EE – 3410 Electric Power
Fall 2003
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TRANSFORMERS

1. MAGNETIC CIRCUIT EXCITED BY ALTERNATING CURRENT

According to the Faraday's experiment the voltage *e* induced in one turn linking a changing magnetic field (see Fig.1) is proportional to the time rate of change of flux Φ:

$$e = \frac{d\Phi}{dt} \tag{1}$$

The polarity of the induced voltage can be determined by the Lenz's law that says:

"The induced voltage is always in such a direction as to tend to oppose the change in flux linkage that produces it"

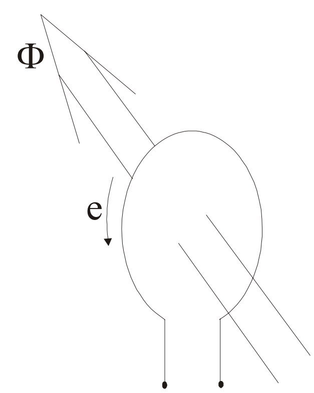


Fig.1 Explanation to equation (1)

This is shown in Fig.1. For multitern coil the induced voltage is:

$$e = N\frac{d\Phi}{dt} = \frac{d\lambda}{dt} \tag{2}$$

where:

N – is the number of coil turns and

λ – is the flux linkage in weber turns.

Suppose we have a coil wound on one leg of a close iron core as shown in Fig.2. To draw an equivalent circuit of such a device called inductor, and then to analyze its behavior under variable supply condition let we consider first an ideal inductor.

1.1. An ideal inductor

An ideal inductor is defined by the following assumptions:

- The coil of inductance L has the resistance R equal to zero,
- There is an ideal magnetic circuit of the iron core with no power losses in it,
- There is no leakage flux, what means that the whole magnetic flux is within the iron core.

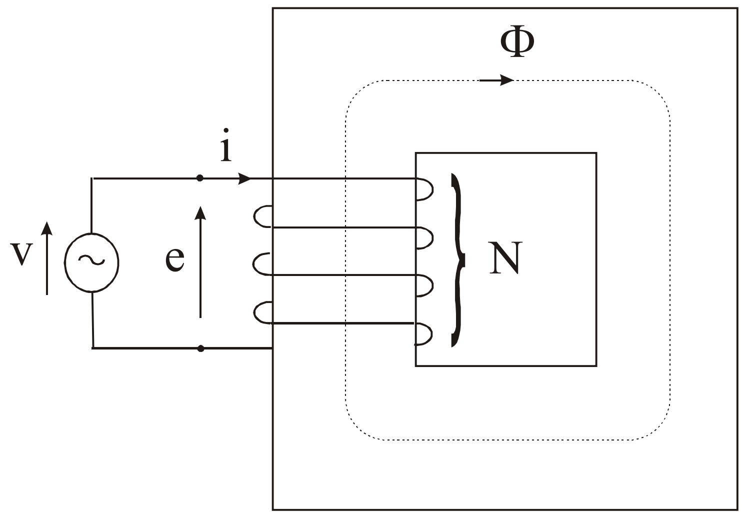


Fig.2 Scheme of the inductor supplied from the ac. source

Assuming a linear relation between current and flux, the sinusoidal current

$$i = I_m \sin(\omega t) \tag{3}$$

produces the sinusoidal flux

$$\Phi = \Phi_m \sin(\omega t) \tag{4}$$

The voltage induced in N-turn coil is

$$e = N\frac{d\Phi}{dt} = N \cdot \omega \cdot \Phi_m \cdot \cos(\omega t) = E_m \cos(\omega t) \tag{5}$$

The effective value of this voltage is:

$$E = \frac{E_m}{\sqrt{2}} = \frac{N\omega\Phi_m}{\sqrt{2}} = \frac{2\pi}{\sqrt{2}} N f \Phi_m = 4.44 N f \Phi_m \tag{6}$$

The voltage expressed in terms of current flowing trough the coil is:

$$e = L\frac{di}{dt} \tag{7}$$

For sinusoidal current:

$$e = L\omega I_m \cos\omega t = E_m \cos\omega t \tag{8}$$

The effective value of the voltage expressed in the complex form is:

$$\underline{E} = jX_\mu \underline{I} \tag{9}$$

where $X_\mu = L\omega$ is the magnetizing reactance.

For an ideal inductor, the induced voltage (*emf* – *E*) is equal to the voltage supply $\underline{E} = \underline{V}$.

The equivalent circuit of such an inductor is shown in Fig.3. The phasor diagram of voltages and current is shown in Fig.4.

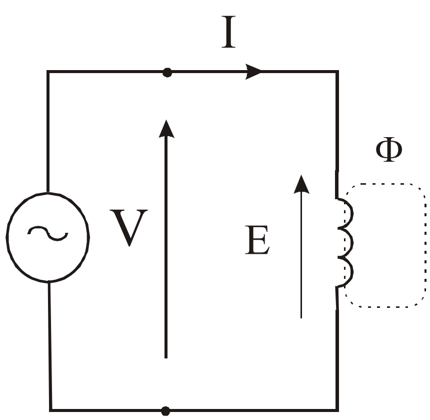


Fig.3 Inductor equivalent circuit

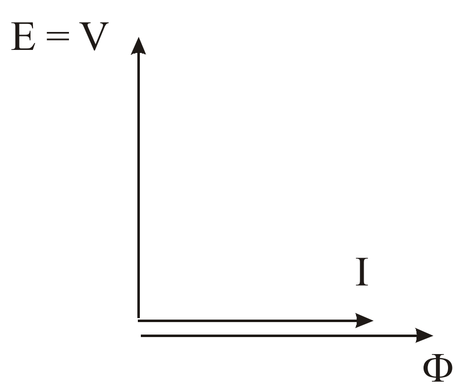


Fig.4 Phasor diagram corresponding with Fig.3

1.2 A real inductor

A real inductor has a real coil and the real magnetic circuit. This magnetic circuit is described by the hysteresis loop of B-H characteristic shown in Fig.5. During the process of magnetization by the alternating flux the energy is lost due to the hysteresis loop. This energy loss, called the hysteresis loss is proportional to the area closed by the hysteresis loop. That means it depends on the material the inductor core is made of. The empirical formula for this loss is:

$$\Delta P_h = K_h f \cdot B_m^n \tag{10}$$

where the constant K_h and n vary with the core material. n is often assumed to be 1.6 – 2. Since, according to equation (6) B is proportional to E we can write for $n=2$

$$\Delta P_h = K_h' \frac{E^2}{f} \tag{11}$$

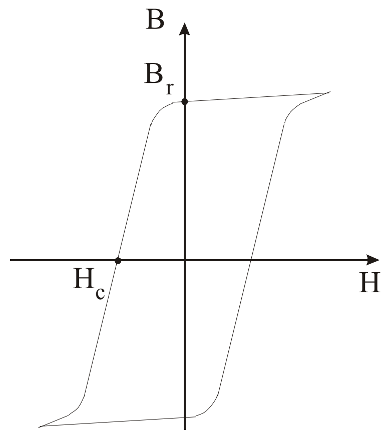


Fig.5 Hysterisis loop of B-H characteristic

There is another source of power loss in the magnetic core too. These are eddy currents induced in the core. To illustrate this phenomenon let us consider the solid core shown in Fig.6.a. If the magnetic flux existing in the core is directed towards the paper and is increasing, it induces the voltages in the core, which, in the case of close electric loops, cause the eddy currents that generate the power losses i^2R as a heat. The power losses can be reduced by decreasing i (increasing R). If, instead of a solid iron core, thin laminations are used (Fig.6.b), the effective induced currents are decreased by the increase of the resistance of the effective paths. The eddy current losses are significantly reduced in that case.

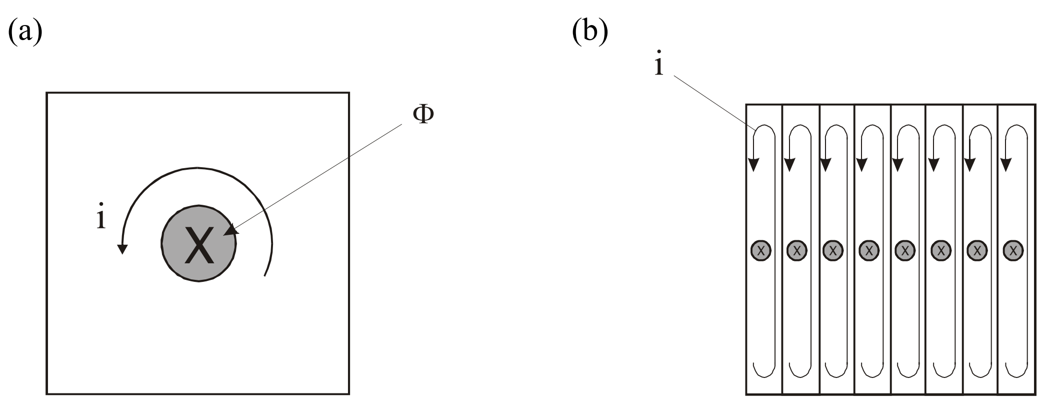


Fig.6. Eddy currents in: (a) solid iron core, (b) laminated iron core

For the given core, the eddy currents power losses are given by

$$\Delta P_e = K_e f^2 B_m^2 \tag{12}$$

Since the voltage induced in the coil is proportional to $f \cdot B_m$, the power losses are equal to:

$$\Delta P_e = K_e' E^2 \tag{13}$$

The constant K_e' depends on the conductivity of the core material and the square of the thickness of the laminations.

Combining the eddy currents and hysteresis power losses the total core power losses are:

$$\Delta P_{Fe} = \Delta P_h + \Delta P_e = K_{Fe} f^{1.3} B^2 \tag{14}$$

For the purpose of the equivalent circuit we intend to build, an equivalent resistance R_{Fe} is introduced. Then the core power losses at constant supply frequency can be expressed as follows:

$$\Delta P_{Fe} = \frac{E^2}{R_{Fe}} \tag{15}$$

The power losses ΔP_{Fe} are proportional to the square of voltage E, which appears across the resistance R_{Fe}. This allows to show the inductor equivalent circuit in form as in Fig.7a.

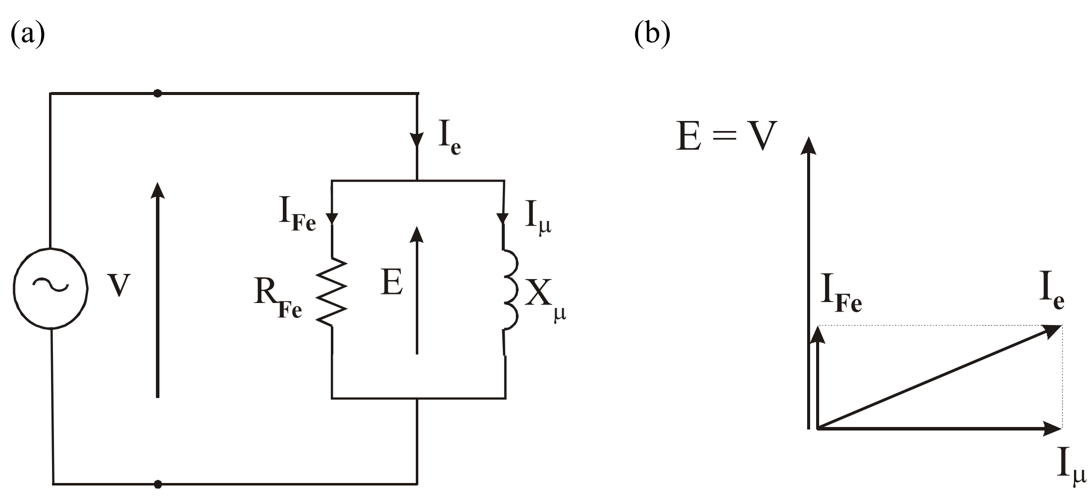


Fig.7 An equivalent circuit (a) and the corresponding phasor diagram (b) of the inductor with core losses

The excitation current $\boldsymbol{I_e}$ is split into two components: the magnetizing current $\boldsymbol{I_\mu}$ and $\boldsymbol{I_{Fe}}$, proportional to the core power losses. Fig.7.b, with the phasor diagram shows the relationship between the voltage E and currents $\boldsymbol{I_\mu}$ and $\boldsymbol{I_{Fe}}$. These currents are displaced from each other by an angle $\pi/2$. This displacement can be explained by means of excitation current waveform shown in Fig.8. If the coil is supplied with sinusoidal voltage the flux Φ must be sinusoidal too according to equation 1. Since the magnetizing characteristic B-H is nonlinear, and has a hysteresis loop, the current waveform obtained from magnetizing curve is far from sinusoidal. If we extract two current components from the current i_e by finding the symmetrical currents with regard to the l line we obtain $\boldsymbol{i_h}$ current being in phase with the voltage E and magnetizing current $\boldsymbol{I_\mu}$ lagging the voltage E by the angle $\pi/2$ (see Fig.7).

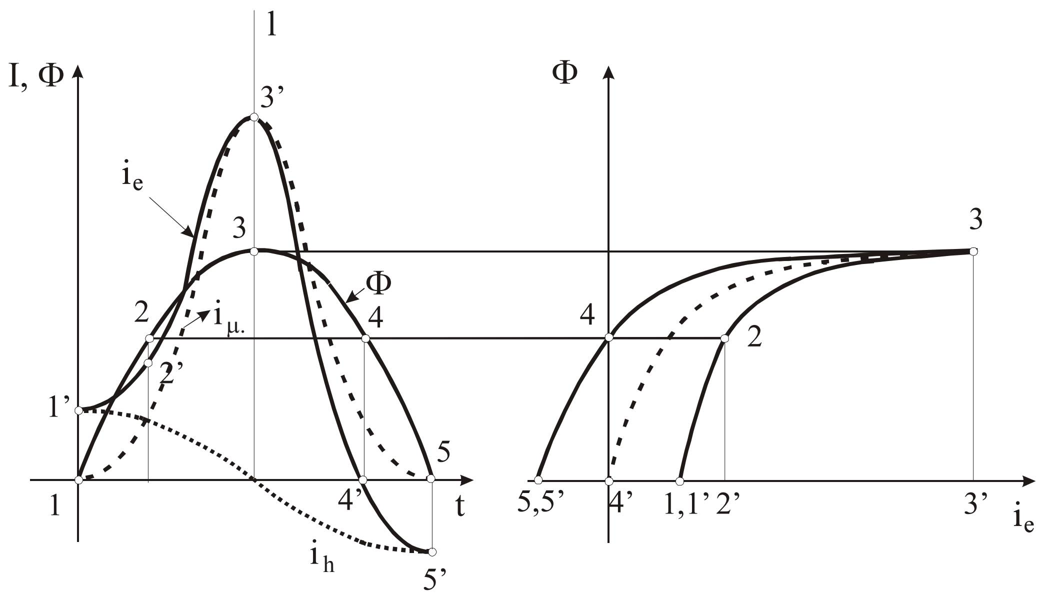


Fig.8 Extraction of hysteresis current component $\boldsymbol{i_h}$ from the excitation current i_e

So far we did not take into account the resistance of the coil R and the leakage flux Φ_s. This is the flux, which goes via air as shown in Fig.9. It induces the voltage $\boldsymbol{E_s}$ in the coil, which is equal

$$e_s = N\frac{d\Phi_s}{dt} \tag{16}$$

If we express the flux Φ_s in terms of the current:

$$\Phi_s = L_s i, \tag{17}$$

then the voltage:

$$e_s = L_s\frac{di}{dt} = L_s \omega I_m \cos \omega t \tag{18}$$

or written in complex notation:

$$\underline{E}_s = j X_s \underline{I} \tag{19}$$

where X_s is the leakage reactance.

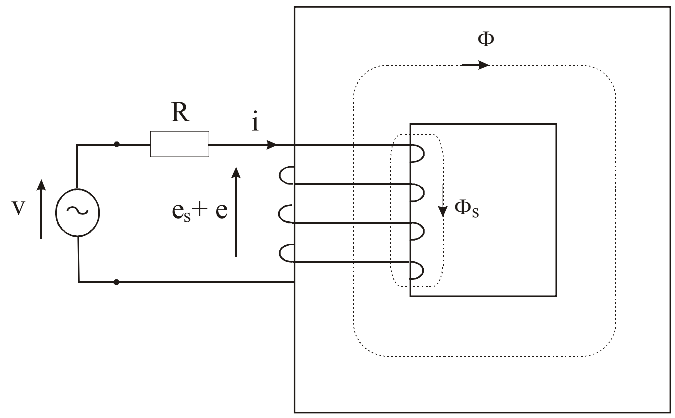


Fig.9 Diagram of the real inductor with the coil resistance R and the leakage flux Φ_s

The equivalent circuit of the real inductor is shown in Fig.10.a. The circuit shows magnetic fluxes associated with their inductances L_s and L_μ. The voltage equation of the circuit is

$$\underline{V} = \underline{E} + (R + jX_s)\underline{I} \tag{20}$$

An adequate phasor diagram is shown in Fig.10.b.

(a) (b)

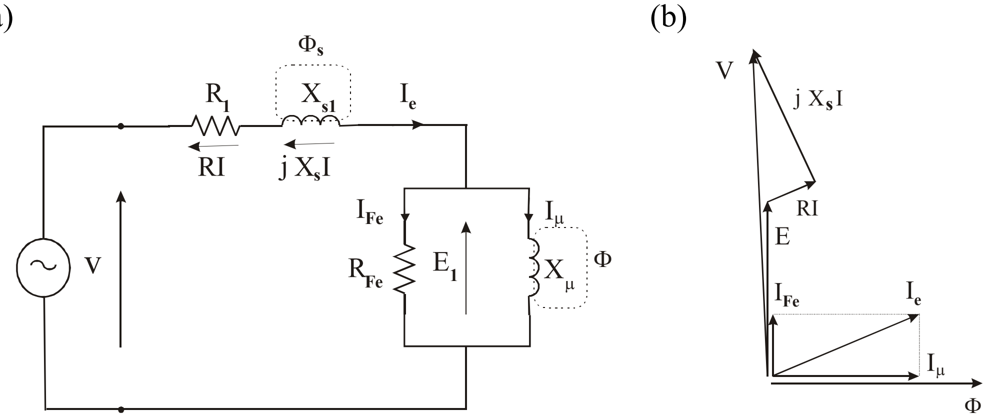


Fig.10 Equivalent circuit (a) and phasor diagram (b) of the real inductor

2. SINGLE-PHASE TRANSFORMER

2.1. Transformer operation and construction

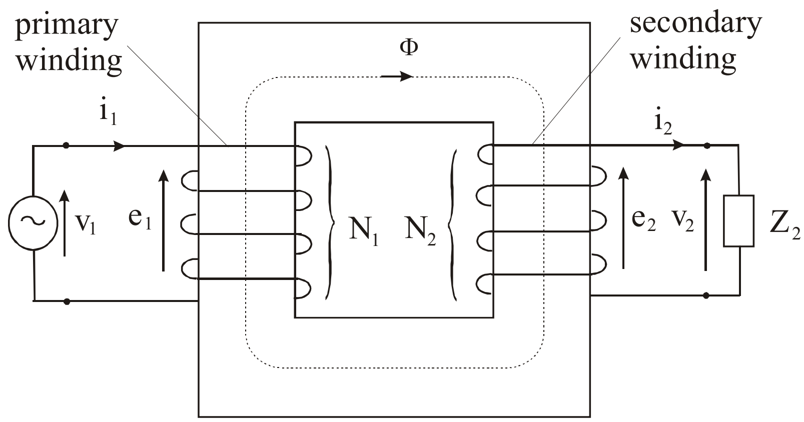


Fig.11 Two-winding single-phase transformer

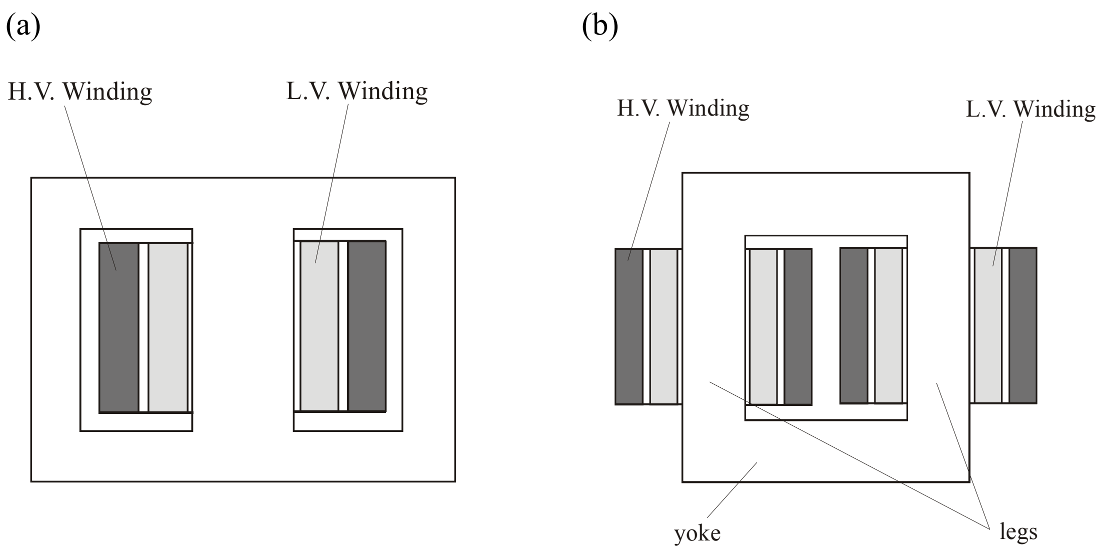


Fig.12.(a) Shell-type transformer, (b) core-type transformer

Two cooling systems:

- air-cooled: small transformers
- oil-cooled: large transformers

Very large transformers are immersed in the tanks with radiators and forced circulation.

(a) (b)

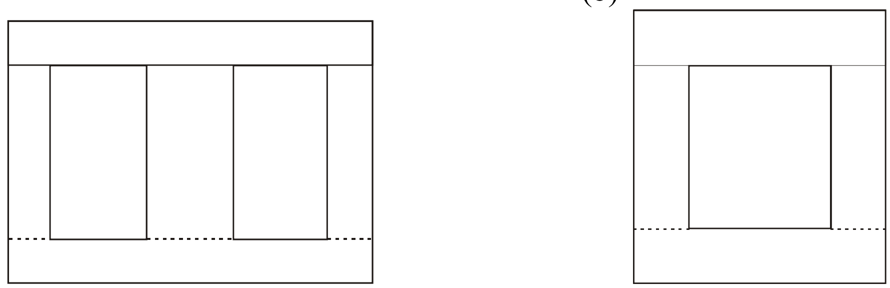


Fig.13. Construction of transformer cores from stampings: (a) shell-type and (b) core-type transformer

2.2. Ideal transformer

Assumptions for an ideal transformer:

- $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are equal to 0
- Φ_{s1} and Φ_{s2} are equal to 0 ($\mu = \infty$ and $\boldsymbol{L} = \infty$)

Assuming a sinusoidal time variation of flux:

$$\Phi = \Phi_{\boldsymbol{m}}\, sin(\omega \boldsymbol{t}) \tag{21}$$

the induced emfs:

$$\boldsymbol{e}_1 = \boldsymbol{N}_1 \frac{\boldsymbol{d\Phi}}{\boldsymbol{dt}} = \boldsymbol{N}_1 \cdot \omega \cdot \Phi_{\boldsymbol{m}} \cdot cos(\omega \boldsymbol{t}) = \sqrt{2}\boldsymbol{E}_1\, cos(\omega \boldsymbol{t}) \tag{22}$$

$$\boldsymbol{e}_2 = \boldsymbol{N}_2 \frac{\boldsymbol{d\Phi}}{\boldsymbol{dt}} = \boldsymbol{N}_2 \cdot \omega \cdot \Phi_{\boldsymbol{m}} \cdot cos(\omega \boldsymbol{t}) = \sqrt{2}\boldsymbol{E}_2\, cos(\omega \boldsymbol{t}) \tag{23}$$

The rms voltages in complex notation:

$$\underline{\boldsymbol{E}}_1 = \boldsymbol{j}4.44\boldsymbol{f}\boldsymbol{N}_1\Phi_{\boldsymbol{m}} \tag{25}$$

$$\underline{\boldsymbol{E}}_2 = \boldsymbol{j}4.44\boldsymbol{f}\boldsymbol{N}_2\Phi_{\boldsymbol{m}} \tag{26}$$

The ratio of induced voltages:

$$\frac{\boldsymbol{e}_1}{\boldsymbol{e}_2} = \frac{\boldsymbol{E}_1}{\boldsymbol{E}_2} = \frac{\boldsymbol{N}_1}{\boldsymbol{N}_2} = \boldsymbol{a} \tag{27}$$

For an ideal transformer:

$$\boldsymbol{V}_1 = \boldsymbol{E}_1 \quad \text{and} \quad \boldsymbol{V}_2 = \boldsymbol{E}_2 \tag{28}$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \tag{29}$$

There are no power losses and

$$S_1 = S_2 \tag{30}$$

$$V_1 I_1 = V_2 I_2 \tag{31}$$

From the above equation:

$$\frac{V_1}{V_2} = \frac{I_1}{I_2} = a \tag{32}$$

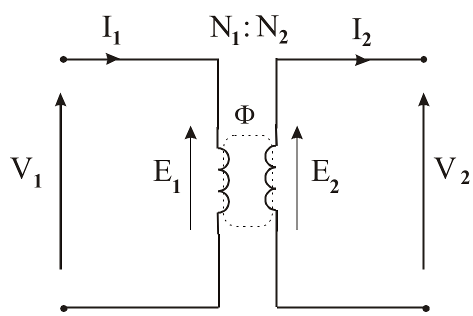


Fig.14 The equivalent circuit of ideal transformer with the magnetic link

To eliminate the magnetic connection let us express the secondary voltage and current as:

$$V_1 = aV_2 = V_2' \tag{33}$$

$$I_1 = \frac{1}{a} I_2 = I_2' \tag{34}$$

V_2' and I_2' are secondary voltage and current referred to a primary side.

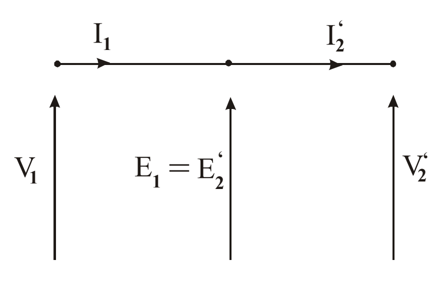


Fig.15 The equivalent circuit of an ideal transformer with electric connection of two sides

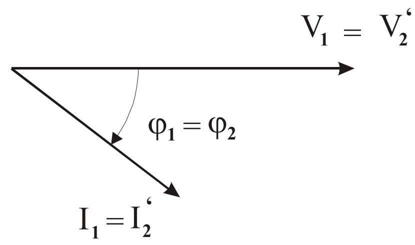


Fig.16 Phasor diagram for an ideal transformer

Transformer can be used for the impedance matching.

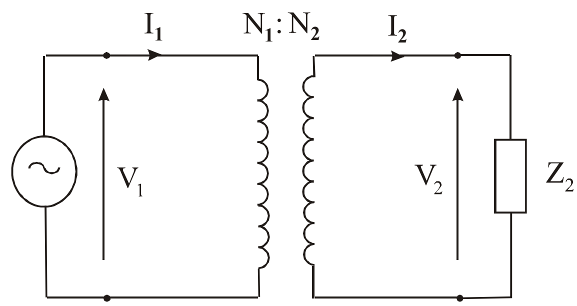


Fig.17 Ideal transformer with secondary load impedance

$$\frac{\boldsymbol{V}_1}{\boldsymbol{I}_1} = \frac{\boldsymbol{a}\boldsymbol{V}_2}{\frac{\boldsymbol{I}_2}{\boldsymbol{a}}} = \boldsymbol{a}^2 \frac{\boldsymbol{V}_2}{\boldsymbol{I}_2} = \boldsymbol{a}^2 \boldsymbol{Z}_2 = \boldsymbol{Z}_2' \tag{35}$$

$\boldsymbol{Z}_2'$ is the output impedance seen at the primary side or the transformer input impedance as shown in Fig.18.

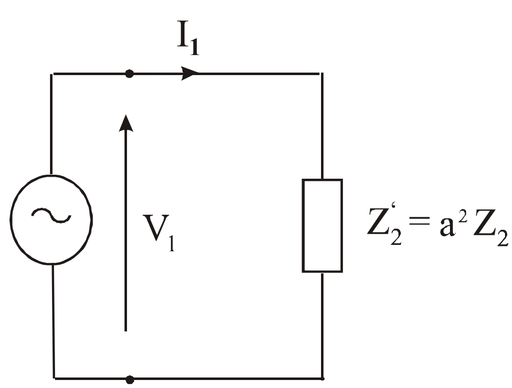


Fig.18. Input impedance for ideal transformer with secondary load impedance $\boldsymbol{Z}_2$

2.3 A real transformer

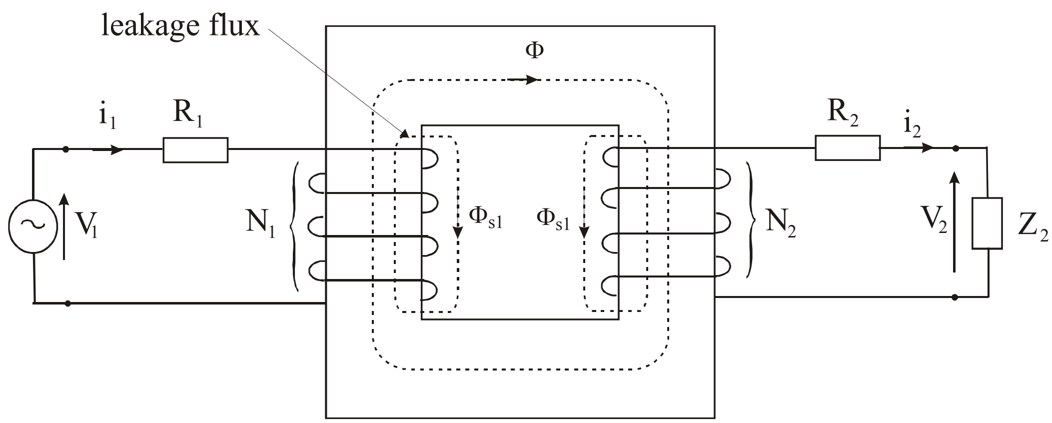


Fig.19 Circuit diagram of a real two winding transformer

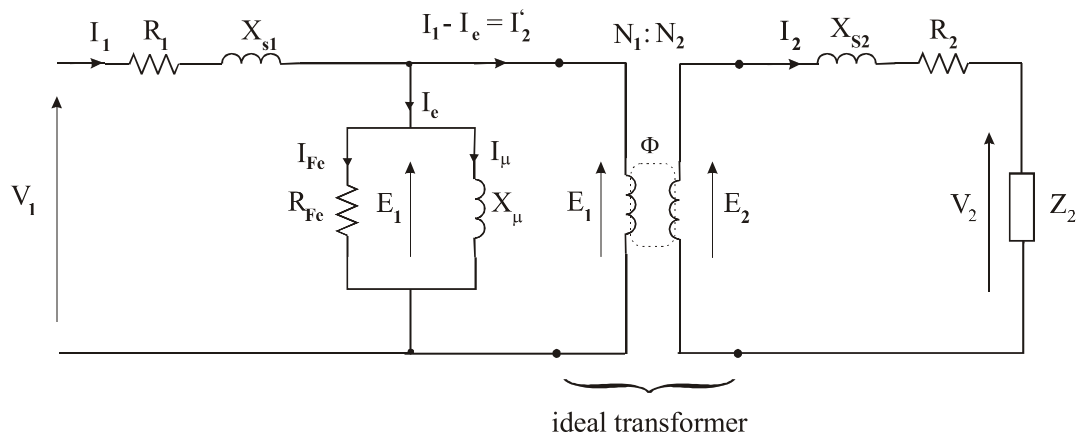


Fig.20 Equivalent circuit of the real transformer with magnetic coupling

Equation of magnetomotive forces in complex notation:

$$\underline{\boldsymbol{F}}_{\boldsymbol{me}} = \underline{\boldsymbol{F}}_{\boldsymbol{m}1} - \underline{\boldsymbol{F}}_{\boldsymbol{m}2} \tag{36}$$

where $\underline{\boldsymbol{F}}_{\boldsymbol{me}}$ is the mmf responsible for generation of flux Φ. The above equation written in other form:

$$\boldsymbol{N}_1 \underline{\boldsymbol{I}}_{\boldsymbol{e}} = \boldsymbol{N}_1 \underline{\boldsymbol{I}}_1 - \boldsymbol{N}_2 \underline{\boldsymbol{I}}_2 \tag{37}$$

Thus:

$$\underline{\boldsymbol{I}}_{\boldsymbol{e}} = \underline{\boldsymbol{I}}_1 - \underline{\boldsymbol{I}}_2 \frac{\boldsymbol{N}_2}{\boldsymbol{N}_1} \tag{38}$$

or

$$\underline{I}_e = \underline{I}_1 - \underline{I}_2' \tag{40}$$

where:
$\underline{I}_e$ - excitation current, and
$\underline{I}_2'$ - secondary current referred to the primary side.

From the equivalent circuit in Fig.20

$$\underline{V}_1 = \underline{E}_1 + (R_1 + jX_{s1})\underline{I}_1 \tag{41}$$

$$\underline{V}_2 = \underline{E}_2 - (R_2 + jX_{s2})\underline{I}_2 \tag{42}$$

Multiplying the above equation by *a*:

$$a \cdot \underline{V}_2 = a \cdot \underline{E}_2 - a^2 R_2 \frac{I_2}{a} - ja^2 X_{s2} \frac{I_2}{a} \tag{43}$$

or written in other form:

$$\underline{V}_2' = \underline{E}_2' - R_2' \underline{I}_2' - jX_{s2}' \underline{I}_2' \tag{44}$$

where:

$\underline{V}_2' = a\underline{V}_2$,
$\underline{E}_2' = a\underline{E}_2$,
$\underline{I}_2' = \frac{\underline{I}_2}{a}$,

Are the secondary voltages and current referred to the primary side

$R_2' = a^2 R_2$,
$X_{s2}' = a^2 X_{s2}$

Are the secondary circuit parameters referred to the primary side

Since $\underline{E}_2' = \underline{E}_1$, we can connect both circuits (Fig.21)

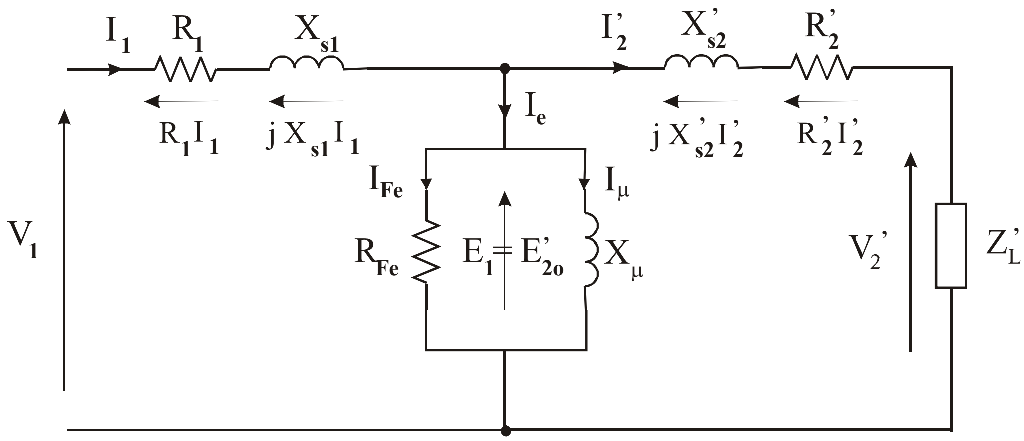


Fig.21 Modified equivalent circuit of a single-phase transformer

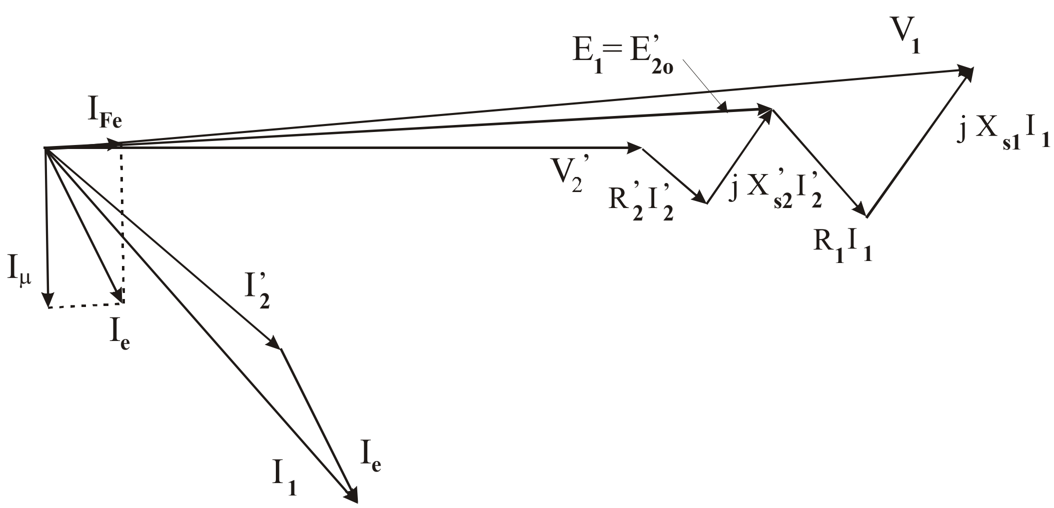


Fig.22 Phasor diagram of the real transformer at the inductive load (load current is lagging the voltage)

The primary impedance together with the shunt parameters can be transferred to the secondary side as shown in Fig.23.

The transferred to the secondary side parameters are as follows:

$$\boldsymbol{R}_1' = \frac{\boldsymbol{R}_1}{\boldsymbol{a}^2}, \quad \boldsymbol{X}_{s1}' = \frac{\boldsymbol{X}_{s1}}{\boldsymbol{a}^2},$$
$$\boldsymbol{R}_{Fe}' = \frac{\boldsymbol{R}_{Fe}}{\boldsymbol{a}^2}, \quad \boldsymbol{X}_{\mu}' = \frac{\boldsymbol{X}_{\mu}}{\boldsymbol{a}^2},$$
$$\boldsymbol{V}_1' = \frac{\boldsymbol{V}_1}{\boldsymbol{a}}, \quad \boldsymbol{I}_1' = \boldsymbol{a}\boldsymbol{I}_1.$$

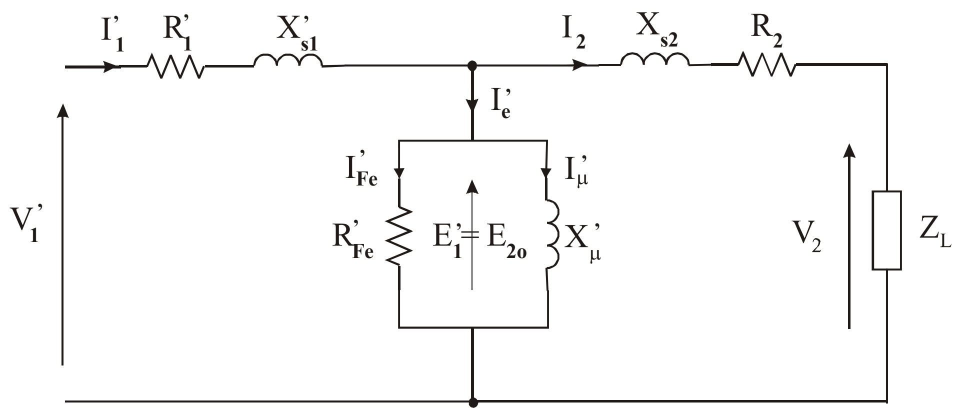


Fig.23 Equivalent circuit with parameters referred to the secondary side.

2.4 Test for determination of circuit parameters

2.4.1 Open-circuit test

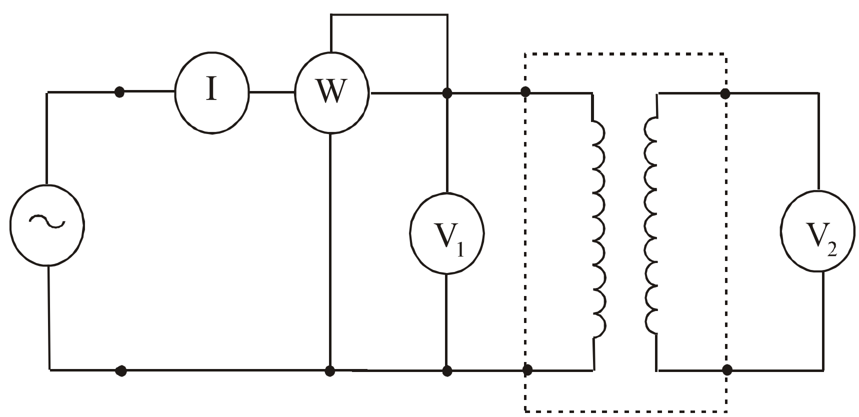


Fig.24 Circuit diagram for the open-circuit test of the transformer

Measured quantities: $\boldsymbol{V}_1$, $\boldsymbol{I_o}$, $\boldsymbol{P_o}$ and $\boldsymbol{V}_2$.

Since $\boldsymbol{I_o} << \boldsymbol{I_n}$ and $\boldsymbol{R}_1 << \boldsymbol{R_{Fe}}$, $\boldsymbol{X}_{s1} << \boldsymbol{X}_{\mu}$ the modified equivalent circuit is as shown in Fig.25. From the measured quantities the following parameters can be calculated:

$$\boldsymbol{P_o} = \frac{\boldsymbol{E}_1^2}{\boldsymbol{R_{Fe}}} = \frac{\boldsymbol{V}_1^2}{\boldsymbol{R_{Fe}}} \tag{45}$$

$$\boldsymbol{R_{Fe}} = \frac{\boldsymbol{V}_1^2}{\boldsymbol{P_o}} \tag{46}$$

$$X_\mu = \frac{E_1}{I_\mu} = \frac{V_1}{I_\mu} \tag{47}$$

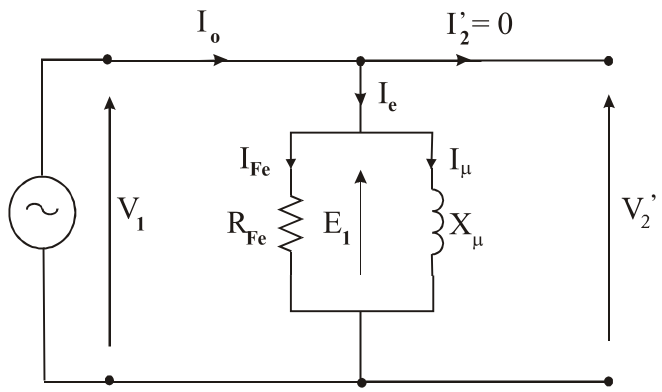


Fig.25 Equivalent circuit of the transformer at the open circuit test

According to the phasor diagram of Fig.26, which corresponds to the equivalent circuit of Fig.25, the magnetizing current:

$$I_\mu = \sqrt{I_o^2 - I_{Fe}^2} \tag{48}$$

where
$$I_{Fe} = \frac{P_o}{V_1} \tag{49}$$

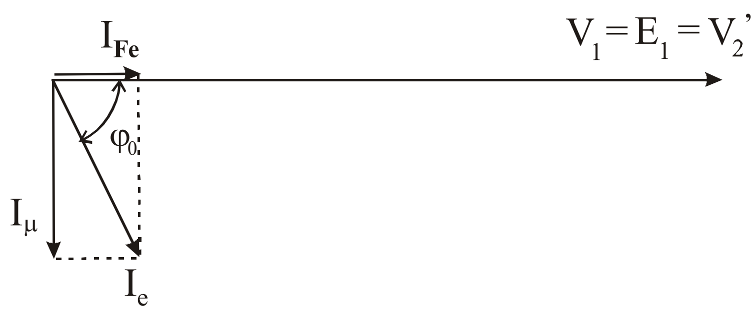


Fig.26 Phasor diagram at open-circuit test corresponding with the equivalent circuit in Fig.25.

From the open-circuit test:

$$a = \frac{V_1}{V_2} \tag{50}$$

2.4.2 Short-circuit test

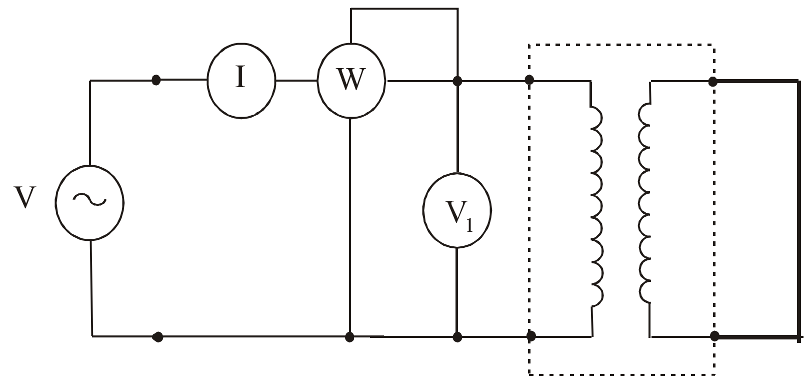


Fig.27 Circuit diagram of the transformer at short-circuit test

Since $\boldsymbol{I_e} \ll \boldsymbol{I_{sc}}$, the equivalent circuit is as in Fig.28.

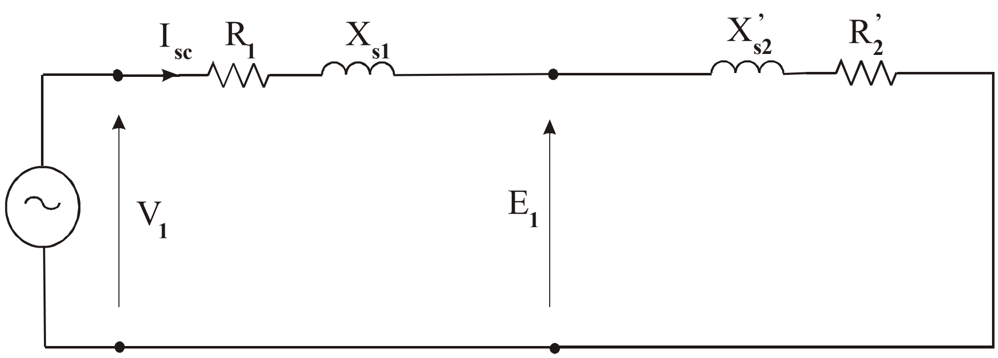


Fig.28 Equivalent circuit of the transformer at short circuit test

Measured quantities: V_1 , I_{sc}, P_{sc}.
Since:

$$P_{sc} = R_{sc} I_{sc}^2 \tag{51}$$

the following parameters can be determined from the measured quantities:

$$R_{sc} = R_1 + R_2' = \frac{P_{sc}}{I_{sc}^2} \tag{52}$$

Since

$$Z_{sc} = \frac{V_{sc}}{I_{sc}} \tag{53}$$

the short-circuit reactance is equal to

$$X_{sc} = X_{s1} + X'_{s2} = \sqrt{Z_{sc}^2 - R_{sc}^2} \tag{54}$$

For most of power transformers:

$$\boldsymbol{R}_1 = \boldsymbol{R}'_2 = \frac{\boldsymbol{R}_{sc}}{2} \quad \text{and} \quad \boldsymbol{X}_{s1} = \boldsymbol{X}'_{s2} = \frac{\boldsymbol{X}_{sc}}{2} \tag{55}$$

The phasor diagram corresponding to the equivalent circuit in Fig.28 is drawn in Fig.29.

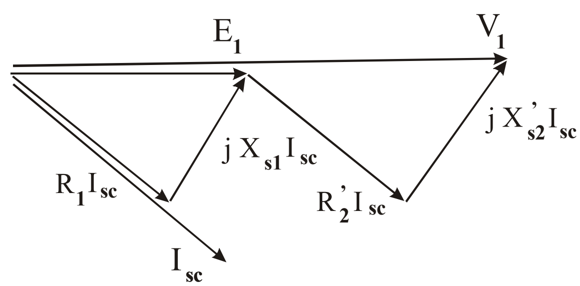


Fig.29 Phasor diagram at short-circuited secondary

2.6 Transformer operation at on-load condition

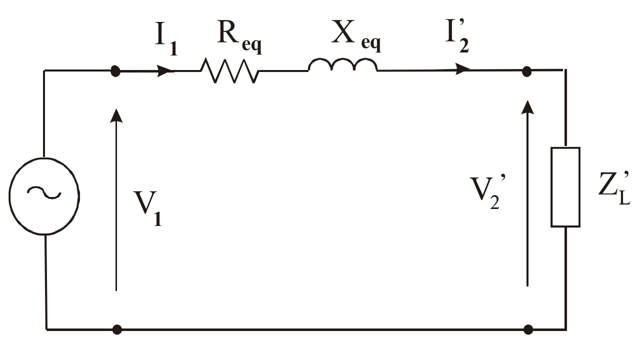
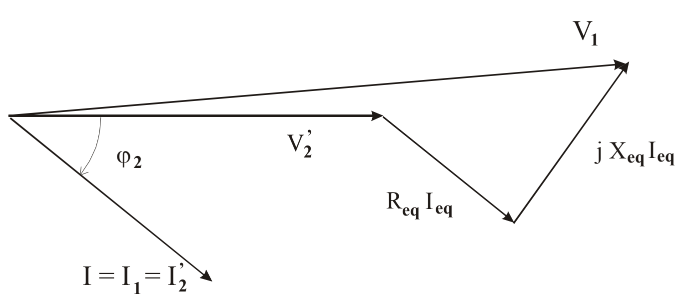


Fig.30 Simplified equivalent circuit of the transformer at on-load operation

(a)



(b)

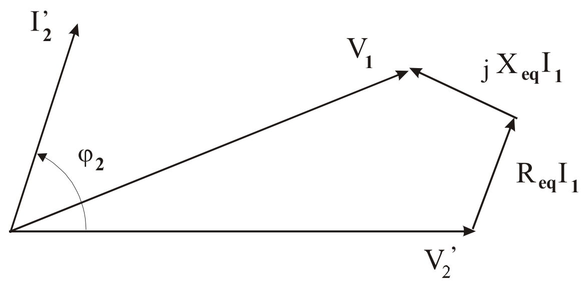


Fig.31 Phasor diagrams at on-load conditions: (a) inductive load, (b) capacitive load

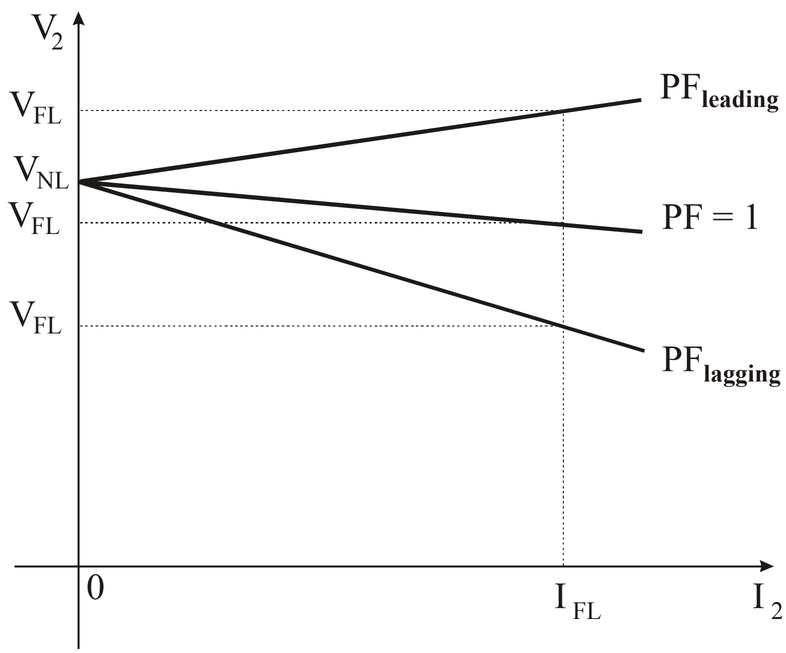


Fig.32 V-I characteristics (external characteristics) at V_1 = const, f=const, PF=const.

Voltage regulation

As the current is drawn trough transformer, the secondary voltage changes because of voltage drop in the internal impedance of the transformer. *Voltage regulation* ($\Delta v_{\%}$) is used to identify this characteristic of voltage change. It is defined as:

$$\Delta v_{\%} = \frac{|V_2|_{NL} - |V_2|_L}{|V_2|_L} 100\% \tag{56}$$

Referring to the equivalent circuit shown in Fig.30 Equ.56 can also be written as:

$$\Delta v_{\%} = \frac{\left|V_2^{'}\right|_{NL} - \left|V_2^{'}\right|_{L}}{\left|V_2^{'}\right|_{L}}$$

The load voltage is normally taken as the rated voltage. Therefore:

$$\left|V_2^{'}\right|_{L} = \left|V_2^{'}\right|_{rated}$$

From equivalent circuit

$$V_1 = V_2^{'} + \mathrm{R}_{eq}\, I_2^{'} + jX_{eq} I_2^{'}$$

If the load is thrown off ($I_1 = I_2^{'} = 0$), the voltage $V_1 = V_2^{'}$. Hence,

$$\left|V_2^{'}\right|_{NL} = \left|V_1\right|$$

Finally:

$$\Delta v_{\%} = \frac{\left|V_1\right| - \left|V_2^{'}\right|_{rated}}{\left|V_2^{'}\right|_{rated}} 100\% \qquad (57)$$

The voltage regulation depends on power factor of the load. This can be appreciated from the phasor diagram (Fig.30). The locus of V_1 is a circle of radius $\left|Z_{eq} I_1^{'}\right|$. The magnitude of V_1 will be maximum if the phasor $\left|Z_{eq} I_1^{'}\right|$ is in phase with V_2. That is

$$\varphi_2 + \varphi_{eq} = 0 \text{, and } \quad \varphi_2 = -\varphi_{eq}$$

where: φ_2 is the angle of the load impedance, and
φ_{eq} is the angle of the transformer equivalent impedance Z_{eq}.

Therefore the maximum voltage regulation occurs if the power factor angle of the load is the same as the transformer equivalent impedance angle and the load power factor is lagging.
To keep the output voltage unchanged i.e. to adjust it to the required value, turns ratio is changed by means of tap-changing switch as shown in Fig.33.

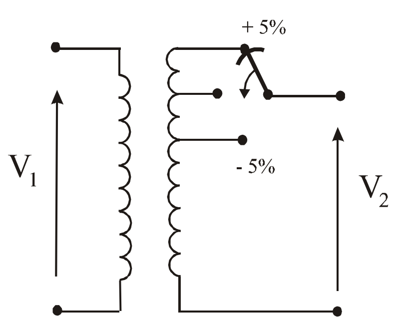


Fig.33 Tap changing switch to vary the secondary winding in the range of ±5% of the rating value

2.7 Efficiency

Definition:

$$\eta = \frac{P_2}{P_1}, \qquad \text{or} \qquad \eta_{\%} = \frac{P_2}{P_1} \cdot 100\% \tag{58}$$

where: P_1 and P_2 are the input and output powers respectively.

If expressed in terms of power losses:

$$\eta = \frac{P_2}{P_2 + \Delta P} \tag{59}$$

The power losses consist of mainly the losses in the core ΔP_{Fe} and in the winding ΔP_w

$$\Delta P = \Delta P_{Fe} + \Delta P_w \tag{60}$$

The latter one is equal to:

$$\Delta P_w = R_{sc} I_2^2 = \left(\frac{I_2}{I_{2n}}\right)^2 \Delta P_{wn} = I_{2pu}^2 \Delta P_{wn} \tag{61}$$

Since the output power is at $V_2 = V_{2n}$:

$$P_2 = V_2 I_2 \cos\varphi_2 = V_{2n} I_{2n} \frac{I_2}{I_{2n}} \cos\varphi_2 = S_n I_{2pu} \cos\varphi_2 \tag{62}$$

The transformer efficiency:

$$\eta = \frac{I_{2pu} S_n \cos\varphi_2}{I_{2pu} S_n \cos\varphi_2 + I_{2pu}^2 \Delta P_{wn} + \Delta P_{Fe}} \tag{63}$$

The efficiency vs. secondary current characteristic is shown in Fig.34. The maximum efficiency is when the iron losses are equal to cooper losses. It comes from

$$\frac{d(\eta)}{d(I_{2pu})} = 0 \tag{64}$$

at constant: S_n, $\cos\varphi_2$, ΔP_{wn} and ΔP_{Fe} the maximum efficiency is at:

$$\cos\varphi_2 = 1, \text{ and when } \quad I_{2pu}^2 \Delta P_{wn} = \Delta P_{Fe} \tag{65}$$

or expressed in other way

$$\Delta P_w = \Delta P_{Fe} \tag{65}$$

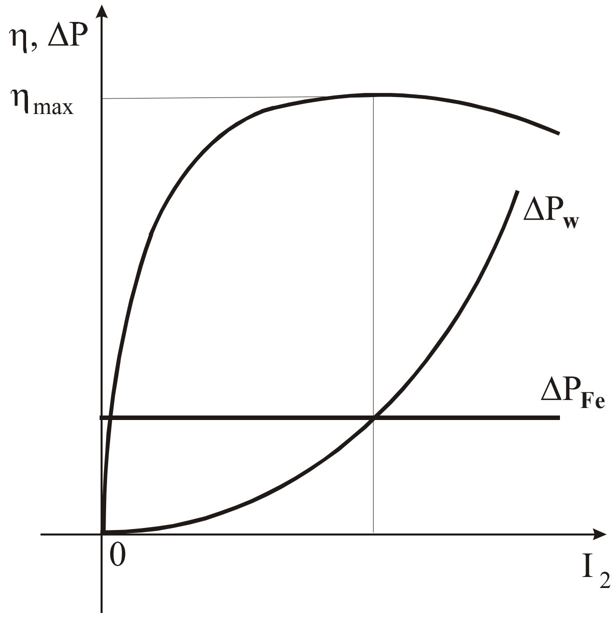


Fig.34 Efficiency and power losses versus secondary current characteristics

2.8 "Per unit" system

In "per unit" (pu) system all quantities and equivalent circuit parameters are expressed not in terms of normal units, but as a proportion of reference or rated value. This is particularly useful in the quantitative description of transformer work.

Let us select a reference value of the voltage equal to the rated value $\boldsymbol{V_n}$. Then the per unit value is

$$\boldsymbol{V}_{1\boldsymbol{pu}} = \frac{\boldsymbol{V}_1}{\boldsymbol{V}_{1\boldsymbol{n}}}, \qquad \boldsymbol{V}_{2\boldsymbol{pu}} = \frac{\boldsymbol{V}_2}{\boldsymbol{V}_{2\boldsymbol{n}}} \tag{66}$$

$$\boldsymbol{I}_{1\boldsymbol{pu}} = \frac{\boldsymbol{I}_1}{\boldsymbol{I}_{1\boldsymbol{n}}}, \qquad \boldsymbol{I}_{2\boldsymbol{pu}} = \frac{\boldsymbol{I}_2}{\boldsymbol{I}_{2\boldsymbol{n}}} \tag{67}$$

If we take as the reference impedance defined as

$$Z_{1n} = \frac{V_{1n}}{I_{1n}}, \qquad Z_{2n} = \frac{V_{2n}}{I_{2n}} \tag{68}$$

then the expression

$$\boldsymbol{V} = \boldsymbol{I} \cdot \boldsymbol{Z}, \tag{69}$$

in the real units can be written in per unit values as follows

$$\frac{V}{V_n} = \frac{I}{I_n}\frac{Z}{Z_n} \tag{70}$$

or

$$V_{pu} = I_{pu} \cdot Z_{pu} \tag{71}$$

We see that

$$Z_{pu} = \frac{V_{pu}}{I_{pu}} = Z\frac{I_n}{V_n} \tag{72}$$

The impedance of the windings of transformers and rotating machines is usually expressed as pu value and is related to the value in Ohms by the equation above.

Let us now consider the pu. system for power *P* or *S*. If the reference value of power

$$S_n = V_{1n} \cdot I_{1n} = V_{2n} I_{2n} \tag{73}$$

$$S_{pu} = \frac{S}{S_n} = \frac{V \cdot I}{V_n I_n} = V_{pu} I_{pu} \tag{74}$$

Having all quantities of one side expressed in pu. system we do not have to transfer them to another side using turns ratio adjustment. They are just equal to the value of another side. For example:

$$\frac{V_{1pu}}{V_{2pu}} = \frac{V}{V_{1n}}\frac{V_{2n}}{V_2} = \frac{N_1}{N_2}\frac{N_2}{N_1} = 1 \tag{75}$$

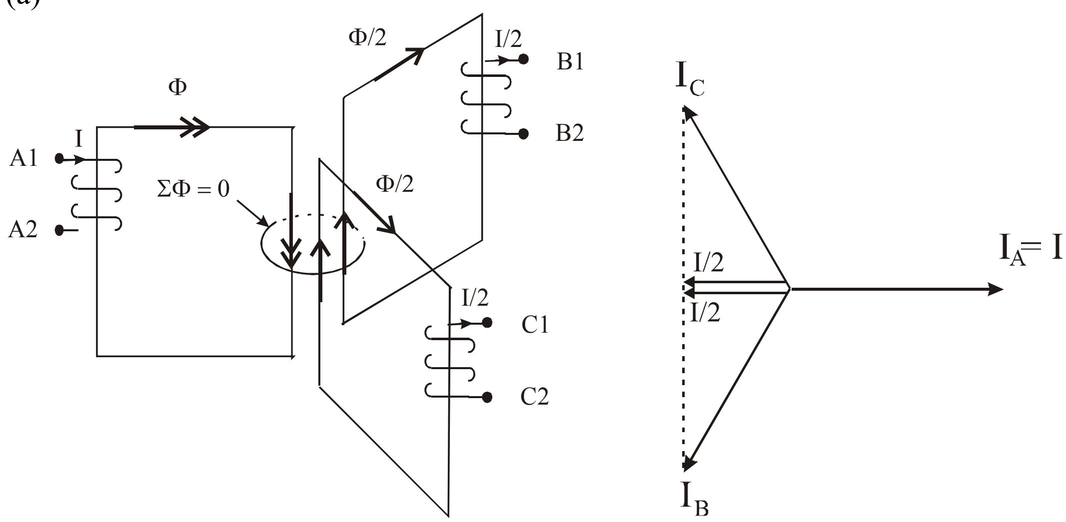
or

$$\frac{I_{1pu}}{I_{2pu}} = \frac{I_1}{I_{1n}}\frac{I_{2n}}{I_2} = \frac{N_2}{N_1}\frac{N_1}{N_2} = 1 \tag{76}$$

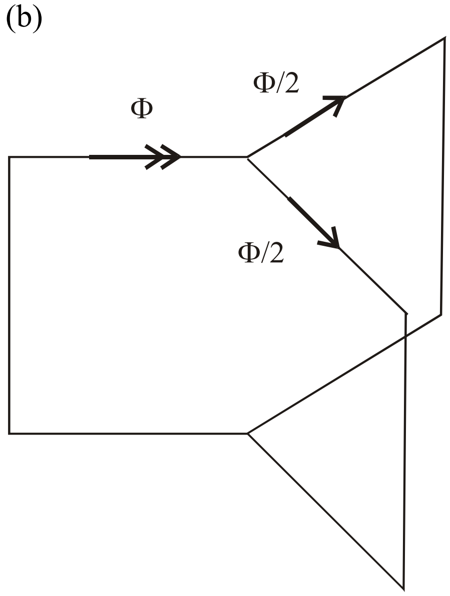
3. THREE-PHASE TRANSFORMERS

3.1 Construction

(a)



(b)



(c)

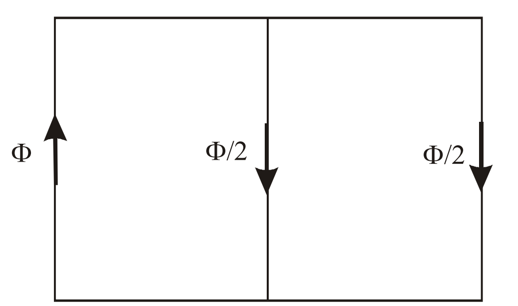


Fig.35.(a) Single-phase transformers supplied from 3-phase symmetrical source, (b) 3-phase transformer core with magnetic symmetry, (c) core of the real 3-phase core-type transformer

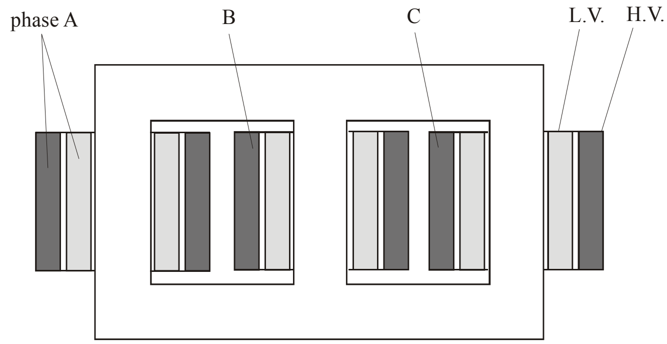


Fig.36 Three-phase core-type transformer

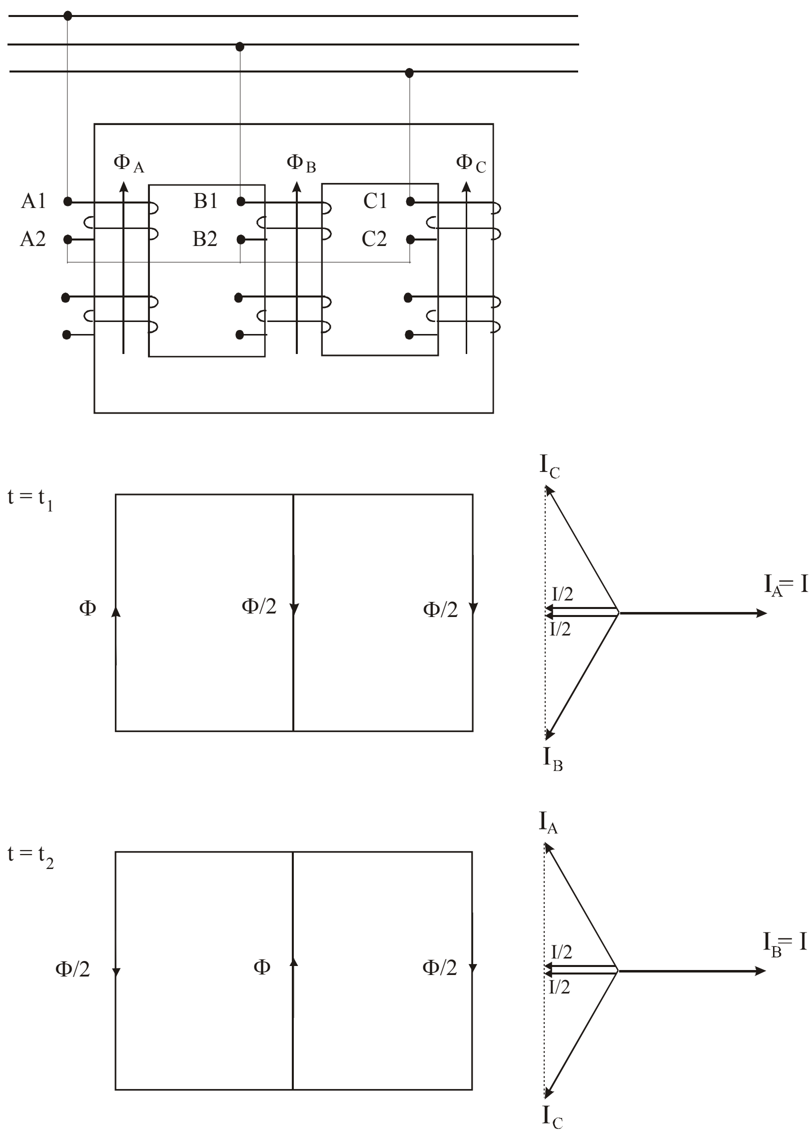


Fig.37 Winding connection of 3-phase transformer and flux distribution in the core legs

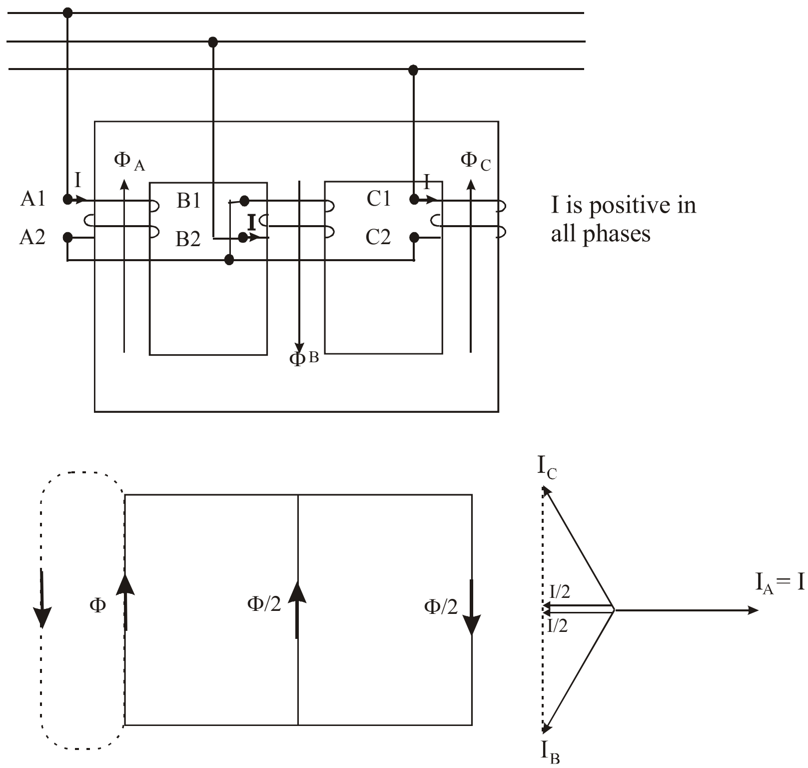
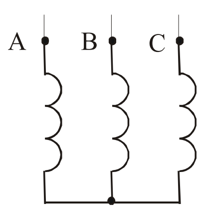
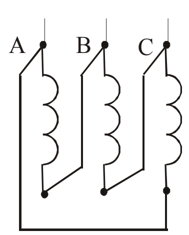
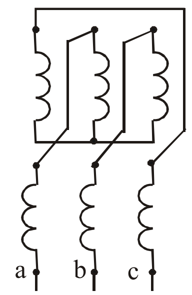
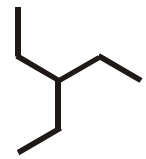


Fig.38 Flux distribution at wrong connection of 3-phase winding

3.2 Connection groups of three-phase winding

Table I. Connections of 3-phase winding

Type of connection	Circuit diagram	Graphic symbol	Symbol	
			H.V.	L.V.
Star		Y	Y	y
Delta		Δ	D	d

Zigzag			-	z

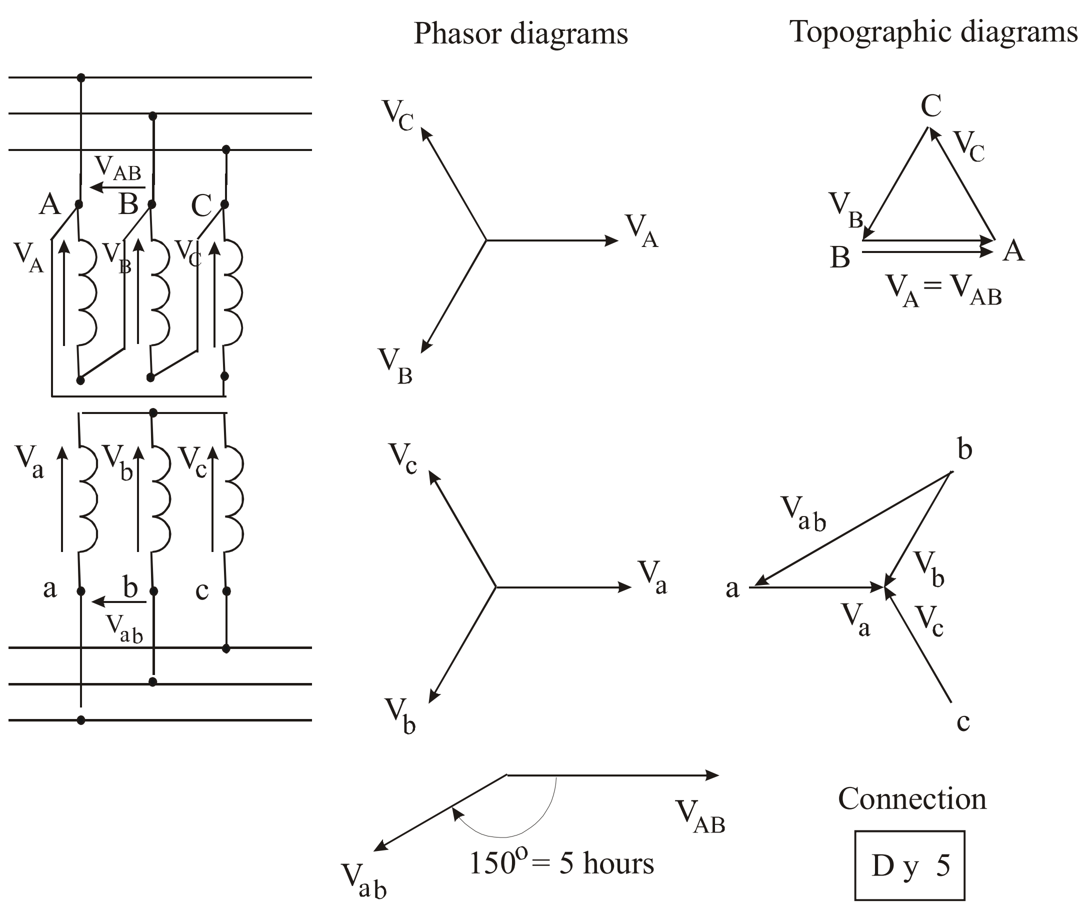


Fig.39. Three-phase transformer connected in $\boldsymbol{Dy_o}5$

3.3 Parallel operation of transformers

Demands put upon the operation of transformers connected in parallel, which must be fulfilled to avoid wrong operation at no-load and on-load conditions:

1) There must be no currents in the secondary windings at no-load conditions,

2) The transformers must load themselves accordingly to their rated powers at on-load operation,
3) The phase angles of the secondary line currents of all in parallel connected transformers must be the same.

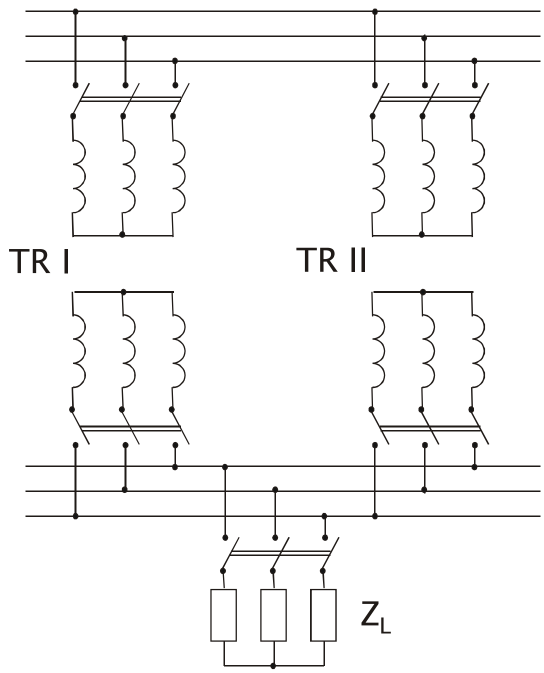


Fig.40 Three-phase transformers connected in parallel

To meet these demands the transformers must satisfy the following requirements:

1) Transformers must have the same voltage ratio,
2) The connection group of transformers must be identical,
3) The rated short-circuit voltages of transformers must be the same,
4) The ratio of rated powers (S^I / S^{II}) should not exceed 1/3.

(a) (b)

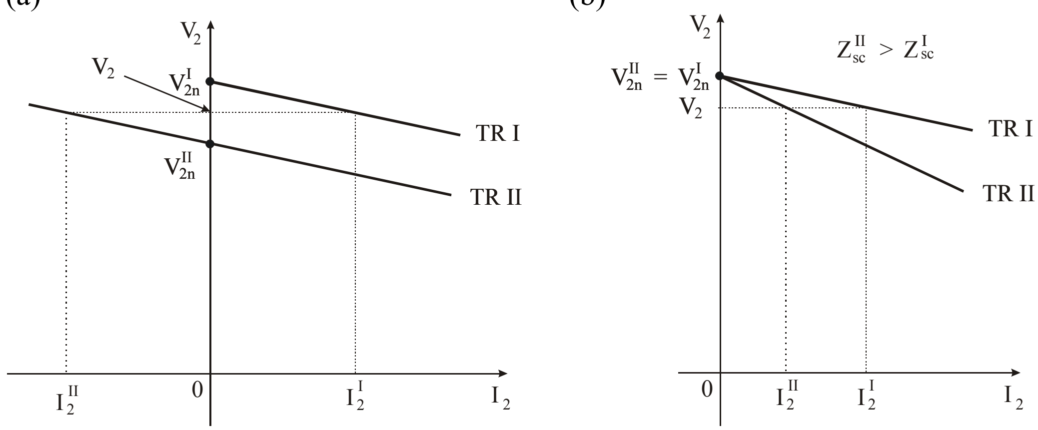


Fig.41.(a) and (b) Illustrations to the requirements 1) and 3) respectively

4. AUTO-TRANSFORMER

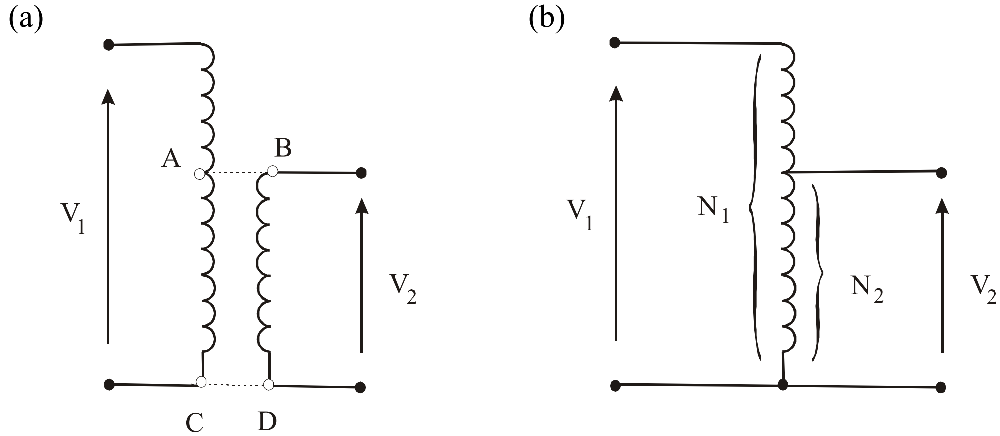


Fig.42 An explanation to the construction of auto-transformer

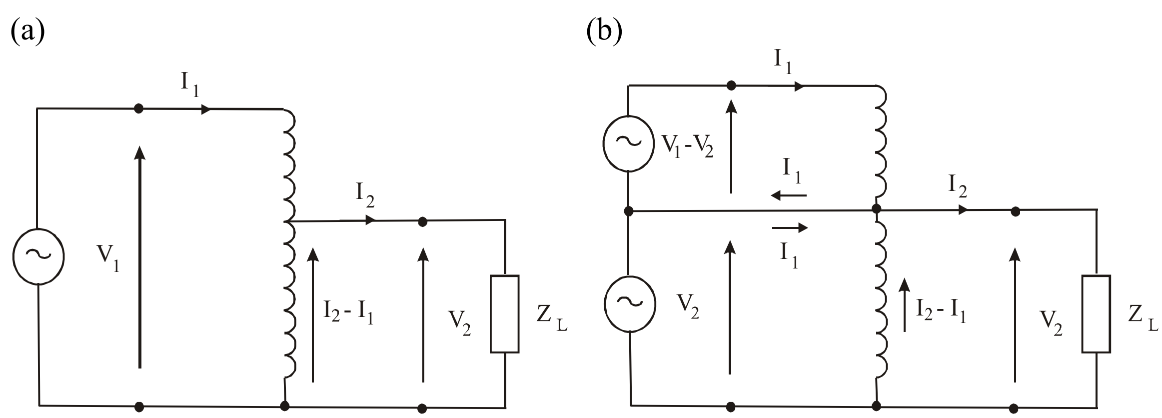
Similar, as for two-winding transformer the turn ratio is defined as follows:

$$\boldsymbol{a} = \frac{\boldsymbol{N}_1}{\boldsymbol{N}_2} \tag{77}$$

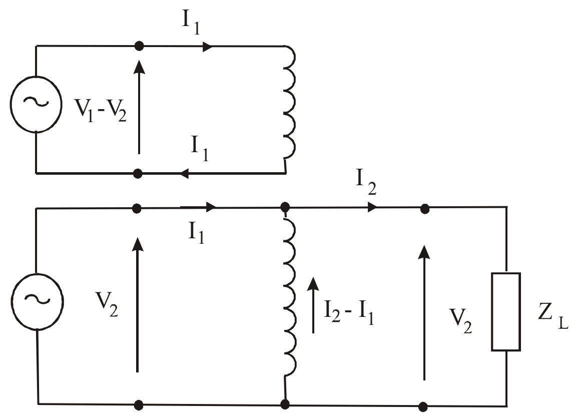
and it is approximately equal to the voltage ratio:

$$\boldsymbol{a} \cong \frac{\boldsymbol{V}_1}{\boldsymbol{V}_2} \tag{78}$$

The power is transferred from the primary side to the secondary side in two ways: by conduction and induction. This is illustrated in Fig.43.



(c)



(d)

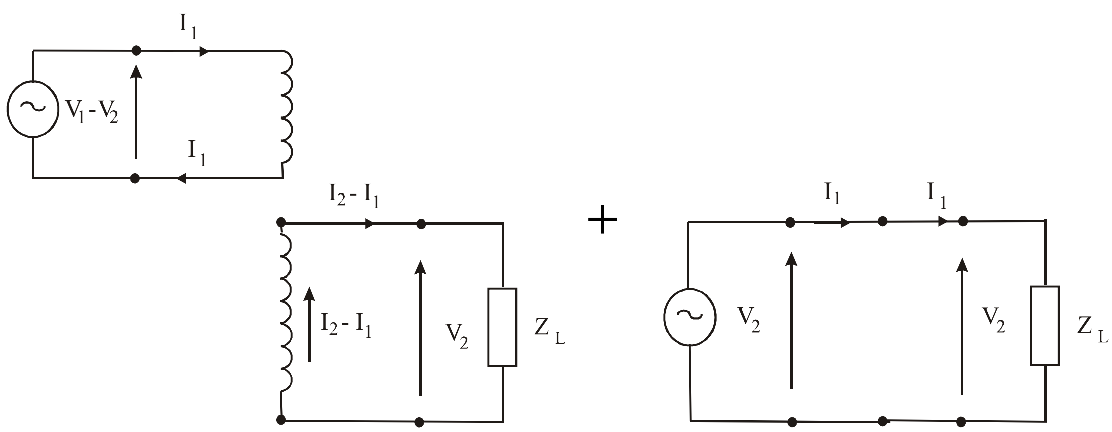


Fig.43 Explanation to the power transfer in the auto-transformer

Ignoring the power losses the total volt-ampere power is the sum of "conduction" power $\boldsymbol{S_c}$ and "induction" power $\boldsymbol{S_i}$.

$$\boldsymbol{S} = \boldsymbol{S_i} + \boldsymbol{S_c} \tag{79}$$

where:

$$\boldsymbol{S_i} = (\boldsymbol{V}_1 - \boldsymbol{V}_2)\boldsymbol{I}_1 = \boldsymbol{V}_1\boldsymbol{I}_1\left(1 - \frac{1}{\boldsymbol{a}}\right) \tag{80}$$

$$\boldsymbol{S_c} = \boldsymbol{V}_2\boldsymbol{I}_1 = \boldsymbol{V}_1\boldsymbol{I}_1\frac{1}{\boldsymbol{a}} \tag{81}$$

Since

$$\boldsymbol{S} = \boldsymbol{V}_1\boldsymbol{I}_1 \tag{82}$$

the two power components expressed in terms of the total power are:

$$\boldsymbol{S_i} = \boldsymbol{S}\left(1 - \frac{1}{\boldsymbol{a}}\right) \tag{83}$$

$$S_c = S\frac{1}{a} \tag{84}$$

The sum:

$$S_i + S_c = S\left(1-\frac{1}{a}\right) + S\frac{1}{a} = S \tag{85}$$

gives the total power S.

The common type of auto-transformer, which can be found in most of laboratories is the variable-ratio auto-transformer in which the secondary connection is movable as shown in Fig.44.

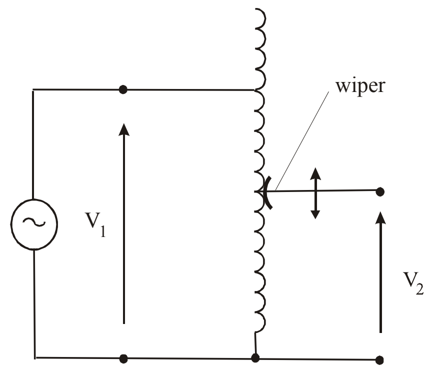


Fig.44 Auto-transformer with variable secondary voltage

5. MEASURING TRANSFORMERS

To measure the voltage and current in high voltage power systems it is necessary to use the measuring voltage and current transformers. Their connection to a three-phase system is shown in Fig.45.

5.1 Voltage transformer

The voltage transformer connected in similar way as ordinary transformer. Due to the high resistance of a voltmeter it operates practically at no-load conditions. Similar to the power transformer it cannot be short-circuited. That would damage the transformer.

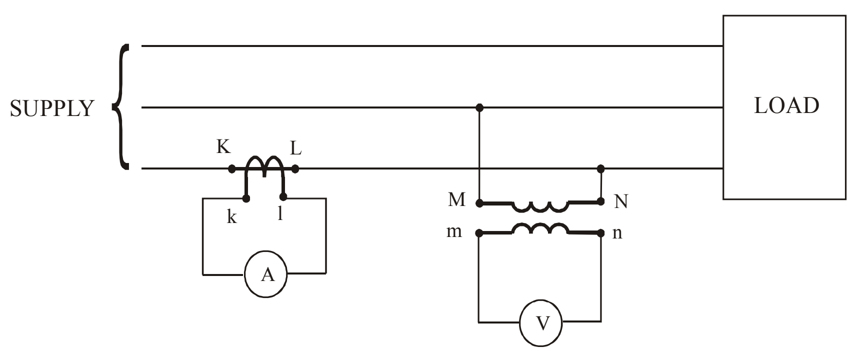
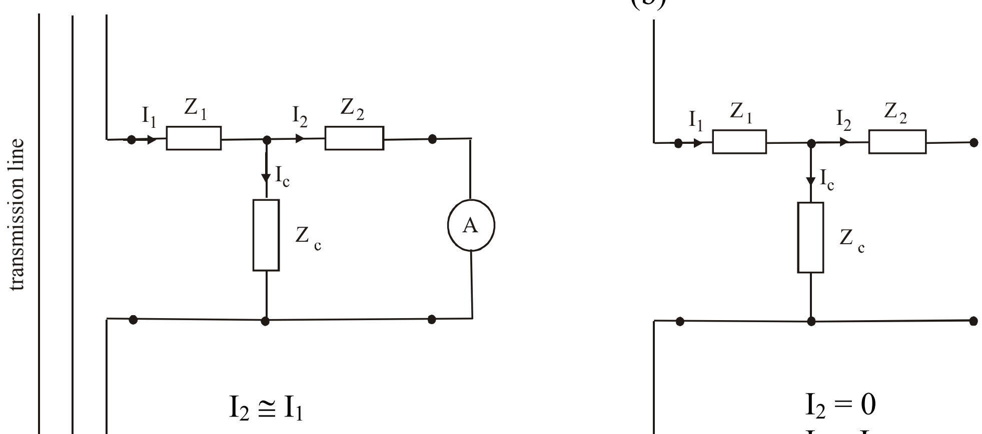


Fig.45. Connection of current and voltage transformers to 3-phase system

5.2 Current transformer

The primary winding labeled K-L is connected in series to a transmission line between the source and the load. Operating in this mode it is essentially excited from the constant-current source. The secondary terminals k-l are connected to the ammeter. Due to the low resistance of ammeter connected to the secondary the current transformer operates practically at short-circuit conditions. This is its normal operation. If the secondary is open the current-transformer can be damaged. The reason why it happens is explained in Fig.46.

(a) (b)



$I_2 \cong I_1$
$I_c \cong 0$

$I_2 = 0$
$I_c = I_1$

Fig.46 Equivalent circuit of current transformer at: (a) normal operation, (b) open-circuited secondary