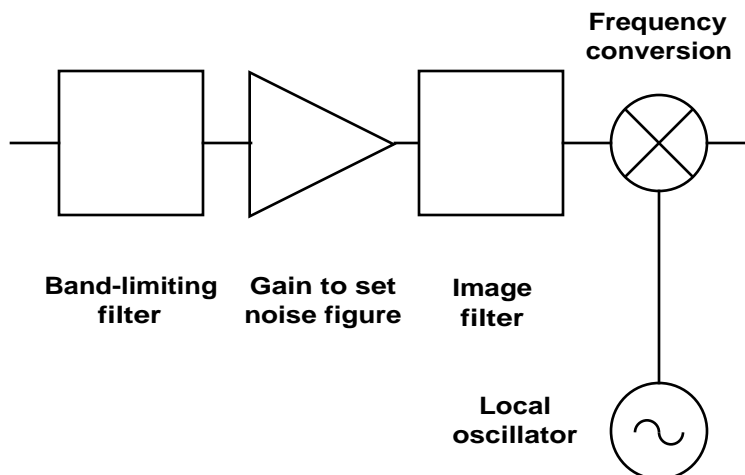
A superheterodyne receiver may have one or more image responses. If we consider a noise measurement in which the noise source is broadband, there will be noise power input at both the signal and image frequency. This can give rise to an error in noise figure measurement.



An equivalent circuit shows both the signal frequency and image frequency contribution to noise. The gain and noise temperature of the preamplifier will typically be different for the two frequencies, with the higher frequency generally exhibiting lower gain and higher noise temperature. In the more general case, there is possible noise contribution at higher order mixer harmonics spurious frequencies.

With a pre-filter, noise from the noise source does not reach the output, so the determination of noise temperature is in error. Without a prefilter, noise measurement is more nearly correct, but can be affected by differences in gain and noise temperature at the two frequencies.

In fact, we can see that the image noise from the preamplifier will degrade the ability of the system to detect small signals at the signal frequency. To preclude this, we need a postfilter to remove the noise from the preamp at the image frequency¹⁰.



Note also that oscillator noise is impressed on signals, which creates a dynamic range issue to be treated elsewhere.

¹⁰ Hayward, L., *Introduction to Radio Frequency Design*, ARRL, 1994, pg. 342

System Noise Temperature

Consider now a composite system consisting of an antenna having noise temperature of T_a (giving account to feedline and other losses) and a receiver having noise temperature T_r . The system noise temperature is simply

$$T_s = T_a + T_r$$

From this we can calculate the equivalent system input noise power and can determine the minimum system input signal that can be detected.

Antenna G/T Defined

For satellite systems, the simple Friis geometric model is sufficient to analyze the propagation path.

$$P_R = \frac{P_T G_T G_R \lambda^2}{(4\pi r)^2} = \frac{\text{EIRP}(G_R \lambda^2)}{(4\pi r)^2}, \text{ where } \text{EIRP} = P_T G_T.$$

Recall that the noise power referred to receiver input is $N = kT_{\text{sys}}B$, we calculate

$$\frac{S}{N} = \frac{P_R}{kT_{\text{sys}}B} = \left(\frac{G_R}{T_{\text{sys}}} \right) \frac{\text{EIRP}(\lambda^2)}{kB(4\pi r)^2}$$

For a given signal bandwidth B , the only parameters controlled by the designer at the receiver end are G_R and T_{sys} , so the factor G/T is used as a figure of merit for receiving systems.