Wind Turbine Blade Analysis using the Blade Element Momentum Method.
Version 1.0

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Nomenclature

a Axial induction factor

a' Angular induction factor

B Number of blades

c Aerofoil chord length

C_L Lift coefficient

C_D Drag coefficient

C_P Power coefficient

D Drag force

F_x Axial force

F_θ Tangential force

L Lift force, angular moment

$\dot{m}$ Massflow

N Number of blade elements

p Pressure

P Power

Q Tip loss correction factor

r radius and radial direction

R Blade tip radius

T Torque

V Absolute velocity

W Relative velocity

x Axial coordinate

β Relative flow angle onto blades

λ Tip speed ratio

λ_r Local Tip speed ratio

η Mechanical/electrical efficiency

ρ Density

σ' Local Solidity

θ Tangential coordinate

Ω Blade rotational speed

ω Wake rotational speed

γ Aerofoil inlet angle

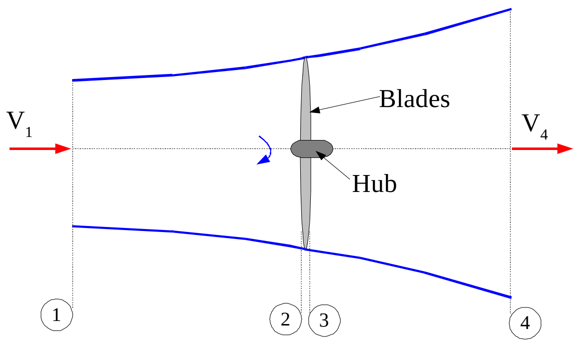

Figure 1: Axial Stream tube around a Wind Turbine

1 Introduction

This short document describes a calculation method for wind turbine blades, this method can be used for either analysis of existing machines or the design of new ones. More sophisticated treatments are available but this method has the advantage of being simple and easy to understand.

This design method uses blade element momentum (or BEM) theory to complete the design and can be carried out using a spreadsheet and lift and drag curves for the chosen aerofoil.

The latest version of this document should be available from the author's website[1] Any comments on the document would be gratefully received. Further details on Wind Turbine Design can be found in Manwell et al. (2002) which provides compreshensive coverage of all aspects of wind energy. Walker and Jenkins (1997) also provide a comprehensive but much briefer overview of Wind Energy.

2 Blade Element Momentum Theory

Blade Element Momentum Theory equates two methods of examining how a wind turbine operates. The first method is to use a momentum balance on a rotating annular stream tube passing through a turbine. The second is to examine the forces generated by the aerofoil lift and drag coefficients at various sections along the blade. These two methods then give a series of equations that can be solved iteratively.

3 Momentum Theory

3.1 Axial Force

Consider the stream tube around a wind turbine shown in Figure 1. Four stations are shown in the diagram 1, some way upstream of the turbine, 2 just before the

[1] http://www.dur.ac.uk/g.l.ingram

blades, 3 just after the blades and 4 some way downstream of the blades. Between 2 and 3 energy is extracted from the wind and there is a change in pressure as a result.

Assume $p_1 = p_4$ and that $V_2 = V_3$. We can also assume that between 1 and 2 and between 3 and 4 the flow is frictionless so we can apply Bernoulli's equation. After some algebra:

$$p_2 - p_3 = \frac{1}{2}\rho(V_1^2 - V_4^2) \tag{1}$$

Noting that force is pressure times area we find that:

$$dF_x = (p_2 - p_3)dA \tag{2}$$

$$\Rightarrow dF_x = \frac{1}{2}\rho(V_1^2 - V_4^2)dA \tag{3}$$

Define a the axial induction factor as:

$$a = \frac{V_1 - V_2}{V_1} \tag{4}$$

It can also be shown that:

$$V_2 = V_1(1-a) \tag{5}$$

$$V_4 = V_1(1-2a) \tag{6}$$

Substituting yields:

$$dF_x = \frac{1}{2}\rho V_1^2[4a(1-a)]2\pi r dr \tag{7}$$

3.2 Rotating Annular Stream tube

Consider the rotating annular stream tube shown in Figure 2. Four stations are shown in the diagram 1, some way upstream of the turbine, 2 just before the blades, 3 just after the blades and 4 some way downstream of the blades. Between 2 and 3 the rotation of the turbine imparts a rotation onto the blade wake.

Consider the conservation of angular momentum in this annular stream tube. An "end-on" view is shown in Figure 3. The blade wake rotates with an angular velocity ω and the blades rotate with an angular velocity of Ω. Recall from basic physics that:

$$\text{Moment of Inertia of an annulus,} I = mr^2 \tag{8}$$

$$\text{Angular Moment,} L = I\omega \tag{9}$$

$$\text{Torque,} T = \frac{dL}{dt} \tag{10}$$

$$\Rightarrow T = \frac{dI\omega}{dt} = \frac{d(mr^2\omega)}{dt} = \frac{dm}{dt}r^2\omega \tag{11}$$

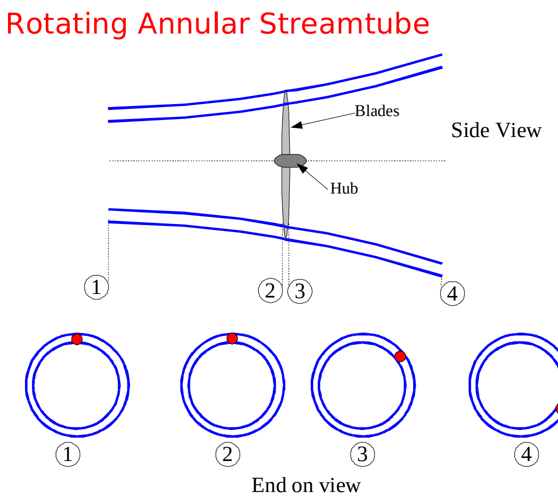

Figure 2: Rotating Annular Stream tube

So for a small element the corresponding torque will be:

$$dT = d\dot{m}\omega r^2 \tag{12}$$

For the rotating annular element

$$d\dot{m} = \rho A V_2 \tag{13}$$
$$d\dot{m} = \rho 2\pi r dr V_2 \tag{14}$$
$$\Rightarrow dT = \rho 2\pi r dr V_2 \omega r^2 = \rho V_2 \omega r^2 2\pi r dr \tag{15}$$

Define angular induction factor a':

$$a' = \frac{\omega}{2\Omega} \tag{16}$$

Recall that $V_2 = V(1-a)$ so:

$$dT = 4a'(1-a)\rho V \Omega r^3 \pi dr \tag{17}$$

Momentum theory has therefore yielded equations for the axial (Equation 7) and tangential force (Equation 17) on an annular element of fluid.

4 Blade Element Theory

Blade element theory relies on two key assumptions:

- There are no aerodynamic interactions between different blade elements
- The forces on the blade elements are solely determined by the lift and drag coefficients

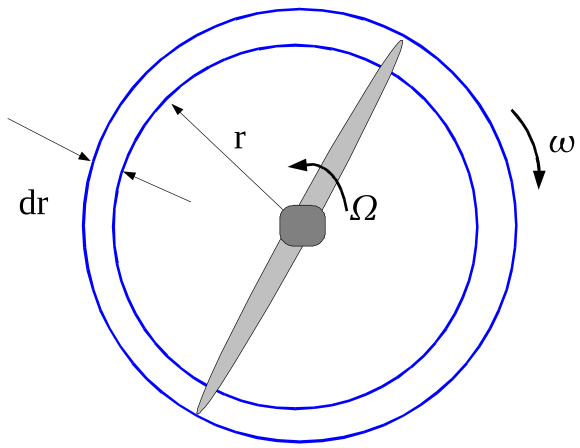

Figure 3: Rotating Annular Stream tube: notation.

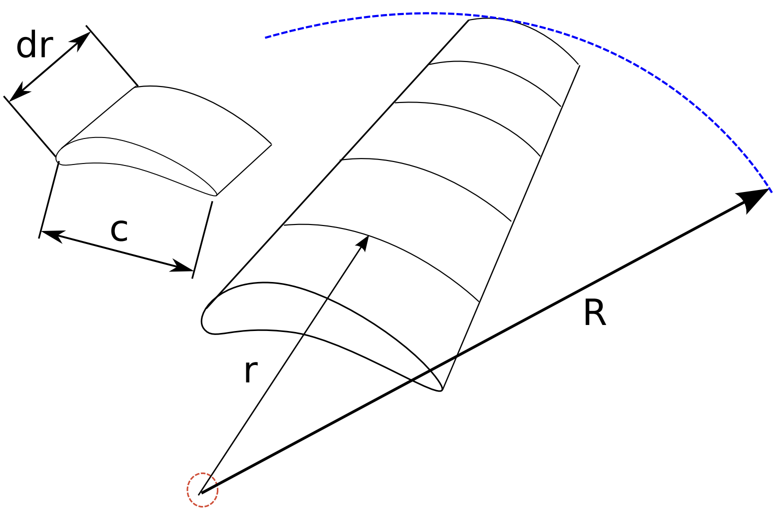

Figure 4: The Blade Element Model

Consider a blade divided up into N elements as shown in Figure 4. Each of the blade elements will experience a slightly different flow as they have a different rotational speed (Ωr), a different chord length (c) and a different twist angle (γ). Blade element theory involves dividing up the blade into a sufficient number (usually between ten and twenty) of elements and calculating the flow at each one. Overall performance characteristics are determined by numerical integration along the blade span.

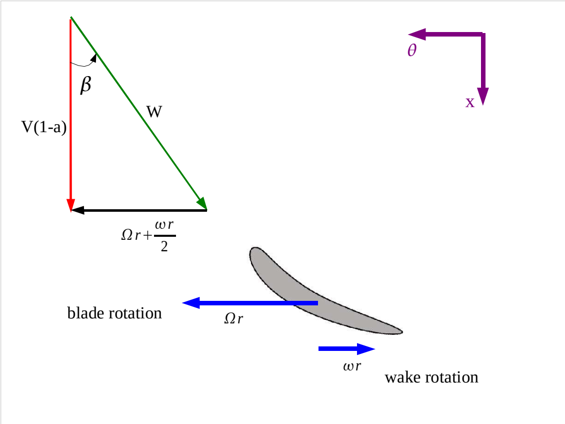

Figure 5: Flow onto the turbine blade

4.1 Relative Flow

Lift and drag coefficient data area available for a variety of aerofoils from wind tunnel data. Since most wind tunnel testing is done with the aerofoil stationary we need to relate the flow over the moving aerofoil to that of the stationary test. To do this we use the relative velocity over the aerofoil. More details on the aerodynamics of wind turbines and aerofoil selection can be found in Hansen and Butterfield (1993).

In practice the flow is turned slightly as it passes over the aerofoil so in order to obtain a more accurate estimate of aerofoil performance an average of inlet and exit flow conditions is used to estimate performance.

The flow around the blades starts at station 2 in Figures 2 and 1 and ends at station 3. At inlet to the blade the flow is not rotating, at exit from the blade row the flow rotates at rotational speed ω. That is over the blade row wake rotation has been introduced. The average rotational flow over the blade due to wake rotation is therefore $\omega/2$. The blade is rotating with speed Ω. The average tangential velocity that the blade experiences is therefore $\Omega r + \frac{1}{2}\omega r$. This is shown in Figure 5.

Examining Figure 5 we can immediately note that:

$$\Omega r + \frac{\omega r}{2} = \Omega r(1+a') \tag{18}$$

Recall that (Equation 5): $V_2 = V_1(1-a)$ and so:

$$\tan\beta = \frac{\Omega r(1+a')}{V(1-a)} \tag{19}$$

Where V is used to represent the incoming flow velocity V_1. The value of β will vary from blade element to blade element. The local tip speed ratio λ_r is defined as:

$$\lambda_r = \frac{\Omega r}{V} \tag{20}$$

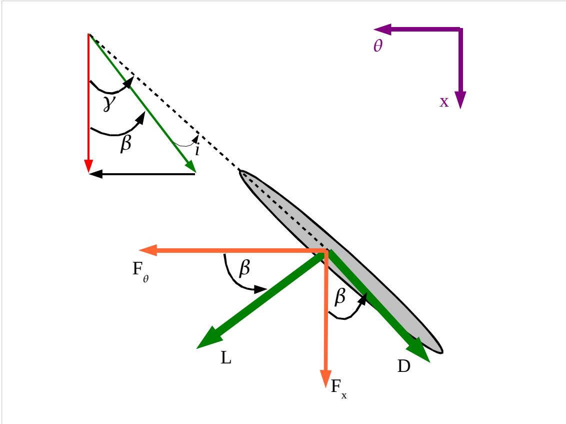

Figure 6: Forces on the turbine blade.

So the expression for $\tan\beta$ can be further simplified:

$$\tan\beta = \frac{\lambda_r(1+a')}{(1-a)} \tag{21}$$

From Figure 5 the following relation is apparent:

$$W = \frac{V(1-a)}{\cos\beta} \tag{22}$$

4.2 Blade Elements

The forces on the blade element are shown in Figure 6, note that by definition the lift and drag forces are perpendicular and parallel to the incoming flow. For each blade element one can see:

$$dF_\theta = dL\cos\beta - dD\sin\beta \tag{23}$$

$$dF_x = dL\sin\beta + dD\cos\beta \tag{24}$$

where dL and dD are the lift and drag forces on the blade element respectively. dL and dD can be found from the definition of the lift and drag coefficients as follows:

$$dL = C_L\frac{1}{2}\rho W^2 cdr \tag{25}$$

$$dD = C_D\frac{1}{2}\rho W^2 cdr \tag{26}$$

Lift and Drag coefficients for a NACA 0012 aerofoil are shown in Figure 7, this graph shows that for low values of incidence the aerofoil successfully produces a

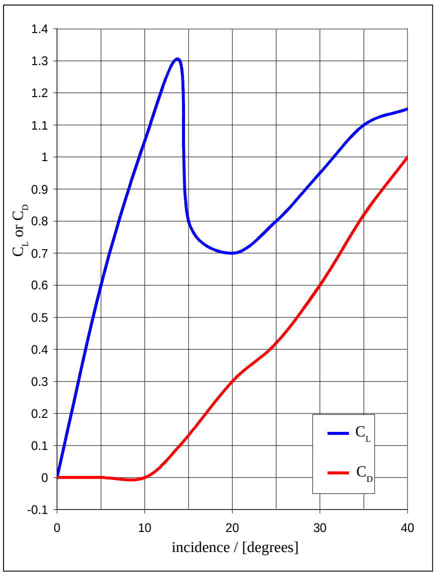

Figure 7: Lift and Drag Coefficients for a NACA 0012 Aerofoil

large amount of lift with little drag. At around $i = 14^{\circ}$ a phenomenon known as stall occurs where there is a massive increase in drag and a sharp reduction in lift.

If there are B blades, combining Equation 23 and equation 25 it can be shown that:

$$dF_x = B\frac{1}{2}\rho W^2 (C_L \sin\beta + C_D \cos\beta)cdr \tag{27}$$

$$dF_\theta = B\frac{1}{2}\rho W^2 (C_L \cos\beta - C_D \sin\beta)cdr \tag{28}$$

The Torque on an element, dT is simply the tangential force multiplied by the radius.

$$dT = B\frac{1}{2}\rho W^2 (C_L \cos\beta - C_D \sin\beta)crdr \tag{29}$$

The effect of the drag force is clearly seen in the equations, an increase in thrust force on the machine and a decrease in torque (and power output).

These equations can be made more useful by noting that β and W can be expressed in terms of induction factors etc. (Equations 21 and 22). Substituting and carrying out some algebra yields:

$$dF_x = \sigma'\pi\rho\frac{V^2(1-a)^2}{\cos^2\beta}(C_L \sin\beta + C_D \cos\beta)rdr \tag{30}$$

$$dT = \sigma'\pi\rho\frac{V^2(1-a)^2}{\cos^2\beta}(C_L \cos\beta - C_D \sin\beta)r^2dr \tag{31}$$

where σ' is called the local solidity and is defined as:

$$\sigma' = \frac{Bc}{2\pi r} \tag{32}$$

5 Tip Loss Correction

At the tip of the turbine blade losses are introduced in a similar manner to those found in wind tip vorticies on turbine blades. These can be accounted for in BEM theory by means of a correction factor.

This correction factor Q varies from 0 to 1 and characterises the reduction in forces along the blade.

$$Q = \frac{2}{\pi}\cos^{-1}\left[exp\left\{-\left(\frac{B/2[1-r/R]}{(r/R)\cos\beta}\right)\right\}\right] \tag{33}$$

The results from $\cos^{-1}$ *must be in radians*. The tip loss correction is applied to Equation 7 and Equation 17 which become:

$$dF_x = Q\rho V_1^2[4a(1-a)]\pi r dr \tag{34}$$

$$dT = Q4a'(1-a)\rho V\Omega r^3\pi dr \tag{35}$$

6 Blade Element Momentum Equations

We now have four equations, two dervied from momentum theory which express the axial thrust and the torque in terms of flow parameters (Equations 35 and 34):

$$dF_x = Q\rho V_1^2[4a(1-a)]\pi r dr \tag{36}$$

$$dT = Q4a'(1-a)\rho V\Omega r^3\pi dr \tag{37}$$

We also have two quations derived from a consideration of blade forces which express the axial force and torque in terms of the lift and drag coefficients of the aerofoil (Equations 30 and 31):

$$dF_x = \sigma'\pi\rho\frac{V^2(1-a)^2}{\cos^2\beta}(C_L\sin\beta + C_D\cos\beta)rdr \tag{38}$$

$$dT = \sigma'\pi\rho\frac{V^2(1-a)^2}{\cos^2\beta}(C_L\cos\beta - C_D\sin\beta)r^2dr \tag{39}$$

To calculate rotor performance Equations 35 and 34 from a momentum balance are equated with Equations 30 and 31. Once this is done the following useful relationships arise:

$$\frac{a}{1-a} = \frac{\sigma' \left[C_L \sin\beta + C_D \cos\beta\right]}{4Q\cos^2\beta} \tag{40}$$

$$\frac{a'}{1-a} = \frac{\sigma' \left[C_L \cos\beta - C_D \sin\beta\right]}{4Q\lambda_r cos^2\beta} \tag{41}$$

Equation 40 and 41 are used in the blade design procedure.

7 Power Output

The contribution to the total power from each annulus is:

$$dP = \Omega dT \tag{42}$$

The total power from the rotor is:

$$P = \int_{r_h}^{R} dPdr = \int_{r_h}^{R} \Omega dT dr \tag{43}$$

Where r_h is the hub radius. The power coefficient C_P is given by:

$$C_P = \frac{P}{P_{wind}} = \frac{\int_{r_h}^{R} \Omega dT}{\frac{1}{2}\rho\pi R^2 V^3} \tag{44}$$

Using Equation 31 it is possible to develop an integral for the power coefficient directly. After some algebra:

$$C_P = \frac{8}{\lambda^2}\int_{\lambda_h}^{\lambda} Q\lambda_r^3 a'(1-a)\left[1 - \frac{C_D}{C_L}\tan\beta\right] d\lambda_r \tag{45}$$

8 Blade Design Procedure

1. Determine the rotor diameter required from site conditions and $P = C_P\eta\frac{1}{2}\rho\pi R^2 V^3$ where:

   - P is the power output
   - C_P is the expect coefficient of performance (0.4 for a modern three bladed wind turbine)
   - η is the expected electrical and mechanical efficiencies (0.9 would be a suitable value)
   - R is the tip radius

λ	B
1	8-24
2	6-12
3	3-6
4	3-4
more than 4	1-3

Table 1: Number of Blades

- V is the expect wind velocity

2. Choose a tip speed ratio for the machine. For water pumping pick $1 < \lambda < 3$ (which gives a high torque) and for electrical power generation pick $4 < \lambda < 10$

3. Choose a number of blades B, using Table 1, which is based on practical experience.

4. Select an aerofoil. For $\lambda < 3$ curved plates can be used.

5. Obtain and examine lift and drag coefficient curves for the aerofoil in question. Note that different aerofoils may be used at different spans of the blade, a thick aerofoil may be selected for the hub to give greater strength.

6. Choose the design aerodynamic conditions for each aerofoil. Typically select 80% of the maximum lift value

7. Divide the blade into N elements. Typically 10 to 20 elements would be used.

8. As a first guess for the blade twist and chord, use the blade shape derived with wake rotation, zero drag and zero tip losses. Note that these equations provide an *initial guess only*. The equations are given as follows:

$$\beta = 90^\circ - \frac{2}{3}\tan^{-1}\left(\frac{1}{\lambda_r}\right) \tag{46}$$

$$a = \left(1 + \frac{4\cos^2\beta}{\sigma' C_L \sin\beta}\right)^{-1} \tag{47}$$

$$a' = \frac{1-3a}{4a-1} \tag{48}$$

9. Calculate rotor performance and then modify the design as necessary. This is an iterative process.

9 Example using BEM Theory

The application of BEM can be confusing as it can be used to either to design i.e. select γ and c or to analyse the performance of a blade. In order to make the theory more tractable an example is given here for the analysis of a small turbine section.

> **Example**
> Calculate the power output from the turbine described in Table 2. The turbine has a tip radius of 5 m, and will operate in a wind speed of 7 m/s, a tip speed ratio of 8 and three blades. Assume that the tip loss and the drag coefficient are zero. The turbine uses a NACA 0012 aerofoil.

The solution for a given blade cannot be found directly from the equations but an iterative solution is required. There is more than one way to carry this out, in this document a solution based on guesses for a and a' and subsequent iteration.

The drag C_D is zero and the tip loss correction Q is one. The equations to be solved therefore reduce to:

$$\tan\beta = \frac{\lambda_r(1+a')}{(1-a)} \tag{49}$$

$$\frac{a}{1-a} = \frac{\sigma'\left[C_L \sin\beta\right]}{4\cos^2\beta} \tag{50}$$

$$\frac{a'}{1-a} = \frac{\sigma' C_L}{4\lambda_r \cos\beta} \tag{51}$$

The algorithm for an iterative solution is as follows:

1. Guess a and a'
2. Calculate λ_r and β
3. Look up C_L and C_D for the appropriate incidence angle
4. Calculate a and a' again.

9.1 5m Radius

To demonstrate the operation of the procedure, conditions at the tip ($r = 5m$) will be calculated first.

For conditions at the flow tip:

$$\sigma' = \frac{Bc}{2\pi r} = \frac{3\times 0.19}{2\pi 5} = 0.01814 \tag{52}$$

r / [m]	γ /[°]	c /[m]
0.2	61.0	0.70
1	74.3	0.71
2	84.9	0.44
3	89.1	0.30
4	91.3	0.23
5	92.6	0.19

Table 2: Example Turbine

Equation 46 will be used to estimate the relative flow angle β, "first guesses" for a and a' will also be calculated.

$$\beta = 90^\circ - \frac{2}{3}\tan^{-1}\left(\frac{1}{8}\right) \tag{53}$$

$$\Rightarrow \beta = 85.2^\circ \tag{54}$$

Now γ is 92.6° so the incidence i is about 7.4°, examining the NACA 0012 Lift and Drag Coefficient plot (See Figure 7, this gives a lift coefficient of around 0.85.

We can then use our two equations for the first guess for a and a' to calculate "first guesses" for these two parameters.

$$a = \left(1 + \frac{4\cos^2\beta}{\sigma' C_L \sin\beta}\right)^{-1} \tag{55}$$

$$\Rightarrow a = \left(1 + \frac{4 \times \cos^2 85.2^\circ}{0.01814 \times 0.85 \times \sin 85.2^\circ}\right)^{-1} = 0.3543 \tag{56}$$

$$a' = \frac{1-3a}{4a-1} = \frac{1 - 3 \times 0.3543}{4 \times 0.3543 - 1} = -0.1592 \tag{57}$$

Now a' being less than zero is illogical, but will suffice as the starting point for our iterative procedure. Having determined a suitable starting point we now begin the iteration proper.

9.1.1 Iteration 1

- Use a and a' calculate β

$$\beta = \tan^{-1}\left(\frac{\lambda_r(1+a')}{1-a}\right) \tag{58}$$

$$\Rightarrow \beta = \tan^{-1}\frac{8 \times (1 - 0.1592)}{1 - 0.3543} = 84.5^\circ \tag{59}$$

- Calculate incidence i and then C_L

$$i = \gamma - \beta = 92.6^\circ - 84.5^\circ = 8.05^\circ \tag{60}$$

So from Figure 7 $C_L = 1$

- Calculate new values of a and a' using Equations 50 and 51 respectively.

$$a = \left(1 + \frac{4\cos^2\beta}{\sigma' C_L \sin\beta}\right)^{-1} \tag{61}$$

$$\Rightarrow a = \left(1 + \frac{4 \times \cos^2 84.5^\circ}{0.01814 \times 1 \times \sin 84.5^\circ}\right)^{-1} = 0.2488 \tag{62}$$

$$a' = \left[\frac{\sigma' C_L}{4\lambda_r \cos\beta}\right](1-a) \tag{63}$$

$$\Rightarrow a' = \left[\frac{0.01814 \times 1}{4 \times 8 \times \cos 84.5^\circ}\right](1 - 0.2488) = 0.0030 \tag{64}$$

- Select new values of a and a', here we simply use the values just calculated, i.e. $a = 0.2488$ and $a' = 0.0030$. Note that although we started off with negative value of a' the solution is rapidly converging to a more sensible value.

9.1.2 Iteration 2

- Use a and a' calculate β

$$\beta = \tan^{-1}\left(\frac{\lambda_r(1+a')}{1-a}\right) \tag{65}$$

$$\Rightarrow \beta = \tan^{-1}\frac{8 \times (1 + 0.0030)}{1 + 0.2488} = 84.6^\circ \tag{66}$$

- Calculate incidence i and then C_L

$$i = \gamma - \beta = 92.6^\circ - 84.6^\circ = 8.0^\circ \tag{67}$$

So from Figure 7 $C_L = 1$

- Calculate new values of a and a' using Equations 50 and 51 respectively.

$$a = \left(1 + \frac{4\cos^2\beta}{\sigma' C_L \sin\beta}\right)^{-1} \tag{68}$$

$$\Rightarrow a = \left(1 + \frac{4 \times \cos^2 84.6^\circ}{0.01814 \times 1 \times \sin 84.6^\circ}\right)^{-1} = 0.3387 \tag{69}$$

$$a' = \left[\frac{\sigma' C_L}{4\lambda_r \cos\beta}\right](1-a) \tag{70}$$

$$\Rightarrow a' = \left[\frac{0.01814 \times 1}{4 \times 8 \times \cos 84.6^\circ}\right](1-0.3387) = 0.0040 \tag{71}$$

- Select new values of a and a', here we simply use the values just calculated, i.e. $a = 0.3387$ and $a' = 0.0040$

9.1.3 Iteration 3

- Use a and a' calculate β

$$\beta = \tan^{-1}\left(\frac{\lambda_r(1+a')}{1-a}\right) \tag{72}$$

$$\Rightarrow \beta = \tan^{-1}\frac{8 \times (1+0.0040)}{1+0.3387} = 85.3^\circ \tag{73}$$

- Calculate incidence i and then C_L

$$i = \gamma - \beta = 92.6^\circ - 85.3^\circ = 7.3^\circ \tag{74}$$

So from Figure 7 $C_L \approx 1$

- Calculate new values of a and a' using Equations 50 and 51 respectively.

$$a = \left(1 + \frac{4\cos^2\beta}{\sigma' C_L \sin\beta}\right)^{-1} \tag{75}$$

$$\Rightarrow a = \left(1 + \frac{4 \times \cos^2 85.3^\circ}{0.01814 \times 1 \times \sin 85.3^\circ}\right)^{-1} = 0.3983 \tag{76}$$

$$a' = \left[\frac{\sigma' C_L}{4\lambda_r \cos\beta}\right](1-a) \tag{77}$$

$$\Rightarrow a' = \left[\frac{0.01814 \times 1}{4 \times 8 \times \cos 85.3^\circ}\right](1-0.3983) = 0.0041 \tag{78}$$

- Select new values of a and a', again we simply use the values just calculated, i.e. $a = 0.3983$ and $a' = 0.0041$

The process should by now be clear. This process is tedious to do by hand and for more than a simple calculation it is preferable to use a spreadsheet or program to carry out the sum. An example spreadsheet should be available in the same location that you found this document for example at the author's website.[2] The spreadsheet

[2] http://www.dur.ac.uk/g.l.ingram

Step	Δa	$\Delta a'$
1	-0.1070	0.1622
2	0.0900	0.0010
3	0.0595	0.0001

Table 3: Convergence of the 5m Solution

r / [m]	c / [m]	γ /[°]	a / [-]	a' / [-]	i / [°]	β / [°]
0.2	0.70	61.0	0.2260	0.8994	22.9	38.1
1.0	0.71	74.3	0.2443	0.0676	8.2	66.1
2.0	0.44	84.9	0.2497	0.0180	7.9	77.0
3.0	0.30	89.1	0.2533	0.0081	7.9	81.2
4.0	0.23	91.3	0.2557	0.0046	7.9	83.4
5.0	0.19	92.6	0.2581	0.0030	7.9	84.7

Table 4: Solutions at all points.

was created using the Openoffice.org "Calc" spreadsheet program version 1.1.5. You can obtain this software from the OpenOffice.org website.[3]

It is worth examining the differences between our successive calculations of a and a' to see if we are actually making progress towards a solution. This is shown in Table 3, which shows that the difference between successive values of a and a' rapidly diminishes as the solution progresses.

9.2 Additional Radial Locations

The example spreadsheet calculates a solution at all blade locations using an identical procedure. The main results are summarised in Table 4 which shows, a, a' and i for each blade span.

Equation 45 shows how the total power can be calculated. Here $Q = 1$ and $C_D = 0$ so Equation 45 becomes:

$$C_P = \frac{8}{\lambda^2} \int_{\lambda_h}^{\lambda} \lambda_r^3 a'(1-a) d\lambda_r \tag{79}$$

Recall that trapezium rule:

$$\int_{x_0}^{x_n} f(x)dx \approx \frac{x_n - x_0}{2n} [(y_o + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})] \tag{80}$$

In this example we set $n = 1$ and repeat for each portion of the blade. So x will be replaced by λ_r and $f(x) = \lambda_r^3 a'(1-a)$ for each element. The calculation of power coefficient is shown in Table 5. The power coeficient is around 0.35.

[3] http://download.openoffi ce.org/1.1.5/index.html

r	λ_r	a	a'	$f(x)$	$\frac{x_0-x_n}{2}$	$\int f(x)$
0.20	0.32	0.2260	0.8994	0.0228	0.4	0.0928
1.00	1.60	0.2443	0.0676	0.2091	0.5	0.3255
2.00	3.20	0.2497	0.0180	0.4419	0.5	0.5572
3.00	4.80	0.2533	0.0081	0.6724	0.5	0.7874
4.00	6.40	0.2557	0.0046	0.9023	0.5	1.0165
5.00	8.00	0.2581	0.0030	1.1306		
					Sum of $\int f(x)$	2.78
					C_P	0.35

Table 5: Calculation of Power Coefficient.

10 Summary

This short report derives equations for the analysis of wind turbines using the blade element method. These equations are then used in an example performance calculation. Although used for analysis these equations could be equally applied to design activities.

References

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