Pulses on Transmission Lines
James L. Drewniak, and Richard E. DuBroff
EMC Laboratory, University of Missouri-Rolla

Objectives:

1. Study pulses on transmission lines including initial voltage, delay time, reflection and matching, and partial waves.
2. Study transmission line discontinuities including fan-out, vias, and bends.
3. Demonstrate the concepts of lumped element versus transmission line behavior of PCB nets.

Equipment:

1. Tektronix 11801B Mainframe with SD-24 TDR/Sampling Head
2. Tektronix 5006A Mainframe with PG508 Pulse Generator
3. Tektronix TDS520 Oscilloscope
4. PCB with test patterns

A pulse generated by a driver connected to a digital load through a PCB trace (microstrip or stripline type geometries) exhibits distributed behavior when the rise time \( t_r \) becomes on the order of

\[ 6t_d = \frac{\ell}{v_p}, \]

where \( t_d \) is the delay time or time of propagation down the line of length \( \ell \), and \( v_p \) is the speed at which the wave is moving down the line (\( v_p = \frac{c_o}{\sqrt{\varepsilon_r}} \), \( c_o \) is the speed of light in free-space, and \( \varepsilon_r \) is the dielectric constant in the medium) [1]. For slower rise times, the voltage and current are approximately constant at all points along the line, and the circuit can be modeled using lumped circuit elements.

A microstrip geometry is used in this experiment to illustrate distributed or transmission line behavior, and compare and contrast it to lumped element concepts. A schematic representation of the test configuration is shown in Figure 1. The TDR is a pulse generator with a rise time of 20 \( ps \) and a \( 50\Omega \) output impedance. The test board is a section of microstrip transmission line with characteristic impedance \( Z_{02} \), wave speed \( v_{p2} \), and length \( \ell \) that is terminated in some load. Consider first a short circuit at the load. The wave traveling down the \( 50\Omega \) cable connecting the TDR to the test board is partially reflected at the discontinuity between the cable and test board. The reflection coefficient is [2], [3].

\[ \rho_{12} = \frac{Z_{02} - 50}{Z_{02} + 50} \]
Note that the reflection coefficient concept in the time-domain is applicable only to discontinuities and terminations that are *purely resistive*, i.e., not to reactive or nonlinear terminations. For reactive terminations, the boundary condition at the load is in terms of a differential equation. Hence, the reflected wave is the solution to a differential equation at the load. The reflection coefficient for a resistive termination can be read directly from the TDR as shown in the data in Figure 2. The characteristic impedance of the test board

\[ Z_0 = 27.04 \Omega \]

Figure 1: Schematic representation of the TDR and terminated microstrip line.

Figure 2: TDR trace of the wave reflected from the junction of the 50Ω cable and test board.
line can be determined from the measurement of the reflection coefficient as given above, or read directly from the TDR. The “jumps” in the data correspond to discontinuities, either at the end of the trace on the test board, or at the cable/test board junction.

A delay time measurement, together with the trace length $\ell$, can be used to estimate the relative dielectric constant from $t_d = \frac{\ell}{c_o / \sqrt{\varepsilon_r}}$. Note that the TDR display as shown in Figure 2 is the signal at the pulse generator. As a result the measured time between the jumps is the down and back transit time or $2t_d$. The measured round-trip time is $2t_d = 3.78\,\text{ns}$, resulting in an approximate $\varepsilon_r \approx 3.5$ for the substrate material used. The measured characteristic impedance, together with the measured delay time and line length can be used to estimate the inductance and capacitance per unit length $L$ and $C$ from $Z_{02} = \sqrt{\frac{L}{C}}$ and $t_d = \frac{\ell}{v_p} = \frac{\ell}{\sqrt{LC}}$, where the wave speed is related to $L$ and $C$ by $v_p = \frac{1}{\sqrt{LC}}$. Then, $L = \frac{Z_{02} t_d}{\ell}$, and $C = \frac{t_d}{Z_{02} \ell}$. For the present $Z_{02} = 27\,\Omega$ line studied, $L = 1.7\,\text{H/cm}$ and $C = 2.3\,\text{pF/cm}$.

![Figure 3: Measured voltage at the sending end for a shorted line.](image)
The voltage at the sending end (generator) with the load end shorted is shown in Figure 3. The measured voltage is comprised of all waves, incident, reflected, re-reflected etc. on the line at any given time. The pulse generator open circuit voltage is 500mV, so the initial wave launched down the 50Ω cable is 

\[ V_i = 500 \frac{Z_0}{Z_0 + Z_g} = 250mV, \]

where \( Z_0 = 50Ω \) is the characteristic impedance of the cable, and \( Z_g = 50Ω \) is the generator output impedance. The reflection coefficient at the discontinuity between the coaxial cable and test board is \( \rho_{12} = -0.3 \), and the transmission coefficient is \( \tau_{12} = 1 + \rho_{12} = 0.7 \). The transmitted voltage is \( (500mV) \tau_{12} = 175mV \). The bouncing waves can be conceptually tracked on a bounce diagram as shown in Figure 4. The total voltage waveform at a particular point (here, the generator \( z=0 \) end) is then the sum of all partial waves on the line at a particular as shown for the sending end voltage.

\[ V_t = \frac{1}{2} (200mV - 100mV) \]

\[ \text{time(s)} \]

\[ \text{time(s)} \]

The voltage at any point on the line is a function of time resulting from the bouncing waves. Ideally, the voltage at any point would be constant with time once the incident wave has passed. In order to eliminate reflections and the voltage jumps; the load must
be matched to the characteristic impedance of the line. The measured TDR reflection for both 27Ω leaded and 1206 SMT parts is shown in Figure 5. The leaded part indicates

![TDR trace for the microstrip line with Z₀ = 27Ω terminated in 27Ω leaded and SMT parts.](image)

Figure 5: TDR trace for the microstrip line with $Z_0 = 27\Omega$ terminated in 27Ω leaded and SMT parts.

...a significant discontinuity as a result of the series inductance of the leads, while the SMT part results in minimal series inductance. This indicates the effect of added trace length in terminating lines on PCBs when using SMT parts. The lack of reflections with the SMT part also corroborates the calculation of $Z_{02}$ from the measured reflection coefficient.

Fan-out results in another type of transmission line discontinuity. In this experiment, a nominal 43Ω characteristic impedance microstrip line is used. Measurements for a single line and a line fanned out into three lines are shown in Figure 6. All lines are terminated in 50Ω. The termination at the fan-out discontinuity is the three 43Ω lines in parallel, or 14.3Ω. The reflection coefficient at this discontinuity is then

$$\rho_{2,3} = \frac{14.3 - 43}{14.3 + 43} = -0.5.$$  

The final values of the voltages are different in the two cases as well. The final voltage is calculated from the DC circuit, since all the bounces have died away. For the particular terminations shown, the initial voltage prior to the pulse is
−125\text{mV} \text{ instead of } 0. \text{ The open circuit voltage at the generator terminals is still } 500\text{mV}. \text{ For a single line, the final value of the voltage is } \frac{50}{50 + 50} = −125 = 125\text{mV}.

For the three fan-out lines, the final voltage on the line is \( \frac{50/30}{50 + 50/30} = −125 = 0\text{mV} \). Thus resistive shunt terminations change the value of the final line voltage, and draw DC current.

![TDR trace for a 43Ω characteristic impedance line fanned out into three such lines.](Figure 6)

The effect of 90° versus mitred corners is shown in Figure 7. The two traces are for nearly identical lines with four 90° corners each, and are terminated in 50Ω. The only difference between the lines is that one has mitred corners and the other does not. The rise time of the TDR pulse is 20\text{ps}, and as a result, the small series \( L \) and shunt \( C \) associated with the un-mitred corner is very clear [4]. The discontinuities appear smaller for successive corners because the energy that is reflected is smaller. The line with mitred corners has little reflection at the corners. Though the reflections are clear, it is worth noting that voltage swing is relatively small in comparison to the peak swing of 250\text{mV} for the terminated line. Further, the discontinuity is significant only for very high frequencies. A comparison of the mitred and un-mitred cases with a nominal 5\text{ns}
pulse generator shows that the sending and receiving end voltages are independent of the mitering, i.e., they are identical.

Transmission-line or distributed behavior is compared with lumped element behavior in Figures 8 (a) and (b). The 27Ω line on the test board is terminated in a nominal 1000pF capacitor. The TDR trace with a 20ps rise time source is shown Figure 8 (a). The reflections at each end of the line are clear, and indicative of transmission-line behavior. The time constant for this termination on the line is longer than the delay time of the line, and, hence, the voltage does not reach its final value between each successive bounce. The total initial voltage at the capacitive load when the incident wave reaches the load must be zero, since the voltage across the capacitor cannot change instantaneously, and is reflected in the downward jump of the trace. The final value must be the open circuit voltage of the pulse generator or 500mV. These details will be pursued in depth in a second experiment.

The distributed behavior is compared and contrasted to the lumped behavior illustrated in Figure 8 (b). Here a nominal 5ns and 50ns rise time is used from a pulse generator. There are no clear reflections as with the distributed case. For the 50ns pulse the response can be modeled as a series RLC, with the series L modeling the properties of the microstrip trace. The total series L in the model can be estimated by multiplying the inductance per unit length for the line $L = 1.7 \frac{nH}{cm}$ by the line length $\ell = 30.2cm$, or 51.3nH. The series resonance is at approximately 22.7MHz. As a result there is little energy in the 50ns rise-time pulse to excite this resonance, and the response is a simple RC-type. Alternatively, significantly below the series resonance, the behavior of the circuit is dominated by the load capacitance, and above, by the series inductance of the microstrip line. It turns out that the RC time constant is 50ns as well. For the 5ns pulse, there is sufficient energy in the pulse at the series resonance frequency to excite the resonance as shown in Figure 8(b). The measured resonance is approximately 26MHz.

Experimental procedures:

- Investigate the reflection coefficient and sending-end voltage for a shorted line and relate to the bounce diagram.
- Terminate the line in a matched load to eliminate reflections and demonstrate the effect of termination interconnect inductance for a matched load.
- Demonstrate the impedance discontinuity associated with fan-out.
- Compare the effects of mitred and un-mitred bends for 20ps, and 5ns rise times.
- Demonstrate the concepts of distributed versus lumped behavior.
Figure 7: TDR trace for two identical lines with four 90° corners, mitered versus unmitered.

Figure 8: Distributed versus lumped behavior. (a) TDR trace (20 ps rise time).
Figure 8: Distributed versus lumped behavior. (b) Sending end voltage with a 50 ns, and 5 ns rise time pulse.

References